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Mark Scheme (Results)
January 2009

GCE

GCE Mathematics (6665/01)

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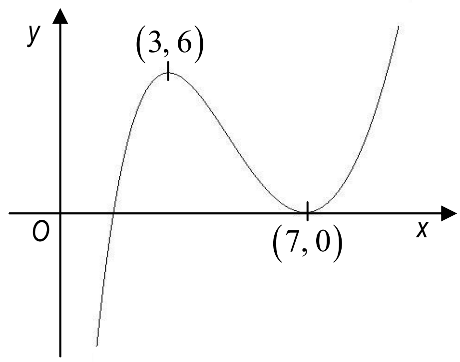
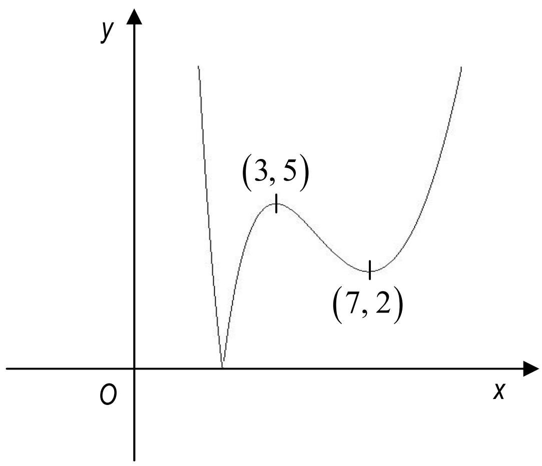
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\frac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\frac{10}{\sqrt{9}}=12+\frac{10}{3}$	M1
	$=\frac{46}{3}$ Accept awrt 15.3	A1 (6)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sin 2x}{x^2}\right)=\frac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\frac{\text{A1+A1}}{\text{A1}}$ (4) **[10]**
	Alternative to (b)	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x \quad \left(=\frac{2\cos 2x}{x^2}-\frac{2\sin 2x}{x^3}\right)$	A1 (4)

Question Number	Scheme	Marks
2 (a)	$\dfrac{2x+2}{x^2-2x-3}-\dfrac{x+1}{x-3}=\dfrac{2x+2}{(x-3)(x+1)}-\dfrac{x+1}{x-3}$	
	$=\dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1
	$=\dfrac{(x+1)(1-x)}{(x-3)(x+1)}$	M1
	$=\dfrac{1-x}{x-3}$ Accept $-\dfrac{x-1}{x-3},\ \dfrac{x-1}{3-x}$	A1 (4)
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1-x}{x-3}\right)=\dfrac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1
	$=\dfrac{-x+3-1+x}{(x-3)^2}=\dfrac{2}{(x-3)^2}$ $\ast$ cso	A1 (3)
		[7]
	Alternative to (a)	
	$\dfrac{2x+2}{x^2-2x-3}=\dfrac{2(x+1)}{(x-3)(x+1)}=\dfrac{2}{x-3}$	M1 A1
	$\dfrac{2}{x-3}-\dfrac{x+1}{x-3}=\dfrac{2-(x+1)}{x-3}$	M1
	$=\dfrac{1-x}{x-3}$	A1 (4)
	Alternatives to (b)	
	① $\mathrm{f}(x)=\dfrac{1-x}{x-3}=-1-\dfrac{2}{x-3}=-1-2(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1
	$=\dfrac{2}{(x-3)^2}$ $\ast$ cso	A1 (3)
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(x-3)^{-1}+(1-x)(-1)(x-3)^{-2}$	M1
	$=-\dfrac{1}{x-3}-\dfrac{1-x}{(x-3)^2}=\dfrac{-(x-3)-(1-x)}{(x-3)^2}$	A1
	$=\dfrac{2}{(x-3)^2}$ $\ast$	A1 (3)

Question Number	Scheme	Marks	
3 (a)	 Shape $(3, 6)$ $(7, 0)$	B1 B1 B1	 (3)
(b)	 Shape $(3, 5)$ $(7, 2)$	B1 B1 B1	 (3) **[6]**

Question Number	Scheme		Marks	
4	$x = \cos(2y + \pi)$			
	$\frac{dx}{dy} = -2\sin(2y + \pi)$		M1 A1	
	$\frac{dy}{dx} = -\frac{1}{2\sin(2y + \pi)}$	Follow through their $\frac{dx}{dy}$ before or after substitution	A1ft	
	At $y = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{1}{2\sin\frac{3\pi}{2}} = \frac{1}{2}$		B1	
	$y - \frac{\pi}{4} = \frac{1}{2}x$		M1	
	$y = \frac{1}{2}x + \frac{\pi}{4}$		A1	(6)
				[6]

Question Number	Scheme	Marks
5 (a)	$\mathrm{g}(x) \geq 1$	B1 (1)
(b)	$\mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\,\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\,\mathrm{e}^{x^2}$ ∗	A1 (2)
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\,\mathrm{e}^{x^2}\right)$	
(c)	$\mathrm{fg}(x) \geq 3$	B1 (1)
(d)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\,\mathrm{e}^{x^2}\right) = 2x + 6x\,\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\,\mathrm{e}^{x^2} = x^2\,\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 (6)
		[10]

Question Number	Scheme	Marks
6 (a)(i)	$\sin 3\theta = \sin(2\theta + \theta)$	
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	
	$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$	M1 A1
	$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ * cso	A1 (4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$	
	$-2\sin 3\theta + 1 = 0$	M1 A1
	$\sin 3\theta = \frac{1}{2}$	M1
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$	A1 A1 (5)
(b)	$\sin 15° = \sin(60° - 45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$	M1
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso	A1 (4)
		[13]
	Alternatives to (b)	
	① $\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$	M1
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$	M1 A1
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso	A1 (4)
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$	
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$	
	$\sin^2 15° = \frac{2-\sqrt{3}}{4}$	M1 A1
	$\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2-\sqrt{3}}{4}$	M1
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ * cso	A1 (4)

Question Number	Scheme	Marks
7 (a)	$f'(x) = 3e^x + 3xe^x$	M1 A1
	$3e^x + 3xe^x = 3e^x(1+x) = 0$	
	$x = -1$	M1 A1
	$f(-1) = -3e^{-1} - 1$	B1 (5)
(b)	$x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 (3)
(c)	Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$f(0.257\,55) = -0.000\,379\,...$	
	$f(0.257\,65) = 0.000\,109\,\ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ∗ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	(3) **[11]**
	Note: $x = 0.257\,627\,65\,\ldots$ is accurate	

Question Number		Scheme		Marks	
8	(a)	$R^2 = 3^2 + 4^2$		M1	
		$R = 5$		A1	
		$\tan\alpha = \frac{4}{3}$		M1	
		$\alpha = 53 \ldots ^\circ$	awrt 53°	A1	(4)
	(b)	Maximum value is 5	ft their R	B1 ft	
		At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1	
		$\theta = \alpha = 53 \ldots ^\circ$	ft their α	A1 ft	(3)
	(c)	$\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)^\circ$			
		Minimum occurs when $\cos(15t - \alpha)^\circ = -1$		M1	
		The minimum temperature is $(10 - 5)^\circ = 5^\circ$		A1 ft	(2)
	(d)	$15t - \alpha = 180$		M1	
		$t = 15.5$	awrt 15.5	M1 A1	(3)
					[12]