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WELSH JOINT EDUCATION COMMITTEE

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MATHEMATICS C1

Pure Mathematics

Specimen Mark Scheme

2005/2006

1. (a) Gradients: $AB \quad \frac{10+2}{-12-4} = -\frac{3}{4}$, B1
- $BC \quad \frac{6-10}{10+12} = -\frac{4}{22}$, B1
- $CA \quad \frac{6+2}{10-4} = \frac{4}{3}$. B1
- (b) gradient $AB \times$ gradient $AC = -1$ (o.e.) M1 ($m_1m_2 = -1$)
(or Pythagoras¹ Theorem)
- $\therefore AB \perp AC$
giving $\hat{BAC} = 90^\circ$ A1
- (c) Equation of AB is
- $y + 2 = -\frac{3}{4}(x - 4)$ M1 ($y - y_1 = m(x - x_1)$)
- so that $4y + 8 = -3x + 12$
- or $3x + 4y - 4 = 0$ (1) A1 (convincing)
- (d) Coordinates of D : $(-1, 8)$. B1, B1
- $\therefore AD = \sqrt{(4+1)^2 + (-2-8)^2}$ M1 (correct formula)
- $= \sqrt{125}$ A1
2. $\frac{(2\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} = \frac{10 + 2\sqrt{10} + \sqrt{10} + 2}{5 - 2}$ M1 (rationalise)
- $= \frac{12 + 3\sqrt{10}}{3}$ A1 (numerator unsimplified)
A1 (denominator unsimplified)
- $= 4 + \sqrt{10}$ A1

3. Equate ys
 $\therefore x^2 - 3x + 2 = 3x - 7$ M1
 giving $x^2 - 6x + 9 = 0$ M1 (standard form of quad
 and attempt to solve)
 $\therefore (x-3)^2 = 0$ A1
 so that $x = 3, y = 2$ A1

The line $y = 3x - 7$ is a tangent to the curve $y = x^2 - 3x + 2$ at the point (3, 2) B2

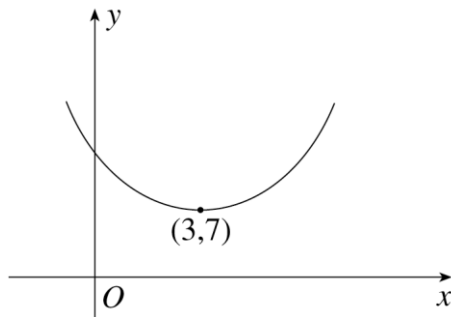
(Allow B1 for (3, 2) is point of intersection of line $y = 3x - 7$ and the curve $y = x^2 - 3x + 2$)

4. Condition for distinct real roots is M1 (Use of $b^2 - 4ac$)
 $16 - 4 \times 2k(k-1) > 0$ A1
 $\therefore 2 - k^2 + k > 0$
 $k^2 - k - 2 < 0$ A1 (convincing)
 giving $(k-2)(k+1) < 0$ B1 (fixed points)
 then $-1 < k < 2$ or $(-1, 2)$. M1 (any method) A1

5. (a) $2(x^2 - 6x) + 25$
 $= 2[(x-3)^2 - 9] + 25$
 $= 2(x-3)^2 + 7$ B1(a), B1(b), B1(c)
 (a, b, c need not be displayed).

- (b) Least value of $2x^2 - 12x + 25 = 7$ B1
 Corresponding value of $x = 3$ B1

- (c) M1 (shape)
 A1 (location of minimum)



6. (a) $-8k + 32 - 6 - 2 = 0$ M1 (use of factor theorem)
 so that $k = 3$ A1 (correct)
 A1 (convincing)
- (b) $3x^3 + 8x^2 + 3x - 2 = (x + 2)(3x^2 + 2x - 1)$ M1 A1
 $= (x + 2)(3x - 1)(x + 1).$ A1
- Roots are $-2, \frac{1}{3}, -1.$ A1
- (c) Remainder $= 3(3)^3 + 8(3)^2 + 9 - 2$ M1 (use of remainder theorem)
 $= 160.$ A1
7. (a) $(2x + 3)^4 = (2x)^4 + 4(2x)^3(3) + 6(2x)^2(3)^2$ M1 (5 terms, 2 correct)
 $+ 4(2x)(3)^3 + 3^4$ A2 (unsimplified, -1 for each error)
 $= 16x^4 + 96x^3 + 216x^2 + 216x + 81$ A1 (simplified)
- (b) ${}^nC_2(3)^2 = 54$ B1
 so that $\frac{n(n-1)3^2}{1.2} = 54$ B1
- $\therefore n^2 - n - 12 = 0$ M1 (standard form of quadratic and attempt to solve)
- $\therefore n = 4$ A1

8. (a) $y + \delta y = (x + \delta x)^2 - 4(x + \delta x) + 2.$ (o.e.) B1
- $\delta y = (x + \delta x)^2 - 4(x + \delta x) + 2$ (o.e.) M1
- $-x^2 + 4x - 2$
- $= 2x\delta x + (\delta x)^2 - 4\delta x.$ (o.e.) A1
- $\therefore \frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} (2x + \delta x - 4)$ (o.e.) M1
- $= 2x - 4.$ A1
- (b) $3(-4)x^{-5} + 4 \cdot \frac{1}{2}x^{-\frac{1}{2}}$ M1 (attempted differentiation of x^{-4})
- $= -\frac{12}{x^5} + \frac{2}{\sqrt{x}}.$ A1
- M1 (attempted differentiation of $x^{\frac{1}{2}}$)
- A1
9. $\frac{dy}{dx} = 4x^3 + 1$ B1
- $= 5$ at (1,3). B1
- Equation is
- $y - 3 = 5(x - 1)$ M1 (for using gradient)
- A1

10. (a) $\frac{dy}{dx} = 3x^2 - 6x - 9.$ B1

$$\frac{dy}{dx} = 0 \Rightarrow 3(x^2 - 2x - 3) = 0. \quad \text{M1}$$

giving $x = 3, -1$ A1

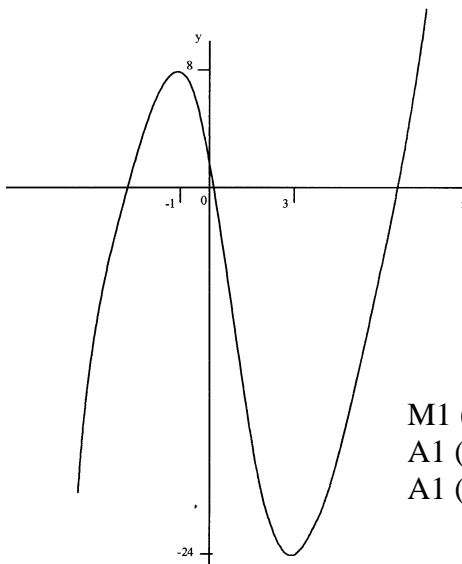
When $x = 3, y = -24,$
 $x = -1, y = 8$ A1, A1

$$\frac{d^2y}{dx^2} = 6x - 6. \quad \text{M1 (any method)}$$

$x = -1, \quad \frac{d^2y}{dx^2} = -6 - 6 < 0. \quad \text{Max. point at } (-1, 8) \quad \text{A1}$

$x = 3, \quad \frac{d^2y}{dx^2} = 6 \times 3 - 6 > 0. \quad \text{Min. point at } (3, -24). \quad \text{A1}$

(b)



M1 (general shape)
 A1 (Position of Max)
 A1 (Position of Min)

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MATHEMATICS C2

Pure Mathematics

Specimen Mark Scheme

2005/2006

1.
$$\text{Integral} \approx \frac{0.25}{2} [0.5 + 2 (0.3386243 + 0.2285714 + 0.1572482) + 0.1111111]$$

$$\approx 0.2575$$

M1 (correct formula)

B1 (3 values)

B1 (other 2 values)

A1 (F.T. one slip)

2. (a)
$$6(1 - \cos^2 \theta) + \cos \theta - 5 = 0$$

$$\therefore 6 \cos^2 \theta - \cos \theta - 1 = 0$$

giving $\cos \theta = \frac{1}{2}, -\frac{1}{3}$.

$$\theta = 60^\circ, 300^\circ, 109.5^\circ, 250.5^\circ$$

(b)
$$3x = 135^\circ, 315^\circ, 495^\circ$$

$$x = 45^\circ, 105^\circ, 165^\circ$$

M1 (correct elimination of $\sin^2 x$)

M1 (standard form of quadratic and an attempt to solve)

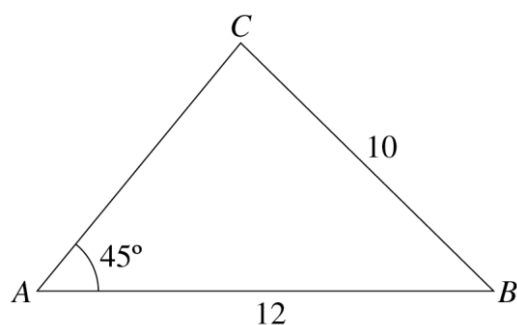
A1

B1 (60°, 109.5°) B1, B1

B1

B1, B1, B1

3.



