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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

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MARKING SCHEMES

SUMMER 2005

MATHEMATICS
(NEW SPECIFICATION)

WJEC
CBAC

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2005 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1

Solutions and Mark Scheme

1.	(a)	Gradient of $AB = \frac{7+1}{1-5} = -2$	M1 (correct attempt to find gradients)
		Gradient of $CD = \frac{3-7}{8-6} = -2$	Al (both gradients)
		$\therefore$ Lines are parallel	B1 (must involve two equal gradients)
	(b)	Equation of AB is	
		$y - 7 = -2(x-1)$	M1 (use of $y - y_1 = m(x - x_1)$ (o.e.) with appropriate values)
			A1 (give mark here. F.T gradient if 2 Ms have been gained)
		$2x + y - 9 = 0$ (1)	
	(c)	Gradient of $L = \frac{1}{2}$	B1 $\left(-\frac{1}{\text{gradient of } AB}, o.e.\right)$
		Equation of L is	
		$y - 7 = \frac{1}{2}(x-6)$	B1 (F.T. gradient of L if B1 gained in (c) and M2 gained in (a), (b))
		$2y - 14x = x - 6$ $x - 2y + 8 = 0$ (2)	B1 (convincing)
	(d)	Solve (1), (2) $x = 2,\ y = 5$	M1, A1 (C.A.O.) (allow only for algebraic solution)
	(e)	Mid-point of AB has coordinates $\left(\frac{1+5}{2}\right), \left(\frac{7-1}{2}\right)$ i.e. (3, 3)	B1, B1
		$EF = \sqrt{(3-2)^2 + (5-3)^2} = \sqrt{5}\ (\approx 2.24)$	M1 (correct formula) A1 (F.T. coordinates of E and F)
			[14]

2.	(a)	$3\sqrt{5}+4\sqrt{5}-5\sqrt{5}=2\sqrt{5}$	M1(attempt to simplify/one correct answer) A1 (all correct) A1 (F.T. one slip with answer of form $k\sqrt{5}$)
	(b)	$\frac{(6+\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}=\frac{12-6\sqrt{2}+2\sqrt{2}-2}{4-2}$	M1 (correct rationalising)
			A1 (numerator with $(\sqrt{2})^2$ = 2, allow 2 × 6)
			A1 (denominator with no ??)
		$=5-2\sqrt{2}$ (allow $\frac{10-4\sqrt{2}}{2}$)	A1 (F.T. one slip)
			[7]
3.	(a)	(f(1) = 0)	M1 (any method)
		$3+5+a-4=0$ $a=-4$	A1
		<u>Special Case</u> B1 if $a=-4$ assumed	
	(b)	$3x^3+5x^2-4x-4=(x-1)(3x^2+8x+4)$	M1 ($3x^2+ax+b$, a or b correct, any method) A1
		$=(x-1)(3x+2)(x+2)$	A1 (F.T. one slip)
		Roots are 1, $-\frac{2}{3}$, –2	A1 (F.T. one slip)
	(c)	Remainder $=3(-1)^3+5(-1)^2-4(-1)-4$	M1 (any method, division must have $3x^2+2x+a$)
		= 2	A1
			[8]

