Mark Scheme (Results)

January 2011

GCE

GCE Core Mathematics C3 (6665) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎿ The second mark is dependent on gaining the first mark

January 2011
Core Mathematics C3 6665
Mark Scheme

Question Number	Scheme		Marks
1. (a)	$7\cos x - 24\sin x = R\cos(x+\alpha)$		
	$7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $7 = R\cos\alpha$ Equate $\sin x$: $24 = R\sin\alpha$		
	$R = \sqrt{7^2+24^2}\,; = 25$	$R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$	$\tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ awrt 1.287	M1 A1
	Hence, $7\cos x - 24\sin x = 25\cos(x+1.287)$		(3)
(b)	Minimum value = $\underline{-25}$	-25 or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x = 10$		
	$25\cos(x+1.287) = 10$		
	$\cos(x+1.287) = \frac{10}{25}$	$\cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$	M1
	PV $= 1.159279481...^c$ or $66.42182152...^\circ$	For applying $\cos^{-1}\left(\frac{10}{\text{their } R}\right)$	M1
	So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$	either 2π + or − their PV^c or 360° + or − their PV$^\circ$	M1
	gives, $x = \{3.836906..., 6.155465...\}$	awrt 3.84 OR 6.16 awrt 3.84 AND 6.16	A1 A1
			(5) **[9]**

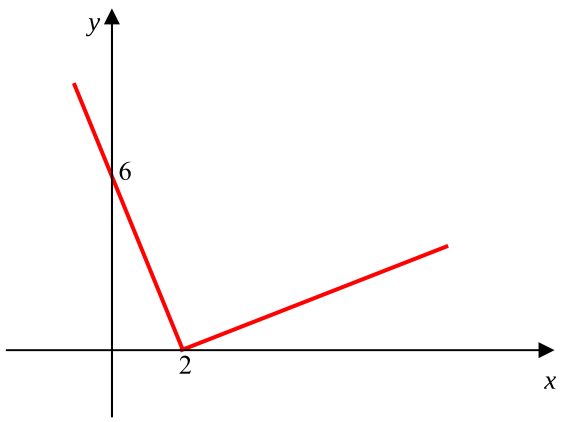
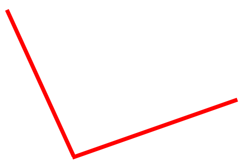
Question Number	Scheme		Marks
2.			
(a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{8x^2 - 6x - 2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1
			(4)
(b)	$\mathrm{f}(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$		
	$\mathrm{f}(x) = \dfrac{(4x+1)}{(2x-1)} - 2$		
	$= \dfrac{(4x+1) - 2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 *
			(2)
(c)	$\mathrm{f}(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$\mathrm{f}'(x) = 3(-1)(2x-1)^{-2}(2)$	$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$\mathrm{f}'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$	Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1
			(3)
			[9]

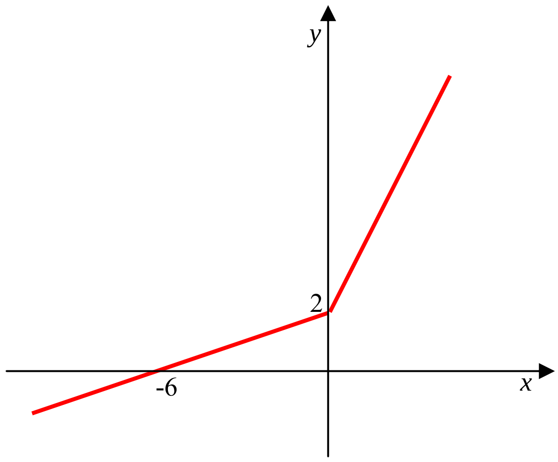
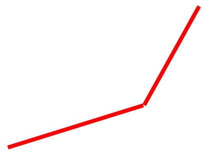
Question Number	Scheme		Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2\left(1-2\sin^2\theta\right) = 1 - 2\sin\theta$	Substitutes either $1-2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$		
	$\theta = \{54, 126, 198, 342\}$	Any one correct answer 180-their pv All four solutions correct.	A1 dM1(*) A1 **[6]**

Question Number	Scheme		Marks
4. **(a)**	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t=0, \theta=90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t=0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t=5, \theta=55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t=5$ and $\theta=55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm\alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2 e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2 e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Question Number	Scheme		Marks
5. (a)	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
(b)	Apply product rule: $\left\{\begin{matrix} u = (8 - x) & v = \ln x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = -1 & \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x} \end{matrix}\right\}$	$vu' + uv'$	M1
	$\mathrm{f}'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$\mathrm{f}'(3.5) = 0.032951317...$ $\mathrm{f}'(3.6) = -0.058711623...$ Sign change (and as $\mathrm{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $\mathrm{f}'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $\mathrm{f}'(x) = 0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55) + 1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both $x_1 =$ awrt 3.529 and $x_2 =$ awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp.	x_1 , x_2 , x_3 all stated correctly to 3 dp	A1 (3) **[13]**

Question Number	Scheme		Marks
6. **(a)**	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \mathrm{f}^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe
			(3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 (1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	g g(2)= g (0) $= -6$, from sketch.	-6	A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)
(e)(i)		Correct shape 	B1
		$(2,\{0\}),(\{0\},6)$	B1

Question Number	Scheme	Marks
(e)(ii)	 Correct shape  Graph goes through $\left(\{0\},\, 2\right)$ and $\left(-6,\, \{0\}\right)$ which are marked.	B1 B1 (4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$ — Either correct answer or a follow through from part (b) answer	B1 $\sqrt{\ }$ (1) **[13]**

Question Number	Scheme		Marks
7			
(a)	$y = \dfrac{3+\sin 2x}{2+\cos 2x}$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = 3+\sin 2x & v = 2+\cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3+\sin\pi}{2+\cos\pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $\left(\frac{\pi}{2}, 3\right)$, $\mathrm{m}(\mathbf{T}) = \dfrac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\mathrm{m}(\mathbf{T}) = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2\left(\frac{\pi}{2}\right) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) **[8]**

Question Number	Scheme		Marks
8.			
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both $\underline{\underline{\text{underlined steps.}}}$	A1 **AG**
			(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4},\ n \in \mathbb{Z}.$		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K \sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]

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