(a) By the Sine Rule,

$$\frac{10}{\sin 45^\circ} = \frac{12}{\sin C}$$

M1 (correct Sine Rule)

$$\therefore \sin C = \frac{12 \sin 45^\circ}{10}$$

$$\therefore C = 58^\circ, 122^\circ \text{ (correct to the nearest degree) A1, A1}$$

(b) When $C = 58^\circ$, $B = 180^\circ - (45^\circ + 58^\circ) = 77^\circ$

Sine Rule, $\frac{10}{\sin 45^\circ} = \frac{AC}{\sin 77^\circ}$

M1 (any correct method)

A1 (correct B)

so that $AC = \frac{10 \sin 77^\circ}{\sin 45^\circ} \approx 13.8\text{cm}$

A1

When $C = 122^\circ$, $B = 180^\circ - (45^\circ + 122^\circ) = 13^\circ$

Sine Rule, $\frac{10}{\sin 45^\circ} = \frac{AC}{\sin 13^\circ}$

A1

so that $AC = \frac{10 \sin 13^\circ}{\sin 45^\circ} \approx 3.2\text{cm}$

A1

4. (a) $S_n = a + ar + \dots + ar^{n-2} + ar^{n-1}$

B1

$rS_n = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n$

M1

$\therefore S_n - rS_n = a - ar^n$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$

A1(convincing)

Sum to infinity = $\frac{a}{1-r}$

B1

(b) $\frac{a}{1-r} = 4a$

M1 ($\frac{a}{1-r} = ka, k = \frac{1}{4}$ or 4)

A1 (correct)

$\therefore 1-r = \frac{1}{4}$

M1 (eliminate a and attempt to solve)

giving $r = \frac{3}{4}$

A1

5. (a) $a + 3d = 11$
 $a + 5d = 17$
 $d = 3, a = 2$

B1

B1

M1(attempt to solve)

A1

(b) $S_8 = \frac{8}{2} [2 \times 2 + 7 \times 3] = 100$

B1

6. $\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2}{x} + C$ M1 (attempt to integrate $x^{\frac{1}{2}}$)
 A1
- $= \frac{2}{3} x^{\frac{3}{2}} + \frac{2}{x} + C$ M1 (attempt to integrate x^{-2})
 A1
7. (a) Coordinates of A $12 - 2x = 2x^2$ M1 (equate ys)
 $x^2 + x - 6 = 0$ M1 (standard form of quadratic and attempt to solve)
 $\therefore x = 2$ A1
 $\therefore A (2, 8)$ A1
- Coordinates of B are (6,0) B1
- (b) Integral = $\int_0^2 2x^2 dx + \int_2^6 (12 - 2x) dx$ M1 (sum up difference or definite integrals)
 A1 (all correct)
- $= \left[\frac{2x^3}{3} \right]_0^2 + \left[12x - x^2 \right]_2^6$ B1, B1, B1
- $= \frac{16}{3} - 0 + 72 - 36 - (24 - 4)$ M1 (use of limits)
- $= \frac{64}{3}$ A1 (C.A.O.)
8. $A\hat{O}B = \theta$
- $2 \times 6 \cdot 8 + 6 \cdot 8 \theta = 23 \cdot 12$ B1
- so that $\theta = 1 \cdot 4$ B1
- shaded area = $\frac{1}{2} r^2 (1 \cdot 4 - \sin 1 \cdot 4)$ B1 (sector area)
 B1 (triangle area)
 M1 (difference of areas)
- $\approx 9 \cdot 58 \text{cm}^2$ A1

9. (a) Let $x = a^y$, $y = \log_a x$ B1
 so that $x^n = (a^y)^n = a^{ny}$ B1
 and $\log_a x^n = ny = n \log_a x$ B1
- (b) $(y + 1) \log 2 = \log 3$ B1
 $\therefore y = \frac{\log 3 - \log 2}{\log 2}$ M1 (attempt to isolate y)
A1
 ≈ 0.585 B1
10. (a) $(5, 7); 5$ B1, B1
- (b) Gradient of radius = $\frac{7-3}{5-2} = \frac{4}{3}$ M1 (correct method of finding gradient of radius)
 \therefore Gradient of tangent = $-\frac{3}{4}$ M1
A1
 \therefore Equation of tangent is

$$y - 3 = -\frac{3}{4}(x - 2)$$
 A1
- (c) (i) $CQ^2 = (13 - 5)^2 + (13 - 7)^2 = 64 + 36 = 100$ M1 (attempt to find CQ)
 $CQ >$ radius and Q lies outside circle A1 (convincing)
- (ii) For circles touching externally,
 sum of radii = CQ M1
 \therefore Radius of required circle = $\sqrt{100} - 5 = 5$ A1
 Equation of circle is
 $(x - 13)^2 + (y - 13)^2 = 25$ (o.e.) A1

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MATHEMATICS C3

Pure Mathematics

Specimen Mark Scheme

2005/2006

1.	x	$x^3 + 10x - 4$	
	0	-4	
	1	7	M1 (attempt to find values)

change of sign of $x^3 + 10x - 4$ indicates that there is a root between 0 and 1.

A1 (correct values and conclusion)

$$x_0 = 0.3, x_1 = 0.3973, x_2 = 0.39372873$$

B1 (x_1)

$$x_3 = 0.39389633, x_4 = 0.39388853$$

$$x_4 \approx 0.39389.$$

B1 (x_4)

	x	$x^3 + 10x - 4$	
	0.393885	-0.00004	Change of sign indicates presence of root which is 0.39389, correct to five decimal places
	0.393895	0.00006	
			M1
			A1
			A1

2.	Integral $\approx \frac{0.25}{3} [1.41421356 + 4.12310563 + 2(2.46221445) + 4(1.85510275 + 3.22163099)]$	M1 (formula with $h = 0.25$)
		B1 (3 values)
		B1 (2 other values)
	≈ 2.56	A1 (C.A.O.)

3.	$9 > 2x - 5 > -9.$	B1, B1
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$$\therefore 14 > 2x > -4$$

$$\text{giving } 7 > x > -2 \quad \text{or } (-2, 7)$$

B1 ($7 > x$)

B1 ($x > -2$)

4. (a) $3y^2 \frac{dy}{dx} - 2x^2 y \frac{dy}{dx} - 2xy^2 = 2x + 3$ B1 $\left(3y^2 \frac{dy}{dx}\right)$
M1 $\left(-2x^2 y \frac{dy}{dx} \pm 2xy^2\right)$
A1
- $\therefore \frac{dy}{dx} = \frac{2x + 3 + 2xy^2}{3y^2 - 2x^2 y}$ A1 (all correct)
- (b) $\frac{dy}{dx} = \frac{2t}{3t^2} = \frac{2}{3t}$ M1 $\left(\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}\right)$
A1
- $\therefore \frac{d^2 y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ M1 (correct formula)
- $= -\frac{\frac{2}{3t^2}}{3t^2} = -\frac{2}{9t^4}$ A1, A1 (convincing)
5. $\operatorname{cosec}^2 \theta - 1 = 7 - 2 \operatorname{cosec} \theta$ M1 (correct elimination of $\cot \theta$)
- $\operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta - 8 = 0$ M1 (standard form of quadratic and attempt to solve)
- giving $\operatorname{cosec} \theta = 2, -4,$ A1
- so that $\sin \theta = \frac{1}{2}, -\frac{1}{4}$ B1
- $\therefore \theta = 30^\circ, 150^\circ, 194.5^\circ, 345.5^\circ$ B1 ($30^\circ, 150^\circ$)
B1 (194.5°) B1 (345.5°)

6. (a) $e^{2x} \cos x + 2e^{2x} \sin x$

M1 ($e^{2x}f(x) + g(x) \sin x$)
 A1 ($f(x) = \cos x$),
 A1 ($g(x) = ke^{2x}$)
 A1 ($k = 2$)

(b)
$$\frac{(x^2 + 3)(4x) - (2x^2 - 4)(2x)}{(x^2 + 3)^2} = \frac{20x}{(x^2 + 3)^2}$$

M1
$$\left(\frac{(x^2 + 3)f(x) - (2x^2 - 4)g(x)}{(x^2 + 3)^2} \right)$$

A1 ($f(x) = 4x, g(x) = 2x$)

A1 (simplification)

(c) $8x \sec^2(4x^2 + 3)$

B1 ($f(x) \sec^2(4x^2 + 3)$)
 B1 ($f(x) = 8x$)

7. (a) (i) $-\frac{1}{4} e^{-4x+1}$

B1 (ke^{-4x+1})
 B1 ($k = -\frac{1}{4}$)

(ii) $\frac{1}{2} \ln |2x+1| - \frac{1}{6(3x+7)^2}$

M1 ($k \ln |2x+1|$)

A1 ($k = \frac{1}{2}$)

M1
$$\left(\frac{k}{(3x+7)^2} \right)$$

A2
$$\left(k = -\frac{1}{6} \right)$$

(A1 for $-\frac{1}{2}$)

(b)
$$\left[\frac{-\cos 2x}{2} \right]_0^{\frac{\pi}{2}}$$

M1 ($k \cos 2x, k = \pm \frac{1}{2}, -2$)

A1 ($k = -\frac{1}{2}$)

$$= -\frac{\cos \pi}{2} + \frac{1}{2} = 1$$

A1

8. (a) $y = \tan^{-1}x$

so that $\tan y = x$

B1

then $\sec^2 y = \frac{dx}{dy}$

B1

$\therefore \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$

B1

(b) $\frac{2x}{x^2 + 1}$

M1 $\left(\frac{f(x)}{x^2 + 1}\right)$ A1 ($f(x) = 2x$)

(c) $\int \frac{3+x}{1+x^2} dx = 3 \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx$

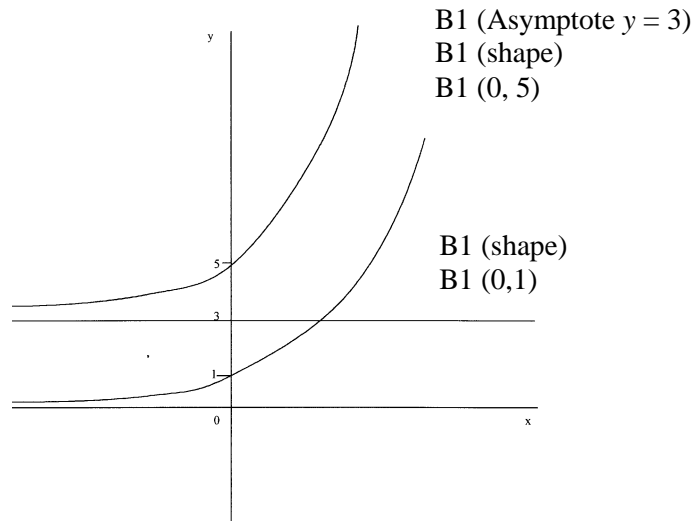
M1 (attempt to rewrite)

A1 (correct)

$= 3 \tan^{-1} x + \frac{1}{2} \ln(1+x^2) + C$

B1, B1

9.