4.	$(1+2x)^6 = 1+6(2x)+\frac{6.5}{1.2}(2x)^2+\frac{6.5.4}{1.2.3}(2x)^3+\ldots\ldots\ldots\ldots$	M1 (substitution of $2x$, n=6 in $(1+x)^n$)
	$=1+12x+60x^2+160x^3+\ldots\ldots\ldots\ldots$	A1 $(1+12x)$ A1 $(60x^2)$ A1 $(160x^3)$
		[4]
5.	$y + \Delta y = (x+\Delta x)^2 - 7(x+\Delta x) + 2$	B1
	$\Delta y = (x+\Delta x)^2 - 7(x+\Delta x) + 2 - (x^2 - 7x + 2)$	M1
	$= 2x\Delta x + (\Delta x)^2 - 7\Delta x$	A1
	$\frac{\Delta y}{\Delta x} = 2x + \Delta x - 7$	M1 (divide by Δx and let $\Delta x \to 0$. Method must involve $\Delta x \to 0$ and some statement about answer being a limit)
	$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x}$	
	$= 2x - 7$	A1 (award for clear presentation and no abuse of notation)
		[5]
6.	(a) $16.\frac{1}{2\sqrt{x}}, -\frac{32}{x^2}$ (o.e)	B1, B1
	$\frac{dy}{dx} = \frac{8}{2} - \frac{32}{16} = 2$	B1 (C.A.O.)
	(b) Slope of normal = $-\frac{1}{2}$	B1 $\left(\frac{-1}{\text{candidate's } \frac{dy}{dx}}\right)$
	When x=4, $y = 16\times 2 + \frac{32}{4} + 2$	
	$= 42$	B1 (C.A.O.)

Equation is $y - 42 = -\frac{1}{2}(x - 4)$ B1 (F.T. slope if first B1 in (c) gained, and candidate's value of y)

[6]

7. (a) $\left(\frac{dy}{dx} = 0\right)$

$3x^2 - 6x = 0$ B1 $\left(\frac{dy}{dx}\right)$

M1 $\left(\frac{dy}{dx} = 0\right)$

$3x(x - 2) = 0$

$x = 0, 2$ A1 (either root)

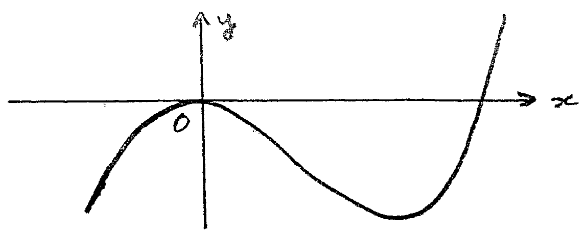
When $x = 0$, $y = 0$; when $x = 2$, $y = -4$ A1 (both, C.A.O.)

$\frac{d^2y}{dx^2} = 6x - 6$ M1 (any method)

$x = 0$, $\frac{d^2y}{dx^2} = -6 < 0$ max. pt A1

$x = 2$, $\frac{d^2y}{dx^2} = 6 > 0$ min. pt A1

(b)



B1 (shape, allow graph not crossing x - axis

B2 (stationary pts on a cubic)

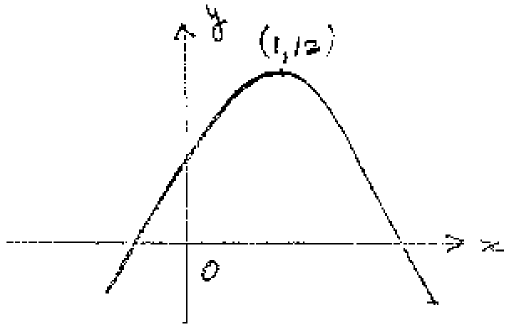
(c) Three solutions for $-4 < k < 0$ B2 (F.T candidate's stationary points)

<u>Special Cases</u> B1 for $k < 0$ or $k > -4$ (or $k < 0$, $k < -4$)

B1 for $-4 \le k \le 0$ [12]