- 10.** (a) (i) Let $y = \ln(2x - 3) + 4$ M1 (attempt to isolate x)
- $\therefore y - 4 = \ln(2x - 3)$
- giving $e^{y-4} = 2x - 3$ A1
- $$x = \frac{e^{y-4} + 3}{2}$$
- A1
- so that $f^{-1}(x) = \frac{e^{x-4} + 3}{2}$ A1
- (ii) domain $[4, \infty)$, range $[2, \infty)$ B1, B1
- (b) $gh(x) = (2x + 2)^2 + 3$, $hg(x) = 2(x^2 + 3) + 2$. B1, B1
- $\therefore (2x + 2)^2 + 3 = 2[2(x^2 + 3) + 2] + 15$ M1 (unsimplified)
- $\therefore 4x^2 + 8x + 4 + 3 = 4x^2 + 16 + 15$ M1 (attempt to solve)
- and $x = 3$ A1

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MATHEMATICS C4

Pure Mathematics

Specimen Mark Scheme

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$$\begin{aligned}
 1. \quad & 1 + \left(-\frac{1}{2}\right)(2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(2x)^2}{1.2} \\
 & + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(2x)^3}{1.2.3} + \dots \\
 & = 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \dots
 \end{aligned}$$

B1 (1 - x)
B1 $\left(\frac{3}{2}x^2\right)$ B1 $\left(-\frac{5}{2}x^3\right)$

$$\frac{(1-x)^2}{(1+2x)^{\frac{1}{2}}} = (1-2x+x^2)\left(1-x+\frac{3}{2}x^2-\frac{5}{2}x^3+\dots\right)$$

$$= 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \dots$$

$$- 2x + 2x^2 - 3x^3 + \dots$$

$$+ x^2 - x^3 + \dots$$

$$= 1 - 3x + \frac{9}{2}x^2 - \frac{13}{2}x^3 + \dots$$

B1 (1 - 3x)
B1 $\left(\frac{9}{2}x^2\right)$
B1 $\left(-\frac{13}{2}x^3\right)$

Valid for $|x| < \frac{1}{2}$

B1

2. (a) Substitution of appropriate value of θ
Convincing conclusion

B1

A1

(b) $3(1 - 2 \sin^2 \theta) = 1 - \sin \theta$

M1 (correct elimination of $\cos^2 \theta$)

$$\therefore 6 \sin^2 \theta - \sin \theta - 2 = 0$$

M1 (Standard form of quadratic and attempt to solve)

$$\therefore (3 \sin \theta - 2)(2 \sin \theta + 1) = 0$$

so that $\sin \theta = \frac{2}{3}, -\frac{1}{2}$

A1

$$\therefore \theta = 41.8^\circ, 138.2^\circ, 210^\circ, 330^\circ$$

B1 (41.8°, 138.2°)

B1 (210°) B1 (330°)

3. $R \cos \alpha = 5, R \sin \alpha = 4$ B1 (both)
- $R = \sqrt{41}, \alpha = 38.66^\circ$ M1 (any method of finding R or α)
A1 (R) A1 (38.7°)
- $\therefore \sin(\theta + 38.7^\circ) = \frac{3}{\sqrt{41}}$ M1
- $\therefore \theta + 38.66^\circ = 152.06^\circ, 387.94^\circ$ A1 (any value)
- $\therefore \theta = 113.4^\circ, 349.3^\circ$ A1 (both)
4. (a) Let $\frac{3x^2 + 2x + 1}{x^2(x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ M1 (correct form)
- $\therefore 3x^2 + 2x + 1 \equiv Ax(x-1) + B(x-1) + Cx^2$ m1 (clearing fraction and attempt to find constants)
- so that $A = -3, B = -1, C = 6$ A2 (3 constants)
A1 (2 constants)
- (b) $\int \frac{3x^2 + 2x + 1}{x^2(x-1)} dx = -3 \ln|x| + \frac{1}{x}$ B1, B1
 $+ 6 \ln|x-1|$ B1
- (| | may be omitted)
5. $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$ M1 $\left(\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} \right)$
A1
- For normal at P , gradient = $-p$ B1
- $\therefore y - 2ap = -p(x - ap^2)$ M1
- giving $px + y - 2ap - ap^3 = 0$ A1 (convincing)
- $Q: y = 0, x = 2a + ap^2$ B1
- $R: y = 0, x = ap^2$ B1
- $\therefore QR = 2a$ B1

6. (a) $\frac{dx}{dt} = -kx$ B1

Now at $t = 0, x = 2, \frac{dx}{dt} = -0.064$ M1 (use of data)

Then $k = \frac{-0.064}{-2} = 0.032$

$\therefore \frac{dx}{dt} = -0.032x$ A1

(b) $\int \frac{dx}{x} = \int -0.032 dt$ M1 (separation of variables
and integration of r.h.s.)

$\therefore \ln x = -0.032t + A$ A1

$t = 0, x = 2$

$\therefore A = \ln 2$ M1

$\therefore \ln x = -0.032t + \ln 2$

$\therefore 0.032t = \ln 2 - \ln x$

$= \ln \frac{2}{x}$ A1

$\therefore t = \frac{1}{0.032} \ln \frac{2}{x}$

$= \frac{125}{4} \ln \frac{2}{x}$ A1

(c) $x = 1, t = \frac{125}{4} \ln 2$ M1

≈ 21.66 years A1

7. Volume $= \pi \int_1^e x^3 \ln x dx$ B1
- $$= \pi \left\{ \left[\frac{x^4}{4} \ln x \right]_1^e - \int_1^e \frac{x^4}{4} \frac{1}{x} dx \right\}$$
- M1 (parts, correct choice of u, v)
- A1
- $$= \pi \left\{ \left[\frac{x^4}{4} \ln x \right]_1^e - \int_1^e \frac{x^3}{4} dx \right\}$$
- m1 (division)
- $$= \pi \left\{ \left[\frac{x^4}{4} \ln x \right]_1^e - \left[\frac{x^4}{16} \right]_1^e \right\}$$
- A1
- $$= \frac{\pi}{16} (3e^4 + 1)$$
- A1
8. (a) $\int_0^{\frac{\pi}{4}} \cos^2 \theta d\theta = \int_0^{\frac{\pi}{4}} \frac{1 + \cos 2\theta}{2} d\theta$ M1 ($a + b \cos 2\theta$)
- A1 ($a = \frac{1}{2}, b = \frac{1}{2}$)
- $$= \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{4}}$$
- B1
- $$= \frac{\pi}{8} + \frac{1}{4} - \frac{0}{2} - \frac{0}{4}$$
- $$= \frac{\pi}{8} + \frac{1}{4}$$
- A1 (convincing)
- (b) $x = 3 \tan \theta, dx = 3 \sec^2 \theta d\theta$ M1 (substitution for dx)
- $x = 0, \theta = 0, x = 3, \theta = \frac{\pi}{4}$ B1 (limits)
- $$27 \int_0^{\frac{\pi}{4}} \frac{3 \sec^2 \theta}{(9 \tan^2 \theta + 9)^2} d\theta = 27 \int_0^{\frac{\pi}{4}} \frac{3 \sec^2 \theta}{81 \sec^4 \theta} d\theta$$
- A1 (unsimplified)
- A1 ($1 + \tan^2 \theta = \sec^2 \theta$)
- $$= \frac{3 \times 27}{81} \int_0^{\frac{\pi}{4}} \cos^2 \theta d\theta$$
- A1 (simplified)
- $$= \left(\frac{\pi}{8} + \frac{1}{4} \right)$$
- A1

9. (a) Where the lines intersect,

$$2\mathbf{i} + \mathbf{j} + \lambda (\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

$$= 2\mathbf{i} + 2\mathbf{j} + t\mathbf{k} + \mu (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

M1 (attempt to equate $\mathbf{i}, \mathbf{j}, \mathbf{k}$ terms)

$$\therefore 2 + \lambda = 2 + \mu,$$

$$1 + \lambda = 2 + 2\mu,$$

$$2\lambda = t + \mu.$$

A1 (correct)

$$\text{Then } \lambda = \mu = -1$$

$$t = -1$$

M1 (attempt to solve)

A1 (λ, μ)

A1 (convincing)

Position vector of point of intersection is $\mathbf{i} - 2\mathbf{k}$

B1

$$(b) \quad \cos \theta = \frac{(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + \mathbf{k})}{|\mathbf{i} + \mathbf{j} + 2\mathbf{k}| |\mathbf{i} + 2\mathbf{j} + \mathbf{k}|}$$

B1 (identification of

appropriate vectors)

B1 (scalar product)

$$\text{Now } (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$= 1 + 2 + 2 = 5$$

M1 (correct method)

A1

$$|\mathbf{i} + \mathbf{j} + 2\mathbf{k}| = \sqrt{6}$$

B1 (for one)

$$|\mathbf{i} + 2\mathbf{j} + \mathbf{k}| = \sqrt{6}$$

$$\cos \theta = \frac{5}{6}$$

$$\theta \approx 34^\circ.$$

B1

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MATHEMATICS FP1

Further Pure Mathematics

Specimen Mark Scheme

2005/2006

1. If $2 - i$ is a root, so is $2 + i$. M1A1
 Also, $x^2 - 4x + 5$ is a factor of the quartic. A1
 Using long division,

$$\begin{array}{r} x^2 - 2x + 5 \\ x^2 - 4x + 5 \overline{) x^4 - 6x^3 + 18x^2 - 30x + 25} \\ \underline{x^4 - 4x^3 + 5x^2} \\ -2x^3 + 13x^2 - 30x \\ \underline{-2x^3 + 8x^2 - 10x} \\ 5x^2 - 20x + 25 \\ \underline{5x^2 - 20x + 25} \\ 0 \end{array}$$

M1A1A1

The other roots are

$$\frac{2 \pm \sqrt{4 - 20}}{2}, \text{ ie } 1 \pm 2i. \quad \text{M1A1}$$

2. $f(x + h) - f(x) = \frac{1}{(x + h)^3} - \frac{1}{x^3}$ M1A1
 $= \frac{x^3 - (x + h)^3}{x^3(x + h)^3}$ A1
 $= \frac{-h(3x^2 + 3hx + h^2)}{x^3(x + h)^3}$ A1
 $f'(x) = \lim_{h \rightarrow 0} \frac{-h(3x^2 + 3hx + h^2)}{x^3(x + h)^3}$ M1
 $= -\frac{3}{x^4}$ A1

3. $\frac{8 - 6i}{1 - 2i} = \frac{(8 - 6i)(1 + 2i)}{(1 - 2i)(1 + 2i)}$ M1A1
 $= \frac{20 + 10i}{5} = 4 + 2i$ A1
 $3(a + ib) - (a - ib) = 4 + 2i$ M1
 $2a = 4$ and $4b = 2$ A1
 $a = 2$ and $b = 1/2$ A1

4. (a) $T_n = n^2 + 2n - (n - 1)^2 - 2(n - 1)$ M1A1
 $= n^2 + 2n - n^2 + 2n - 1 - 2n + 2$ A1
 $= 2n + 1$ (AG)

(b) $S_n = 4 \sum_{r=1}^n r^2 + 4 \sum_{r=1}^n r + \sum_{r=1}^n 1$ M1
 $= \frac{4n(n + 1)(2n + 1)}{6} + \frac{4n(n + 1)}{2} + n$ A1A1A1
 $= \frac{n}{3}(4n^2 + 6n + 2 + 6n + 6 + 3)$ A1
 $= \frac{n}{3}(4n^2 + 12n + 11)$ A1

5. Assume the proposition is true for $n = k$. M1

Letting $T_k = 3^{2k} + 7$, consider

$$\begin{aligned} T_{k+1} - T_k &= 3^{2k+2} + 7 - (3^{2k} + 7) && \text{m1} \\ &= 3^{2k}(9 - 1) && \text{A1} \\ &= 3^{2k} \cdot 8 && \text{A1} \end{aligned}$$