8.	(a)	$x^2 - 16x + 16 = (x - 3)^2 + 7$	B1 ($(x - 3)^2$) B1 (7) (b)
		Least value = 7	B1 (F.T. candidate's *b*, least value must be mentioned)
		No marks for answer derived by calculus.	
	(b)	$x^2 + 2x + 1 \leq 4x + 9$	M1 (correct method of rearranging)
		$x^2 - 2x - 8 \leq 0$	
		$(x - 4)(x + 2) \leq 0$	A1 (fixed points 4, –2 identified, C.A.O.)
		$-2 \leq x \leq 4$	M1 (any method) A1 (F.T. fixed points)
		Allow B1 for $x \geq x \geq 4$	M1 (any method) A1 (F.T. fixed points)
		Allow B1 for $x \geq -2$, or $x \leq 4$ MO AO for $x \leq 4$ and $x \leq -2$	
			[7]
9.	(a)	$2x + c = x^2 + 6x + 7$	B1
		$x^2 + 4x + 7 - c = 0$	M1 (arranging quadratic)
		$4^2 - 4(7 - c) = 0$	M1 (condition for real roots)
		$c = 3$	A1 (C.A.O.)
	(b)	Then $(x + 2)^2 = 0$ so that $x = -2$ (twice)	B1 (F.T. derived *c*)
		Point of contact is $x = -2$, $y = -4 + 3 = 1$	B1 (F.T. derived *c*)
		<u>or</u>	
		The line intersects the curve where gradient = 2	M1 (equate to 2)
		$2x + 6 = 2$	M1 (attempt to diff.) A1 (correct)
		$x = -2$ Point of contact is $x = -2$, $y = -1$ Then $-1 = 4 + c$, $c = 3$	A1 (C.A.O.) B1 (F.T. one slip) B1
			[6]

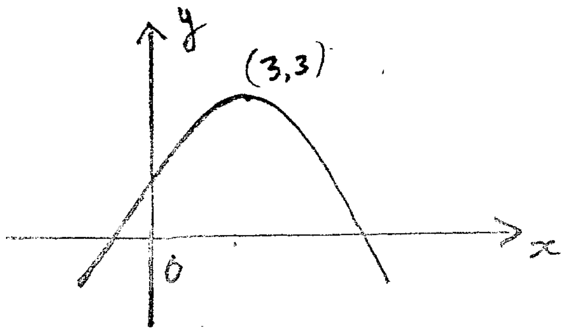
10. (a)



M1 (stationary point coordinates)

A1 (+ve y, –ve x intercepts)

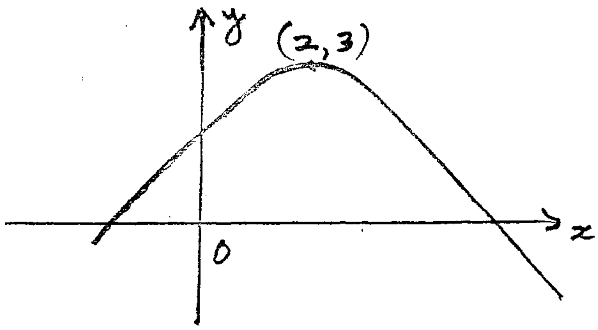
(b)



M1 ($y = 3, x \neq 1$)

A1 (x = 3)

(c)



M1 (stationary point correct)

A1 (+ve y, –ve x intercepts)

[6]

Mathematics C2

Solutions and Mark Scheme

1. $h = 0.2$

Integral $\approx \dfrac{0.2}{2}\left[\begin{array}{l}1+1.41421356+\\ \quad 2(1.01980390+1.07703296+1.16619038+1.28062484)\end{array}\right]$

M1 (correct formula h=0.2)
B1 (4 values)
B1 (2 values)

≈ 1.150 — A1 (F.T. one slip)

Special Case 7 ordinates (six intervals taken)

$h = \dfrac{1}{6}$

Integral $\approx \dfrac{1}{12}\left[\begin{array}{l}1+1.41421356\\ \quad 2(1.01379376+1.05409255+1.1803399\\ \quad +1.20185043+1.30170828)\end{array}\right]$

M1 (correct formula h=$\frac{1}{6}$)
B1 (all values)

≈ 1.149 — A1 (F.T. one slip)

[4]

2. (a) $8\,(1 - \sin^2 x) + 2\sin x - 7 = 0$ — M1 (use of $\sin^2 x + \cos^2 x = 1$, allow $8\cos^2 x + 2\sqrt{1-\cos^2 x} - 7 = 0$)

$8\sin^2 x - 2\sin x - 1 = 0$

$(4\sin x + 1)(2\sin x - 1) = 0$ — M1 (attempt to solve quadratic in $\sin x$, correct formula or $(a\sin x + b)(c\sin x + d)$ with $ac = \sin^2 x$ coefft, bd = constant term)

$\sin x = -\dfrac{1}{4}, \dfrac{1}{2}$ — A1 (C.A.O.)

$x = 194.5^\circ, 345.5^\circ, 30^\circ, 150^\circ$ — B1 (194° – 194.5°)
B1 (345.5° – 346°)
B1 (30°, 150°)

Full F.T for sin $x = a, b$ (one +, –)
2 marks for sin x = –, –
1 mark for sin x = +, +

Subtract 1 for each additional angle in each branch within range. Ignore additional values outside range.

(b) $2x = 45°, 225°$ — B1 (one value)
$x = 22½°, 112½°$ — B1, B1

Subtract 1 for each additional value in the range. Ignore additional values outside range.

[9]

3. (a) n^{th} term $= a + (n - 1)d$ — (must be displayed)

$S_n = a + (a + d) + \text{............} + a - (n - 2)d + a + (n - 1)d$

B1 (at least 3 terms, one at each end)

$S_n = a + (n - 1)d + a + (n - 2)d + \text{.................} + (a + d) + a$

M1 (reverse and add)

$2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + \text{..........} + (2a + (n - 1)d) + (2a + (n - 1)d)$

(n terms)

$= n\left[2a + (n-1)d\right]$

$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ — A1 (convincing)

(b) $a + 6d = 2(a + 2d)$ (1) — M1 ($a + 6d = k(a + 2d)$)
($k = \dfrac{1}{2}$, 2)
A1 ($k = 2$)

$\dfrac{10}{2}[2a + 9d] = 195$ (2) — B1

Solve (1), (2) $d = 3$ — M1 (reasonable attempt to solve equations)
A1 (C.A.O.)

$a = 6$ — A1 (F.T. if either of first M1 or B1 gained)

		$S_{60} = \frac{60}{2}[2\times6 + 59\times3] = 5670$	B1 (F.T. candidate's values if either first M1 or B1 gained)
			[11]
4.	(a)	$a + ar = 6.4, \quad \frac{a}{1-r} = 10$	B1, B1
		Eliminate a $\quad 10(1 - r)(1 + r) = 6.4$	M1 (reasonable attempt to eliminate a)
			A1 (C.A.O.) (any correct expression in r or a)
		$1 - r^2 = 0.64$	
		$r = 0.6$	A1 (F.T. one slip if one B earned)
	(b)	$a = 10 \times (1 - 0.6) = 4$	A1 (F.T. value of r if one B earned)
		$S_{11} = \frac{4}{0.4}\left(1 - (0.6)^{11}\right) = 9.964$	M1 (use of correct formula with derived a, r)
			A1 (C.A.O.)
			[8]
5.	(a)	Centre (4, –2); radius = $\sqrt{4^2 + 2^2 + 5}$ = 5	B1 (centre) M1 (correct attempt to find radius) A1 (radius)
	(b)	(i) $1^2 + 36 - 8 - 24 - 5 = 0$ (so that P lies on C)	B1
		(ii) Gradient of radius = $\frac{-2+6}{4-1} = \frac{4}{3}$	B1
		Gradient of tangent = $-\frac{3}{4}$	M1 $\left(\frac{-1}{\text{candidate's gradient of radius}}\right)$ *o.e.*
			A1 (correct simplified, F.T. one slip)
		Equation is $y + 6 = -\frac{3}{4}(x - 1)$	A1 (F.T. one slip)

Alternative for gradient

$2x + 2y\frac{dy}{dx} - 8 + 4\frac{dy}{dx} = 0$	M1 (attempt to diff.)
	A1 (all correct)
$\frac{dy}{dx} = -\frac{3}{4}$	A1(F.T. one slip) [8]

6. (a)