So, if the proposition is true for $n = k$, it is also true for $n = k + 1$. A1

It is true for $n = 1$ since $T_1 = 16$ which is divisible by 8. A1

The result is therefore proved by mathematical induction A1

[The final A1 is for a concluding statement plus satisfactory lay-out]

6. $\ln f(x) = \frac{1}{2} \ln(x^3 + 1) - \frac{3}{2} \ln(x^2 + 1)$ M1A1

$$\frac{f'(x)}{f(x)} = \frac{1}{2} \cdot \frac{3x^2}{(x^3 + 1)} - \frac{3}{2} \cdot \frac{2x}{(x^2 + 1)} \quad \text{M1A1A1}$$

Putting $x = 2$,

$$\frac{f'(2)}{3/5\sqrt{5}} = \frac{1}{2} \cdot \frac{12}{9} - \frac{3}{2} \cdot \frac{4}{5} \quad \text{M1}$$

$$f'(2) = -\frac{8}{25\sqrt{5}} \quad (-0.143) \quad \text{A1}$$

7. $\alpha + \beta = -3$ and $\alpha\beta = 3$ B1

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \alpha\beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + \alpha\beta \quad \text{M1 A1 A1}$$

$$= 4 \quad \text{A1}$$

$$\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} + \frac{\alpha}{\beta} \cdot \alpha\beta + \frac{\beta}{\alpha} \cdot \alpha\beta = 1 + (\alpha + \beta)^2 - 2\alpha\beta \quad \text{M1 A1}$$

$$= 4 \quad \text{A1}$$

$$\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} \cdot \alpha\beta = \alpha\beta = 3 \quad \text{M1 A1}$$

Required equation is

$$x^3 - 4x^2 + 4x - 3 = 0 \quad \text{M1 A1}$$

8. (a) $\text{Det} = 1(10 - 1) - 1(5 - \lambda) + 2(1 - 2\lambda) = 6 - 3\lambda$ M1A1
Matrix is singular when $\lambda = 2$. A1

- (b) (i) Using reduction to echelon form,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ \mu - 4 \end{bmatrix}$$

Comparing the second and third rows, $\mu = 4$ for consistency.

- (ii) Put $z = a$ so that $y = a$ and $x = 2 - 3a$. M1A1A1

9. (a) Matrix of translation = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Matrix of rotation = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$$T\text{-Matrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

- (b) Consider the image of the point (a, a^2) . This is given by M1

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ a^2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 - a^2 \\ a + 2 \\ 1 \end{bmatrix}$$

So, $x = 1 - a^2$, $y = a + 2$. Eliminating a , the required equation is

$$1 - x = (y - 2)^2.$$

M1

A1

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MATHEMATICS FP2

Further Pure Mathematics

Specimen Mark Scheme

2005/2006

1. For the continuity of g at $x = 1$,

$$1 + b = 2 + a$$
M1A1
Since $g'(x) = 1 + 2bx$ for $x < 1$ and $3ax^2$ for $x > 1$, it follows that
M1

$$1 + 2b = 3a$$
A1
The solution is $a = 3, b = 4$.
M1A1
2. The equation can be rewritten

$$-\sin 3\theta + 2\sin 3\theta \cos 2\theta = 0$$
M1A1

$$\sin 3\theta(2\cos 2\theta - 1) = 0$$
A1
Either

$$\sin 3\theta = 0$$
M1
from which $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ$.
A2 (–1 each error or extra root in the range)
Or

$$\cos 2\theta = 1/2$$
M1
from which $\theta = 30^\circ, 150^\circ$.
A1
3. Converting to trigonometric form

$$3 - 2i = r(\cos \theta + i \sin \theta)$$
M1
where $r = \sqrt{13}$ and $\theta = \tan^{-1}\left(\frac{-2}{3}\right)$
A1, A1

$$(r(\cos \theta + i \sin \theta))^{1/3} = r^{1/3}(\cos \frac{\theta}{3} + i \sin \frac{\theta}{3})$$
M1
So $(3 - 2i)^{1/3} = (\sqrt{13})^{1/3}(\cos \frac{-0.5880}{3} + i \sin \frac{-0.5880}{3})$
A1, A1

$$= 1.50 - 0.30i$$
A1
or

$$= (\sqrt{13})^{1/3}(\cos \frac{2\pi - 0.5880}{3} + i \sin \frac{2\pi - 0.5880}{3})$$
A1

$$= -0.49 + 1.45i$$
A1
or

$$= (\sqrt{13})^{1/3}(\cos \frac{4\pi - 0.5880}{3} + i \sin \frac{4\pi - 0.5880}{3})$$
A1

$$= -1.01 - 1.15i$$
A1
4. (a) $f'(x) = \frac{2(x-1) - (2x+1)}{(x-1)^2} = -\frac{3}{(x-1)^2}$
M1A1
This is < 0 for all x in the domain so f is strictly decreasing.
A1
- (b) The range is $(2, \infty)$.
B1B1
- (c) (i) $f(3) = 7/2$ and $f(4) = 3$ so $f(S) = [3, 7/2]$.
M1A1
- (ii) $f(x) = 3 \Rightarrow x = 4$ (from above) and $f(x) = 4 \Rightarrow x = 5/2$.
B1M1A1
So $f^{-1}(S) = [5/2, 4]$.
A1

5. Using de Moivre's Theorem,

$$z^n = \cos n\theta + i \sin n\theta \quad \text{M1A1}$$

$$z^{-n} = \cos n\theta - i \sin n\theta \quad \text{A1}$$

So, $z^n - \frac{1}{z^n} = 2i \sin n\theta \quad \text{A1}$

$$\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5} \quad \text{M1A1}$$

$$= z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right) \quad \text{A1}$$

$$(2i \sin \theta)^5 = 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta \quad \text{M1}$$

whence

$$\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta) \quad \text{A1}$$

6. (a) Completing the square,

$$(y + 2)^2 = 8(x - 1) \quad \text{M1A1}$$

(i) Vertex is (1, -2). A1

(ii) Focus is (3, -2) A1

(b) (i) For the point P,

$$(y + 2)^2 = 16p^2$$

and $8(x - 1) = 16p^2$. M1A1

confirming that P lies on the parabola.

(ii) $\frac{dy}{dx} = \frac{4}{4p} = \frac{1}{p}$ M1A1

Equation of tangent is

$$y - 4p + 2 = \frac{1}{p}(x - 2p^2 - 1) \quad \text{M1A1}$$

(iii) This passes through the origin if

$$-4p^2 + 2p = -2p^2 - 1$$

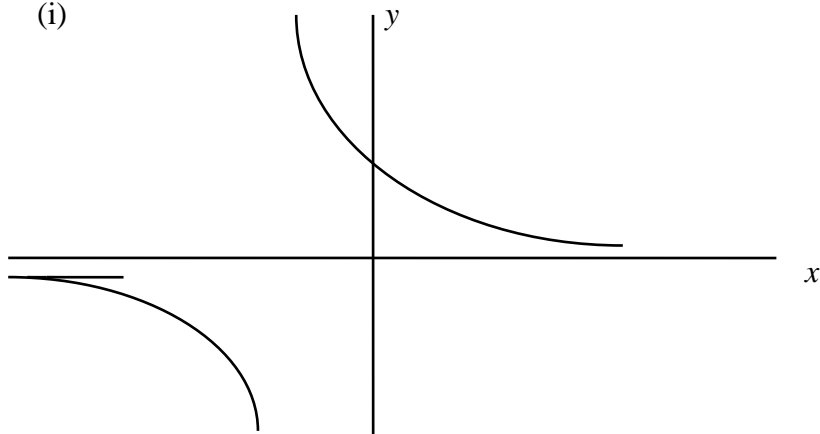
$$2p^2 - 2p - 1 = 0 \quad \text{M1A1}$$

$$p = \frac{2 \pm \sqrt{4 + 8}}{4} = \frac{1 \pm \sqrt{3}}{2} \quad \text{A1}$$

In view of line 1 in (ii),

$$\text{Gradients of tangents} = \frac{2}{1 \pm \sqrt{3}} \quad \text{A1}$$

7. (a) (i)



G1 (shape)
G1 (asymptotes)

(ii) The asymptotes are $x = -1$ and $y = 0$. B1B1

(b) Let
$$\frac{1}{(x+1)(x^2+4)} \equiv \frac{A}{x+1} + \frac{Bx+C}{x^2+4} = \frac{A(x^2+4) + (x+1)(Bx+C)}{(x+1)(x^2+4)}$$

M1

$x = -1$ gives $A = \frac{1}{5}$. A1

Coeff of x^2 gives $B = -\frac{1}{5}$. A1

Const term gives $C = \frac{1}{5}$. A1

(c)
$$\text{Int} = \frac{1}{5} \int_0^1 \frac{1}{x+1} dx + \frac{1}{5} \int_0^1 \frac{1}{x^2+4} dx - \frac{1}{5} \int_0^1 \frac{x}{x^2+4} dx$$
 M1

$$= \frac{1}{5} [\ln(1+x)]_0^1 + \frac{1}{10} \left[\tan^{-1}\left(\frac{x}{2}\right) \right]_0^1 - \frac{1}{10} [\ln(x^2+4)]_0^1$$
 A1A1A1

$$= \frac{1}{5} \ln 2 + \frac{1}{10} \tan^{-1}\left(\frac{1}{2}\right) - \frac{1}{10} (\ln 5 - \ln 4)$$
 A1A1A1

$$= 0.163$$
 A1

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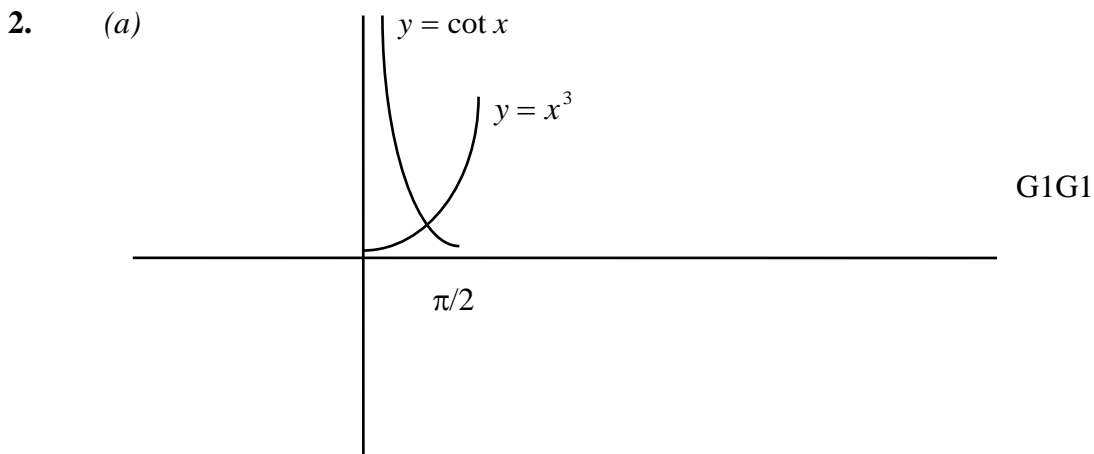
MATHEMATICS FP3