$x = a^m, y = a^n$ $\log_a x = m, \log_a y = n$	B1 (properties of $a^n = x$ and $log_a x$)
$\frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$	B1 (laws of indices)
$\log_a\left(\frac{x}{y}\right) = m - n = \log_a x - \log_a y$	B1 (convincing)

(b)

(i) $\log 5^{2x+1} = \log 7$	M1 (take logs, one correct)
$(2x + 1) \log 5 = \log 7$	A1 (all correct)
$2x + 1 = \frac{\log 7}{\log 5}$	m1 (attempt to start isolating x)
$x = \frac{\log 7 - \log 5}{2\log 5} \approx 0.1045$	A1 (C.A.O.)
(ii) $\log_{10} = \frac{2 \times 18^2}{36^{3/2}} = \log_{10} 3$	B1 (use of addition law)
	B1 (subtraction law) B1 (power law) B1 (C.A.O., simplified answer)

[11]

7. (a)

$\frac{2x^{7/4}}{7/4} + \frac{7x^{1/2}}{1/2}$ $(+ C)$	B1, B1

(b)

(i) $6 - x^2 = 5$	M1 (equating ys)
$x^2 - 6x + 5 = 0$	
$(x - 1)(x - 5) = 0$	M1 (correct method of solving quadratic equations)
$\therefore x = 1, 5$	
$A\ (1, 5), B\ (5, 5)$	A1

(ii) area = $\int_1^5 (6x - x^2)dx$ — M1 (use of integration to find area)

$-\int_1^5 5dx$ — M1 (substitution of correct areas)

$= \left[3x^2 - \frac{x^3}{3}\right]_1^5 - [5x]_1^5$ — B3 (integration)

$= 75 - \frac{125}{3} - 3 + \frac{1}{3} - 20$ — M1 (use of candidate's limits, any order)

$= \frac{32}{3}$ — A1 (C.A.O.)

[12]

8. (a) $(x - 1)^2 = (x - 3)^2 + x^2 - 2x(x - 3).\frac{1}{2}$ — M1 (correct cosine rule with cos 60°)

A1 (cos 60° = $\frac{1}{2}$)

$x^2 - 2x + 1 = x^2 - 6x + 9 + x^2 - x^2 + 3x$ — M1 (both binomial expansions correct, all terms present)

$x = 8$ — A1 (convincing)

Special Case If $x = 8$ assumed and

either cos 60 = $\frac{1}{2}$ derived

or cos 60° = $\frac{1}{2}$

also assumed and work shown to be consistent,

allow M1 (correct use of cosine rule)

A1

(b) area of triangle = $\frac{1}{2} \times 5 \times 8 \times \sin 60° = 10\sqrt{3}$ — M1 (use of formula)

A1 (C.A.O.)

[6]

9.	(a)	$\frac{1}{2}4^2(\pi-\theta)-\frac{1}{2}.4^2\theta=5$ (o.e.)	B1 (one correctly identified area)
			B1 (correct equation)
		$8\pi-8\theta-8\theta=5$	
		$\theta=\frac{8\pi-5}{16}$	B1 (convincing)
	(b)	Difference = $4(\pi-\theta)-4\theta$	B1 (arc *AOC* = 4θ) B1 (arc *BOC* = $4(\pi-\theta)$)
		Diff = $\frac{5}{2}$ (allow any answer rounding to 2.50)	B1
			[6]

Mathematics C3

Solutions and Mark Scheme

1. $h = 0.25$

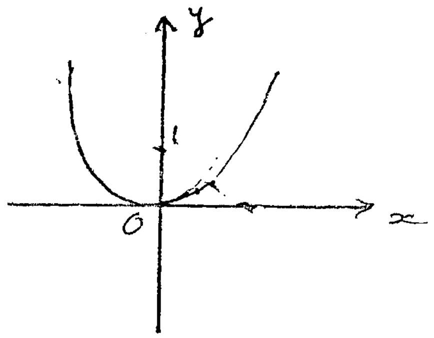
Integral $\approx \frac{0.25}{3}\left[1 + 1.4142136 + 4(1.0004882 + 1.1123420) + 2 \times 1.0155049\right]$