Further Pure Mathematics

Specimen Mark Scheme

2005/2006

1. $1 + \sinh^2 x = 3 + \sinh x$ M1
 leading to $\sinh^2 x - \sinh x - 2 = 0$ A1
 $(\sinh x - 2)(\sinh x + 1) = 0$ M1
 $x = \sinh^{-1}(2) = \ln(2 + \sqrt{5})$ A1A1
 or $x = \sinh^{-1}(-1) = \ln(-1 + \sqrt{2})$ A1A1



The one point of intersection confirms that there is only one root in $(0, \pi/2)$. B1

- (b) The Newton-Raphson iteration is

$$x \rightarrow x - \frac{x^3 - \cot x}{3x^2 + \operatorname{cosec}^2 x}$$
 M1A1

The iterates are
 1
 0.9188838545 M1
 0.9158148137
 0.9158112781 A1
 $x_3 = 0.915811$ correct to 6 decimal places. A1
 Let $f(x) = x^3 - \cot x$ M1
 $f(0.9158105) = \text{negative}$
 $f(0.9158115) = \text{positive}$ A1
 So the root is 0.915811 correct to 6 decimal places. A1

3. (a)
$$\text{Area} = 2\pi \int_0^1 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \text{M1}$$

$$= 2\pi \int_0^1 x^3 \sqrt{1 + 9x^4} dx \quad \text{A1}$$
- (b)
$$u = 1 + 9x^4, du = 36x^3 dx \quad \text{B1}$$

$$[0,1] \rightarrow [1,10] \quad \text{B1}$$

$$\text{Area} = 2\pi \cdot \frac{1}{36} \int_1^{10} \sqrt{u} du \quad \text{M1A1}$$

$$= \frac{\pi}{18} \left[\frac{2}{3} u^{3/2} \right]_1^{10} \quad \text{A1}$$

$$= \frac{\pi}{27} (10\sqrt{10} - 1) \quad \text{A1}$$
4.
$$I = \int_0^{\pi/2} e^{-2x} d \sin x \quad \text{M1}$$

$$= \left[e^{-2x} \sin x \right]_0^{\pi/2} - \int_0^{\pi/2} \sin x \cdot -2e^{-2x} dx \quad \text{A1A1}$$

$$= e^{-\pi} + 2J \quad \text{A1}$$

$$J = \int_0^{\pi/2} e^{-2x} d(-\cos x) \quad \text{M1}$$

$$= -\left[e^{-2x} \cos x \right]_0^{\pi/2} + \int_0^{\pi/2} \cos x \cdot -2e^{-2x} dx \quad \text{A1A1}$$

$$= 1 - 2I \quad \text{A1}$$

Substituting,

$$I = e^{-\pi} + 2(1 - 2I)$$

so
$$I = \frac{2}{5} + \frac{1}{5} e^{-\pi} \quad \text{M1A1}$$

It follows that

$$J = \frac{1}{5} - \frac{2}{5} e^{-\pi} \quad \text{A1}$$
5. (a)
$$f(x) = \ln(1 + \sin x) : f(0) = 0 \quad \text{B1}$$

$$f'(x) = \frac{\cos x}{1 + \sin x} : f'(0) = 1 \quad \text{B1B1}$$

$$f''(x) = -\frac{1}{1 + \sin x} : f''(0) = -1 \quad \text{B1B1}$$

$$f'''(x) = \frac{\cos x}{(1 + \sin x)^2} : f'''(0) = 1 \quad \text{B1B1}$$

The series is

$$x - \frac{x^2}{2} + \frac{x^3}{6} \quad \text{M1A1}$$

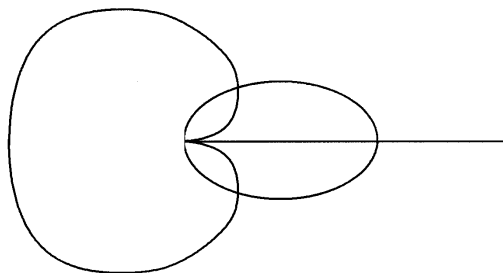
(b)
$$\text{Int} \approx \left[\frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \right]_0^{1/3} \quad \text{M1A1}$$

$$= \frac{1}{18} - \frac{1}{27 \times 6} + \frac{1}{81 \times 24} \quad \text{A1}$$

$$= 0.05 \quad \text{A1}$$

6. (a)

C1



G1G1

(b)
$$\begin{aligned} \text{Area} &= \frac{1}{2} \int_0^{2\pi} (1 - \cos \theta)^2 d\theta && \text{M1} \\ &= \frac{1}{2} \int_0^{2\pi} (1 - 2 \cos \theta + \cos^2 \theta) d\theta && \text{A1} \\ &= \frac{1}{2} \int_0^{2\pi} \left(\frac{3}{2} - 2 \cos \theta + \frac{1}{2} \cos 2\theta \right) d\theta && \text{A1} \\ &= \frac{1}{2} \left[\frac{3\theta}{2} - 2 \sin \theta + \frac{1}{4} \sin 2\theta \right]_0^{2\pi} && \text{A1} \\ &= \frac{3\pi}{2} && \text{A1} \end{aligned}$$

(c) The curves meet where

$$\begin{aligned} \cos 2\theta &= 1 - \cos \theta && \text{M1} \\ 2 \cos^2 \theta + \cos \theta - 2 &= 0 && \text{A1} \\ \cos \theta &= \frac{-1 \pm \sqrt{17}}{4} = 0.780776\dots && \text{A1} \end{aligned}$$

giving (0.219, 0.675) and (0.219, -0.675) A1A1

The curves also intersect at the pole (origin). B1

7. (a) $\sin n\theta - \sin(n-2)\theta = 2 \cos(n-1)\theta \sin \theta$ M1A1
 The given result follows by division by $\sin \theta$.

(b) Using this result,

$$\begin{aligned} \int_0^\pi \frac{\sin n\theta - \sin(n-2)\theta}{\sin \theta} d\theta &= 2 \int_0^\pi \cos(n-1)\theta d\theta && \text{M1} \\ I_n - I_{n-2} &= \frac{2}{n-1} [\sin(n-1)\theta]_0^\pi && \text{A1A1} \\ &= 0 && \text{A1} \end{aligned}$$

(c) (i) If n is even,

$$\begin{aligned} I_n &= I_0 = \int_0^\pi 0 \cdot d\theta && \text{M1A1} \\ &= 0 && \text{A1} \end{aligned}$$

(ii) If n is odd,

$$\begin{aligned} I_n &= I_1 = \int_0^\pi d\theta && \text{M1A1} \\ &= [\theta]_0^\pi && \\ &= \pi && \text{A1} \end{aligned}$$

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MATHEMATICS M1

Mechanics

Specimen Mark Scheme

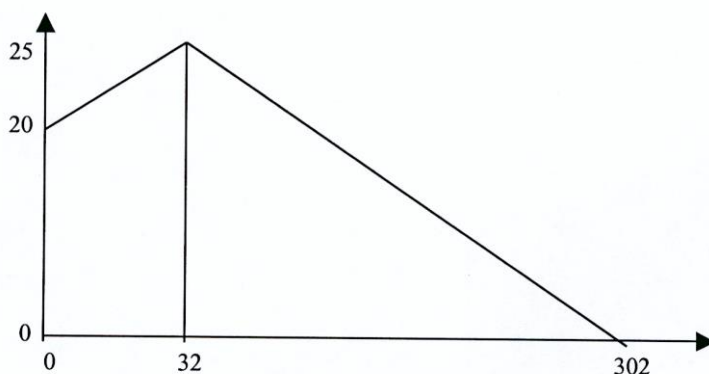
2005/2006

1. (a) Attempt to apply N2L to lift, dimensionally correct equation M1
 $600g - T = 600a$ A1
 $a = 0.4, T = 600(9.8 - 0.4) = 5640 \text{ N}$ A1

- (b) $a = 0, T = 600 \times 9.8 = 5880 \text{ N}$ B1

2. (a) Attempted use of $s = \frac{1}{2}(u+v)t$ with $u = 20, v = 25, s = 720$ M1
 $720 = 0.5(20 + 25)t$ A1
time taken during acceleration = $t = 32 \text{ s}$ A1

(b)



M1 ($v-t$ graph)
G1 (1st 2 points correct)
G1 (last part correct)

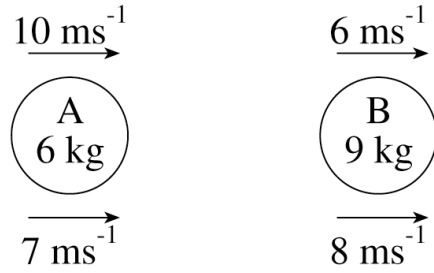
- (c) Use of distance = area under graph M1
Distance = $720 + 0.5 \times 270 \times 25$ A1
= 4095 m A1

3. (a) Use of $v^2 = u^2 + 2as$ with $u = 0, v = 1.4, a = 9.8, s = h$ M1
 $14^2 = 0^2 + 2 \times 9.8h$ A1
 $h = 10 \text{ m}$ A1

- (b) Use of $v = u + at$ with $u = 0, v = 1.4, a = 9.8$ M1
 $14 = 9.8t$ A1
 $t = \frac{10}{7} \text{ s}$ A1

- (c) Required speed = 0.6×14 M1
= 8.4 ms^{-1} A1

4. (a)



Attempt at restitution equation

$$8 - 7 = -e(6 - 10)$$

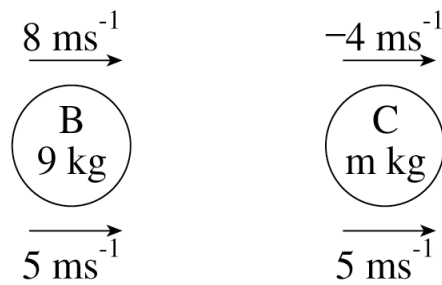
$$e = 0.25$$

M1

A1

A1

(b)



Attempt at use of conservation of momentum, dimensionally correct equation

$$9 \times 8 - 4m = (9 + m) \times 5$$

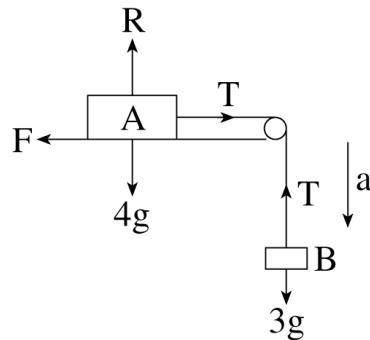
$$m = 3 \text{ kg}$$

M1

A1

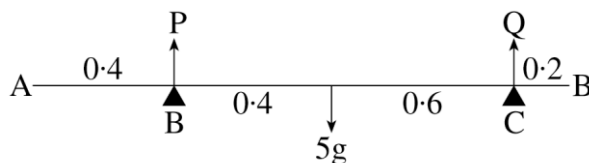
A1

5.



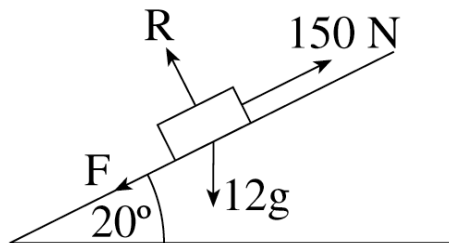
- (a) N2L applied to each particle, dimensionally correct equation
 For A, $T = 4a$ M1 A1
 For B, $3g - T = 3a$ M1 A1
 Substitute for T, $3g - 4a = 3a$ m1
 $a = \frac{3 \times 9.8}{7} = 4.2 \text{ ms}^{-2}$ A1
 $T = 4 \times 4.2 = 16.8 \text{ N}$ A1
- (b) Vertically for B, $T = 3g$ B1
 Vertically for A, $R = 4g$ B1
 Horizontally for a, $F = T = 3g$ B1
 Use of $F \leq \mu R$ M1
 $\mu \geq \frac{F}{R} = \frac{3g}{4g} = 0.75$ A1

6.



- (a) Attempt to take moments about C/B to obtain dimensionally correct equation M1
 $-P \times 1 + 5g \times 0.6 = 0$ A1
 $P = 3g = 29.4 \text{ N}$ A1
 Attempt at second equation with P and Q M1
 $P + Q = 5g$ (o.e.) A1
 $Q = 5g - 3g = 2g = 19.6 \text{ N}$ A1
- (b) Attempt to take moments about C, dimensionally correct equation M1
 $-P \times 1 + 5g \times 0.6 = W \times 0.2$ A1
 Plank does not tilt iff $P = 3g - 0.2W \geq 0$ m1
 Greatest $W = \frac{3g}{0.2} = 15g = 147 \text{ N}$ A1

7.



- (a) Resolve perpendicular to plane $R = 12g \cos 20^\circ$ M1 A1
 $F = 0.6 R$ M1
 $F = 0.6 \times 12 \times 9.8 \cos 20^\circ$ A1
 Attempt to apply N2L // to plane, dimensionally correct M1
 $150 - F - 12g \sin 20^\circ = 12a$ A2 (-1 each error)
 $a = 3.62(28) \text{ ms}^{-2}$ A1

- (b) e.g. Body modelled as particle
No air resistance B1
8. (a) attempt at use of Impulse = change of momentum M1
 $-2 = 0.25(v - 5)$ A1
 $v = -3$ A1
- (b) Attempt at use of Impulse = force \times time M1
 $-2 = F \times 0.2$ A1
 Magnitude of force = 10 N A1
9. (a)
- | | Area | Dist of c of m
from <i>AE</i> | Dist of c of m
from <i>AB</i> | |
|-------------|------|----------------------------------|----------------------------------|------------------------------------|
| <i>DEF</i> | 36 | 2 | 10 | B1 Areas |
| <i>ABCF</i> | 12 | 1 | 3 | B1 Distance from <i>AE</i> |
| Lamina | 48 | x | y | <i>B1</i> Distances from <i>AB</i> |
- Take moments M1
 $48x = 36 \times 2 + 12 \times 1$ A1
 $x = 1.75$ A1
 $48y = 36 \times 10 + 12 \times 3$ A1
 $y = 8.25$ A1
- (b) Required angle identified M1
 $\tan \theta = \frac{1.75}{18 - 8.25}$ A1
 $\theta = 10.18^\circ$ A1

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MATHEMATICS M2

Mechanics

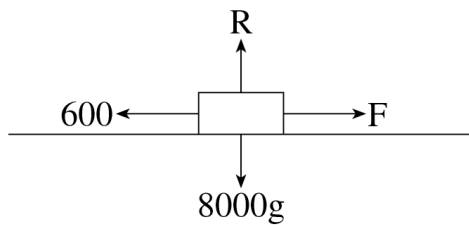
Specimen Mark Scheme

2005/2006

1. (a) Use of $a = \frac{dv}{dt}$ M1
 $a = 6t + 40t^3$ A1

(b) Use of $x = \int v dt$ M1
 $x = t^3 + 2t^5 + c$ A1
 Use of $t = 0, x = -3$ m1
 $x = t^3 + 2t^5 - 3$
 when $t = 2, x = 8 + 64 - 3 = 69$ m A1

2. (a)



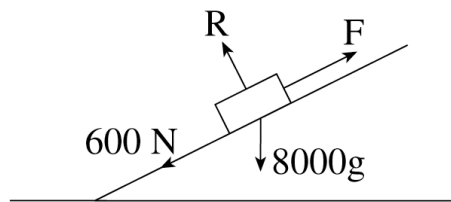
Use of $F = \frac{P}{25}$ B1

N2L $F - 600 = 0$ M1

$\frac{F}{25} = 600$

$P = 600 \times 25 = 15000$ W A1

(b)



$F = \frac{36 \times 1000}{3} = 12000$ B1

$F - 600 - 8000g \sin \alpha = 8000a$ M1 A1

$12000 - 600 - 8000 \times 9.8 \times \frac{1}{14} = 8000a$ A1

$a = 0.725 \text{ ms}^{-2}$ A1

3. (a) Tension in string, $T = \frac{19.6 \times 0.5}{0.7}$ M1

$T = 14$ N A1

(b) Energy stored in string = $\frac{1}{2} \times \frac{19.6 \times 0.5^2}{0.7}$ M1

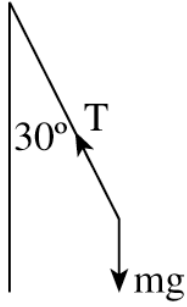
= 3.5 J A1

- (c) Attempt at conservation of energy (at least 3 terms) M1
 Gain in potential energy = $0.4 \times 9.8 \times 0.5 \sin \alpha$ M1
 $= 0.4 \times 9.8 \times 0.5 \times 0.6 = 1.176 \text{ J}$ A1
 Gain in kinetic energy = $0.5 \times 0.4 v^2$ B1
 $1.176 + 0.2v^2 = 3.5$ A1
 $v = 3.4(088) \text{ ms}^{-1}$ A1
4. (a) $y = ut - \frac{1}{2}gt^2$ M1
 $0 = 24.5 \sin 30^\circ t - \frac{1}{2} \cdot 9.8 \cdot t^2$ A1,A1
 $t = 2.5 \text{ seconds}$ A1
- (b) $AB = 24.5 \cos 30^\circ \times 2.5$ M1
 $= 53.0 \text{ m}$ FT student's time
 A1
- (c) Greatest Height = $24.5 \sin 30^\circ \times 1.25 - \frac{1}{2} \cdot 9.8(1.25)^2$ M1
 $= 7.656 \text{ m}$ FT student's time
 A1
- (d) $\dot{y} = 24.5 \sin 30^\circ - 9.8 \times 2 = -7.35$ B1 cao
 $\dot{x} = 24.5 \cos 30^\circ = 21.218$ B1 cao
 Speed = $\sqrt{(-7.35)^2 + (21.218)^2} = 22.45 \text{ m/s}$ M1,A1

 Angle with horizontal = $\tan^{-1} \left(\frac{-7.35}{21.218} \right) = 19.1^\circ$ M1,A1
5. (a) $\mathbf{a} = \frac{1}{2}((3\mathbf{i} - 2\mathbf{j}) - (\mathbf{i} + 2\mathbf{j}))$ M1
 $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$ A1
- (b) Use of $\mathbf{v} = \int \mathbf{a} \, dt$ M1
 $\mathbf{v} = t\mathbf{i} - 2t\mathbf{j} + \mathbf{c}$ A1
 When $t = 0$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ m1
 $\mathbf{c} = \mathbf{i} + 2\mathbf{j}$ A1
 So $\mathbf{v} = (t + 1)\mathbf{i} - (2t - 2)\mathbf{j}$ (o.e.)
- (c) Use of $\mathbf{a} \cdot \mathbf{v} = 0$ M1
 $(t + 1) - 2(2 - 2t) = 0$ A1
 $t = 0.6$ A1

- (d) Use of $\mathbf{r} = \int \mathbf{v} dt$ M1
 $\mathbf{r} = \frac{1}{2}t^2 \mathbf{i} - t^2 \mathbf{j} + t \mathbf{i} + 2t \mathbf{j} + \mathbf{c}$ A1
 when $t = 0$, $\mathbf{r} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{c} = 2\mathbf{i} - \mathbf{j}$ m1
 $\mathbf{r} = (\frac{1}{2}t^2 + t + 2)\mathbf{i} + (-t^2 + 2t - 1)\mathbf{j}$ A1
- (e) when $t = 2$, $\mathbf{r} = (2 + 2 + 2)\mathbf{i} + (-4 + 4 - 1)\mathbf{j}$ M1
 $\mathbf{r} = 6\mathbf{i} - \mathbf{j}$
 $|\mathbf{r}| = \sqrt{6^2 + 1^2}$ M1
 $|\mathbf{r}| = \sqrt{37}$ A1
6. (a) Attempt at conservation of energy M1
 $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(a - a\cos\theta)$ A2 (-1 each error)
 $v^2 = u^2 + 2ag(1 - \cos\theta)$ A1
- (b) Attempt at N2L towards centre M1
 $R - mg \cos\theta = m \times \text{acceleration}$ A1
 $R - mg \cos\theta = -\frac{mv^2}{a}$ A1
 $R = mg \cos\theta - \frac{m}{a}(u^2 + 2ag - 2ag\cos\theta)$ m1
 $R = mg \cos\theta - 2mg + 2mg\cos\theta - \frac{mu^2}{a}$
 $R = mg(3\cos\theta - 2) - \frac{mu^2}{a}$ A1
- (c) $R = 0$ when ball bearing leaves bowl. M1
 $0.05 \times 9.8(3\cos\theta - 2) - \frac{0.05 \times 2^2}{0.5} = 0$ A1
 $\cos\theta = 0.938(77551)$
 $\theta = 20.15(3)^\circ$ A1

7. (a)

Resolve vertically. $T \cos 30^\circ = mg$

M1 A1

$$T = \frac{0.2 \times 9.8}{\cos 30^\circ} = 2.26$$

A1

(b) N2L towards centre

M1

$$T \sin 30^\circ = \frac{mu^2}{0.8 \sin 30^\circ}$$

A1 L.H.S.