M1 (formula with $h = 0.25$)
B1 (3 values)
B1 (2 values)

$\approx$ 1.075 A1
(accept any answers rounding to 1.075)

[4]

2. (a)



G1 (straight line with +ve intercept, –ve slope)
G2 (correct shape and position of curve)
(Two intersections), two roots E1

(b)

x	$x^4 + 3x - 1$
0	–1
1	3

M1 (attempt to find signs or values)
A1 (correct signs or values, correct conclusion)

Change of sign indicates presence of root

$x_0 = 0.3$, $x_1 = 0.330633$ (accept 0.331) B1 (x_1)

$x_2 = 0.329350$, $x_3 = 0.32941$

$x_4 = 032941$
(accept any answer rounding to 0.32941) B1 (x_4)

Try 0.329405, 0.329415

x	$x^4 + 3x - 1$
0.329405	–0.00001
0.329415	0.00002

M1 (attempt to find signs or values)
A1 (correct signs or values)

(Change of signs indicates presence of root which is 0.32941, correct to five decimal places) A1 (conclusion)

[10]

3.	(a)	$\theta = \dfrac{\pi}{4}$, for example (45º)	B1
		$\cot^2\theta = 1$	
		$1 + \text{cosec}^2\theta = 1 + 2 = 3$	B1
		$(\therefore \cot^2\theta \neq \text{cosec}^2\theta + 1)$	
	(b)	$10(\tan^2\theta + 1) = 11\tan\theta + 16$	M1 ($\sec^2\theta = 1 + \tan^2\theta$)
		$10\tan^2\theta - 11\tan\theta - 6 = 0$	M1 (grouping terms and attempt to solve quadratic ($a\tan\theta + b$)($c\tan\theta + d$) with ac = coefficient of $\tan^2\theta$ bd = constant term, or correct formula)
		$(2\tan\theta - 3)(5\tan\theta + 2) = 0$	
		$\tan\theta = \dfrac{3}{2}$, $\tan\theta = -\dfrac{2}{5}$	A1
		θ = 56.3º, 236.3º, 158.2º, 338.2º (56 – 56.5) (236–236.5) (158 – 158.5) (338 – 338.5)	B1 (56.3º, 236.3º) B1 (158.2º) B1 (338.2º)
		Subtract 1 for each additional value in each branch	
			[8]

4.	(a)	$2x + 2x\dfrac{dy}{dx} + 2y + 6y\dfrac{dy}{dx} = 0$	B1 ($2x\dfrac{dy}{dx} + 2y$) B1 ($6y\dfrac{dy}{dx}$)
		$\dfrac{dy}{dx} = -\dfrac{x+y}{x+3y}$ (o.e)	B1 (all correct)
	(b)	$\dfrac{dy}{dx} = \dfrac{6t}{8t^3} = \dfrac{3}{4t^2}$	M1 $\left(\dfrac{\dot{y}}{\dot{x}}\right)$ A1 (correct)
		$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \dfrac{-\frac{3}{2t^3}}{8t^3}$	M1 (correct formula)
		$= -\dfrac{3}{16t^6}$	A1 (F.T one slip in $\dfrac{dy}{dx}$)
			[7]

Alternative scheme for Q. 4(b)

(b) (i) $y^2 = \frac{9}{2}x$ — M1 (correct cartesian form and attempt to differentiate) (or differentiate $y = \sqrt{\frac{9}{2}}x^{\frac{1}{2}}$)

$2y\frac{dy}{dx} = \frac{9}{2}$

$\frac{dy}{dx} = \frac{9}{4y} = \frac{9}{12t^2}$ — A1 (unsimplified form)