A1 R.H.S.

Substitute for T

m1

$$u = 1.50(43979) \text{ ms}^{-1}$$

A1

(c) Tension is constant throughout the rope.

B1

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MATHEMATICS M3

Mechanics

Specimen Mark Scheme

2005/2006

1. (a) N2L applied to particle M1

$$4a = -4g - \frac{gv^2}{9}$$
 A1
 Magnitude of retardation = $\frac{g}{36}(36 + v^2)$ A1 (convincing)
- (b)
$$\frac{dv}{dt} = -\frac{g}{36}(36 + v^2)$$
 M1

$$36 \int \frac{dv}{36 + v^2} = -g \int dt$$
 A1

$$36 \times \frac{1}{6} \tan^{-1} \frac{v}{6} = -gt + c$$
 A1
 When $t = 0, v = 6$ m1

$$C = 6 \times \frac{\pi}{4} = \frac{3\pi}{2}$$
 A1
 At maximum height, $v = 0$ M1

$$t = \frac{3\pi}{2g} = 0.48\text{s}$$
 A1
- (c)
$$v \frac{dv}{dx} = -\frac{g}{36}(36 + v^2)$$
 M1

$$36 \int \frac{v dv}{36 + v^2} = -g \int dx$$
 A1

$$10 \ln(36 + v^2) = -gx + c$$
 A1
 When $t = 0, v = 6, x = 0$ m1

$$c = 18 \ln(72)$$
 A1
 At maximum height. $v = 0$ M1

$$x = \frac{18}{9.8} \ln(2) = 1.27 \text{ m}$$
 A1
2. (a) Max. speed = $a\omega$ M1

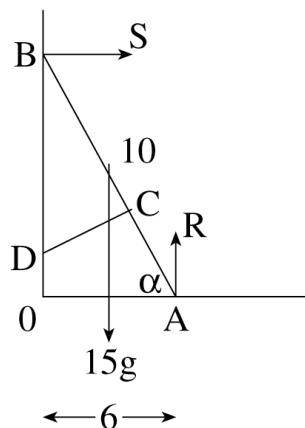
$$a\omega = 8$$
 A1

$$\omega = \frac{8}{5}$$
 A1
- Period = $\frac{2\pi}{\omega} = \frac{5\pi}{4}$ B1 (F.T.)
- Time for 9 oscillations = $\frac{45\pi}{4}$ (or 35.34s) B1

- (b) Using $v^2 = \omega^2(a^2 - x^2)$ with $a = 5, x = 4, \omega = \frac{8}{5}$ o.e. M1
- $v^2 = 1 \cdot 6^2(5^2 - 4^2)$ A1
- $v = 4 \cdot 8 \text{ms}^{-1}$ A1
- (c) $|a| = \omega^2 x$ M1
- $= 1 \cdot 6^2 \times 3$ A1
- $= 7 \cdot 68 \text{ms}^{-2}$ A1
- (d) $x = 5 \sin(1 \cdot 6t)$ B1
- When $x = -2 \cdot 4, t_A = -0 \cdot 3129$ B1
- When $x = 3 \cdot 6, t_B = 0 \cdot 5024$ B1
- Required time $= t_B - t_A$ M1 (overall strategy)
- $= 0 \cdot 8153$ A1 (C.A.O.)

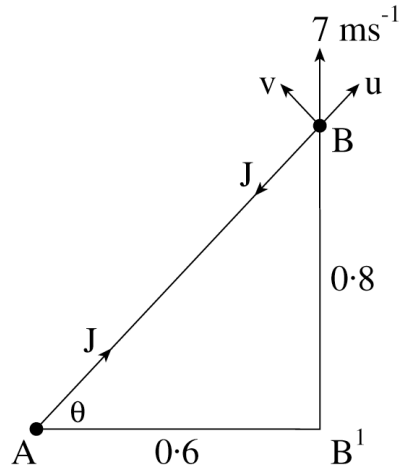
3. Auxiliary equation $m^2 + 8m + 12 = 0$ M1
- $(m + 2)(m + 6) = 0$
- $m = -2, -6$ A1
- Complementary function $x = Ae^{-2t} + Be^{-6t}$ A1
- For particular integral, try $x = at + b$ M1
- $\frac{dx}{dt} = a$
- Substitute into differential equation
- $8a + 12(at + b) = 12t + 20$ A1
- Compare coefficients m1
- $12a = 12$
- $a = 1$
- $8a + 12b = 20$
- $b = 1$ both values A1
- General solution $x = Ae^{-2t} + Be^{-6t} + t + 1$ A1
- When $t = 0, x = 0$ M1
- $0 = A + B + 1$ B1
- When $t = 0, \frac{dx}{dt} = 3$
- $\frac{dx}{dt} = -2Ae^{-2t} - 6Be^{-6t} + 1$ B1
- $-2A - 6B + 1 = 3$ A1
- Solving $A = -1, B = 0$ A1
- General solution is $x = -e^{-2t} + t + 1$
- When $t = 2, x = 2 \cdot 98 \text{ m}$ A1

4. (a)



- | | |
|---|----|
| Resolve horizontally | M1 |
| $S = T \sin \alpha$ | A1 |
| Moments about A, dimensionally correct | M1 |
| $15g \times 3 + 4 \times T = S \times 8$ | A1 |
| Substitute for $S = 0.8T$ | m1 |
| $45g + 4T = 6.4T$ | |
| $T = 183.75 \text{ N}$ | A1 |
| $S = 147 \text{ N}$ | A1 |
| Resolve vertically | M1 |
| $R = 15g + T \cos \alpha$ | A1 |
| $R = 15 \times 9.8 + 183.75 \times 0.6 = 257.25 \text{ N}$ | A1 |
| | |
| (b) Using $T = 2000 \text{ N}$ and adding extra term in moment equation | M1 |
| $441 + 4T + 80g \times x \cos \alpha = 6.4 T$ | A1 |
| $470.4x = (6.4 - 4) \times 2000 - 441$ | |
| $x = 9.27 \text{ m}$ | A1 |
| Man will not reach top of ladder | A1 |
| | |
| (c) Any reasonable assumption | B1 |
| (e.g. man is a particle, ladder is a rigid rod) | |

5. (a)



B1

$$v = 7 \cos\theta = 4.2 \text{ ms}^{-1}$$

B1

For B, $-J = 5 \times 7 \sin\theta - 5u$

M1 A1

For A, $J = 2u$

B1

$$-J = 28 - 5u = -2u$$

$$u = 4 \text{ ms}^{-1}$$

A1

Speed of A = 4 ms^{-1}

$$\text{Speed of B} = \sqrt{4^2 + 4 \cdot 2^2}$$

M1

$$\text{Speed of B} = \sqrt{33.64} = 5.8 \text{ ms}^{-1}$$

A1

$$J = 2u = 8 \text{ Ns}$$

A1

(b) Loss in K.E. = Initial K.E. - Final K.E.

M1

$$\text{Loss in K.E.} = 0.5 \times 5 \times 7^2 - 0.5 \times 5 \times 33.64 - 0.5 \times 2 \times 4^2$$

m1 A1

$$\text{Loss in K.E.} = 22.4 \text{ J}$$

A1

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MATHEMATICS S1

Statistics

Specimen Mark Scheme

2005/2006

1. (a) $\text{Prob} = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times 6 = \frac{2}{7}$ (or $\frac{\binom{4}{1} \times \binom{3}{1} \times \binom{2}{1}}{\binom{9}{3}}$) M1M1A1
- (b) $P(3 \text{ reds}) = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21}$ B1
 $P(3 \text{ blues}) = \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{1}{84}$ B1
 $P(3 \text{ yellow}) = 0$ B1
 $P(3 \text{ same colour}) = \text{Sum} = \frac{5}{84}$ B1
(or $\frac{\binom{4}{3} + \binom{3}{3}}{\binom{9}{3}}$)
2. $E(Y) = 4 \times 4 + 1 = 17$ M1A1
 $\text{Var}(X) = 4$ B1
 $\text{Var}(Y) = 16 \times 4 = 64$ M1A1
 $\text{SD}(Y) = 8$ A1
3. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.7 + 0.4 - 0.28 = 0.82$ (or $= 1 - P(A' \cap B') = 1 - 0.18$) M1A1A1
- (b) (i) $\text{Prob} = P(A)P(B') + P(A')P(B)$ M1A1
 $= 0.7 \times 0.6 + 0.3 \times 0.4 = 0.54$ A1
- (ii) $P(A' \text{ and } B') = 0.3 \times 0.6 = 0.18$ M1A1A1
4. (a) (i) $P(X \geq 4) = 0.8488$ (or $1 - 0.1512$) M1A1
- (ii) $P(X = 6) = 0.6063 - 0.4457$ (or $0.5543 - 0.3937$) M1A1A1
 $= 0.1606$
- (b) (i) $P(Y = 2) = e^{-1.12} \times \frac{1.12^2}{2} = 0.205$ M1A1
- (ii) $P(Y \geq 2) = 1 - P(Y \leq 1)$ M1
 $= 1 - e^{-1.12} \times (1 + 1.12)$ A1
 $= 0.308$ A1
5. (a) $P(\text{White}) = 0.5 \times 0.15 + 0.3 \times 0.2 + 0.2 \times 0.25 = 0.185$ M1A1A1
- (b) $P(A | \text{White}) = \frac{0.5 \times 0.15}{0.185} = 0.405$ M1A1A1

6. (a) Mean = $10 \times 0.8 = 8$; SD = $\sqrt{10 \times 0.8 \times 0.2} = 1.26$ B1M1A1
- (b) (i) $P(X = 8) = \binom{10}{8} \times 0.8^8 \times 0.2^2 = 0.302$ M1A1
(or using tables 0.6242 - 0.3222 or 0.6778 - 0.3758)
- (ii) Y is B(10,0.2) $\rightarrow P(4 \leq X \leq 7) = P(3 \leq Y \leq 6)$ M1A1
 $= 0.9991 - 0.6778 = 0.321$ m1A1
(or 0.3222 - 0.0009)
7. (a) Mean = $1 \times 0.2 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.3 = 2.6$ M1A1
 $E(X^2) = 1 \times 0.2 + 4 \times 0.3 + 9 \times 0.2 + 16 \times 0.3 = 8.0$ M1A1
 $\text{Var} = 8 - 2.6^2 = 1.24$ A1
- (b) $E\left(\frac{1}{X}\right) = 1 \times 0.2 + \frac{1}{2} \times 0.3 + \frac{1}{3} \times 0.2 + \frac{1}{4} \times 0.3 = 0.492$ M1A1A1
- (c) Possibilities are (2,2), (1,3), (3,1). (si) B1
 $\text{Prob} = 0.3 \times 0.3 + 2 \times 0.2 \times 0.2$ M1A1
 $= 0.17$ A1
8. (a) $E(X) = \frac{1}{12} \int_0^2 x(8x - x^3) dx$ M1A1
 $= \frac{1}{12} \left[\frac{8}{3} x^3 - \frac{x^5}{5} \right]_0^2$ A1
 $= 1.24$ A1
- (b) (i) $F(x) = \frac{1}{12} \int_0^x (8y - y^3) dy$ M1A1
 $= \frac{1}{12} \left[4y^2 - \frac{y^4}{4} \right]_0^x$ A1
 $= \frac{1}{12} \left(4x^2 - \frac{x^4}{4} \right)$ A1
- (ii) $P(X \geq 1) = 1 - F(1) = 0.6875$ M1A1A1
- (iii) The median satisfies $F(m) = 0.5$, ie M1
 $4m^2 - \frac{m^4}{4} = 6$ leading to the given equation. A1
Solving,
 $m^2 = \frac{16 \pm \sqrt{256 - 96}}{2}$ M1A1
giving $m = 1.29$ A1

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MATHEMATICS S2

Statistics

Specimen Mark Scheme

2005/2006

1. (a) (i) $z = \frac{200 - 195}{5} = 1$ M1A1
 $\Rightarrow P(\text{overflow}) = 0.1587$ A1
- (ii) $\mu = 200 - 2.326 \times 5 = 188.4$ M1A1A1
- (b) T is $N(980, 125)$ M1A1
 $z = \frac{1000 - 980}{\sqrt{125}} = 1.79$ m1A1
 Prob = 0.0367 A1
2. (a) Distribution of T is $\text{Poi}(2.5)$. B1
 Prob = $e^{-2.5} \cdot \frac{2.5^2}{2} = 0.257$ M1A1
- (b) T is now $\text{Poi}(125) \approx N(125, 125)$ M1A1
 $z = \frac{99.5 - 125}{\sqrt{125}} = -2.28$ M1A1A1
 Prob = 0.0113 A1
3. (a) $\bar{x} = 73.5$ M1A1
 95% confidence limits are
 $73.5 \pm 1.96 \sqrt{\frac{16}{10}}$ M1A1A1
 giving [71.0, 76.0]. A1
- (b) A 95% confidence interval is an interval determined by a method which would ensure that the parameter lies within the interval 95% of the time. B2 (Allow B1 if not completely convinced)
4. (a) $P(\pi R^2 > 36\pi) = P(R > 6)$ M1
 $= \frac{(10 - 6)}{(10 - 4)}$ A1
 $= \frac{2}{3}$ A1
- (b) (i) The density of R is $f(r) = 1/6$ (si) B1
 $E(A) = \pi \int_4^{10} r^2 \cdot \frac{1}{6} dr$ M1A1
 $= \frac{\pi}{18} [r^3]_4^{10}$ A1
 $= 52\pi$ A1

$$\begin{aligned}
 \text{(ii)} \quad E(A^2) &= \pi^2 \int_4^{10} r^4 \cdot \frac{1}{6} dr && \text{M1A1} \\
 &= \frac{\pi^2}{30} [r^5]_4^{10} && \text{A1} \\
 &= 3299.2 \pi^2 && \text{A1} \\
 \text{Var}(A) &= 3299.2 \pi^2 - 52^2 \pi^2 \\
 &= 595.2 \pi^2 && \text{A1}
 \end{aligned}$$

5. The appropriate test statistic is

$$\begin{aligned}
 \text{TS} &= \frac{\bar{x} - \bar{y}}{\sigma \sqrt{\frac{1}{m} + \frac{1}{n}}} && \text{M1} \\
 &= \frac{52.6 - 49.8}{5 \sqrt{\frac{1}{10} + \frac{1}{10}}} && \text{A1A1} \\
 &= 1.25 && \text{A1}
 \end{aligned}$$

EITHER

$$\begin{aligned}
 p\text{-value} &= 2 \times 0.1056 \\
 &= 0.2112 && \text{M1A1} \\
 \text{This is greater than } 0.01 &\text{ so accept that concentrations are equal.} && \text{A1}
 \end{aligned}$$

OR

$$\begin{aligned}
 \text{Critical value} &= 2.576 && \text{M1A1} \\
 \text{The calculated value is less than this so accept that concentrations} &&& \\
 \text{are equal.} &&& \text{A1}
 \end{aligned}$$

6. (a) (i) X is $B(20,p)$ (si) B1
 Sig level = $P(X \geq 14 \mid p = 0.5)$ M1A1
 $= 0.0577$ A1
- (ii) We require M1A1
 $P(X \geq 14 \mid p = 0.7) = P(Y \leq 6 \mid p = 0.3)$ A1
 $= 0.608$
- (b) Under H_0 , X is $B(200,0.5) \approx N(100,50)$ M1A1
 $z = \frac{119.5 - 100}{\sqrt{50}}$ m1A1
 $= 2.76$ A1
 $p\text{-value} = 0.00289$ A1
 Strong evidence to support Dafydd's theory. B1

7. (a) (i) $H_0: \mu = 3$ versus $H_1: \mu > 3$ B1
- (ii) In 5 days, number sold Y is Poi(15) under H_0 . B1
 p -value = $P(Y \geq 20)$ M1
 $= 0.1248$ A1
 We cannot conclude that the mean has increased. B1
- (b) Under H_0 the number sold in 100 days is Poi(300) \approx N(300,300) B1B1
- $$z = \frac{329.5 - 300}{\sqrt{300}}$$
- $= 1.70$ M1A1
- p -value = 0.0446 A1
- Significant at the 5% level because $0.0446 < 0.05$. B1B1

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MATHEMATICS S3

Statistics

Specimen Mark Scheme

2005/2006

1. (a) The possible combinations are given in the following table.

Combination	Sum
1 2	3
1 3	4
1 3	4
2 3	5
2 3	5
3 3	6

M1A1 (combs)A1(sum)

The sampling distribution is

Sum	3	4	5	6
Prob	1/6	2/6	2/6	1/6

M1A1

- (b) The possibilities are

Possibility	Sum
1 1	2
1 2	3
1 3	4
1 3	4
2 1	3
2 2	4
2 3	5
2 3	5
3 1	4
3 2	5
3 3	6
3 3	6
3 1	4
3 2	5
3 3	6
3 3	6

M1A2(Poss)A1(sum)
(-1 each error)

The distribution is

Sum	2	3	4	5	6
Prob	1/16	2/16	5/16	4/16	4/16

M1A1

2. (a) $\hat{p} = \frac{930}{1500} = 0.62$ B1
- (b) $SE \approx \sqrt{\frac{0.62 \times 0.38}{1500}}$ M1
 $= 0.0125...$ A1
 The value of p is not known and has to be estimated. B1
- (c) 90% confidence limits are
 $0.62 \pm 1.645 \times 0.0125$ M1A1
 giving [0.60,0.64]. A1
- (d) No we are not. The confidence interval has a level of confidence associated with it and it is not certain. B1B1
3. (a) $H_0: \mu_x = \mu_y$ versus $H_1: \mu_x \neq \mu_y$ B1
- (b) Test statistic = $\frac{2.51 - 2.45}{\sqrt{\frac{0.082}{120} + \frac{0.094}{80}}}$ M1A1A1
 $= 1.39..$ A1
 $p\text{-value} = 0.0823 \times 2$ M1A1
 $= 0.1646$ A1
 We conclude that there is no difference in mean weight. B1
4. (a) (i) $\Sigma x = 30.4$; $\bar{x} = 3.8$ B1B1
 $\Sigma x^2 = 117.14$, B1
 $\hat{\sigma}^2 = \frac{117.14}{7} - \frac{30.4^2}{7 \times 8}$ M1
 $= 0.231(428..)$ A1
- (ii) 99% confidence limits are
 $3.8 \pm 2.365 \times \sqrt{\frac{0.231428..}{8}}$ M1A1A1
 giving [3.4, 4.2]. A1
- (b) $\bar{x} = \frac{431.2}{110} = 3.92$ B1
 $\hat{\sigma}^2 = \frac{1712.1}{109} - \frac{431.2^2}{110 \times 109}$ M1
 $= 0.2$ A1
 95% confidence limits are
 $3.92 \pm 1.96 \times \frac{0.2}{110}$ M1A1
 giving [3.84,4.00] A1

5. (a)
$$b = \frac{8 \times 17882.5 - 220 \times 638.1}{8 \times 7100 - 220^2}$$
 M1A1

$$= 0.319$$
 A1

$$a = \frac{638.1 - 220 \times 0.319}{8}$$
 M1

$$= 71.0$$
 A1
- (b) Test statistic =
$$\frac{0.319 - 0.3}{0.2} \sqrt{7100 - 220^2 / 8}$$
 M1A1A1

$$= 3.08$$
 A1

$$p\text{-value} = 2 \times 0.001$$
 M1

$$= 0.002$$
 A1
 Very strong evidence that the value of β is not 0.3. B1
- (c) (i) Est length =
$$71.0 + 0.319 \times 35$$
 M1

$$= 82.165$$
 A1

$$SE = 0.2 \sqrt{\frac{1}{8} + \frac{(35 - 27.5)^2}{7100 - 220^2 / 8}}$$
 M1

$$= 0.085$$
 A1
- (ii) 95% confidence limits are

$$82.165 \pm 1.96 \times 0.085$$
 M1
 giving [82.0, 82.3] A1
6. (a)
$$E(\hat{\theta}) = 2E(\bar{X})$$
 M1

$$= 2E(X)$$
 A1

$$= 2 \cdot \frac{\theta}{2} = \theta$$
 A1

$$\text{Var}(\hat{\theta}) = 4\text{Var}(\bar{X})$$
 M1

$$= 4 \cdot \frac{\text{Var}(X)}{48}$$
 A1

$$= \frac{\theta^2}{144}$$
 A1

$$SE(\hat{\theta}) = \frac{\theta}{12}$$
 A1
- (b)
$$\hat{\theta} - \theta \approx N\left(0, \frac{\theta^2}{144}\right)$$
 B1
 We require

$$P\left(|\hat{\theta} - \theta| > 0.05\theta\right)$$
 M1

$$z = \frac{0.05\theta}{\theta/12}$$
 m1A1

$$= 0.6$$
 A1
 Prob = $0.2743 \times 2 = 0.5286$ A1