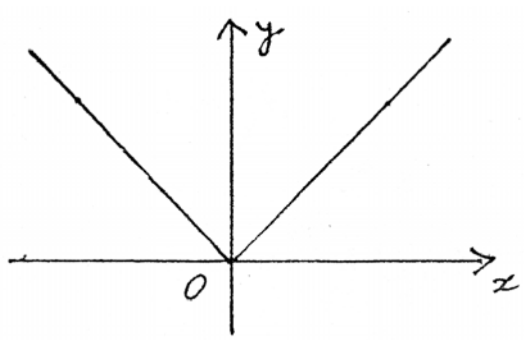
$= \frac{3}{4t^2}$

(ii) $\frac{dy}{dx} = \frac{9}{4y} = \frac{9}{4\sqrt{\frac{9}{2}}x^{\frac{1}{2}}} = \frac{1}{2}\sqrt{\frac{9}{2}}x^{-\frac{1}{2}}$ — M1(correct cartesian and attempt to diff.)

$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{\frac{9}{2}}x^{-\frac{3}{2}} = -\frac{1}{4}\sqrt{\frac{9}{2}}\frac{1}{2^{3/2}t^6}$ — A1 (unsimplified F.T. form; one slip in $\frac{dy}{dx}$)

$= -\frac{3}{16t^6}$

5. (a)



B1 (left hand side)

B1 (right hand side, award only if whole graph above x-axis)

(b) $x = \pm\frac{1}{2}$ — B1 (both)

(c) $x > \frac{1}{3}$, $x < -3$ — B1 ($x > \frac{1}{3}$) M1 ($3x + 4 < -5$) A1

[6]

6.	(a)	(i)	$2e^{2x-5}$	M1 (ke^{2x-5}) A1 ($k = 2$)
		(ii)	$x^2.\frac{1}{x} + 2x\ln x$	M1 (x^2 f(x) + ln$x$ g(x))
				A1 (f(x) = $\frac{1}{x}$, g(x) = $2x$)
			$= x + 2x\ln x$	A1
		(iii)	$4(3x^2 + 2)^3.6x$	M1 ($4(3x^2 + 2)^3$ f(x), f(x) $\neq$ 1) A1 (f(x) = $6x$)
			$= 24x(3x^2 + 2)^3$	A1
	(b)		$\frac{d}{dx}(\tan x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$	M1 $\left(\frac{\cos f(x) - \sin x g(x)}{\cos^2 x}\right)$ A1 (f(x) = cos x, g(x) = –sin x)
			$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$	
			$= \frac{1}{\cos^2 x} = \sec^2 x$	A1 (convincing)
	(c)		$x = \tan y$, $\frac{dx}{dy} = \sec^2 y$	B1
			$\frac{dy}{dx} = \frac{1}{\sec^2 y}$	B1
			$= \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$	B1 (convincing, must see $1 + \tan^2 y$ for $\sec^2 y$)

[14]

<u>Alternative</u>	$1 = \sec^2 y \frac{dy}{dx}$	B1
	$\frac{dy}{dx} = \frac{1}{\sec^2 y}$	B1 ($\frac{dy}{dx}$, subject of formula)
	$= \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$	B1 (convincing)

7. (a) (i) $\frac{1}{3}\ln|3x+7|$ (+ C) M1 ($k\ \ln|3x+7|$, $|\ |$ may be omitted)
A1 ($k = \frac{1}{3}$)

(ii) $\frac{1}{3}e^{3x+2}$ (+ C) M1 ($k\,e^{3x+2}$) A1 ($k = \frac{1}{3}$)

(iii) $-\frac{1}{5(5x+2)^3}$ (+ C) M1 $\left(\frac{k}{(5x+2)^3}, (o.e)k \neq 3\right)$ A1 $\left(k = -\frac{1}{5}\right)$

(b) $\left[-\frac{1}{4}\cos\left(4x+\frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$ M1

$\left(k\cos\left(4x+\frac{\pi}{6}\right), k = -1, -4, \frac{1}{4}, -\frac{1}{4}\right)$

A1 ($k = -\frac{1}{4}$)

$= \left(-\frac{1}{4}\cos\frac{5\pi}{6} + \frac{1}{4}\cos\frac{\pi}{6}\right)$ M1

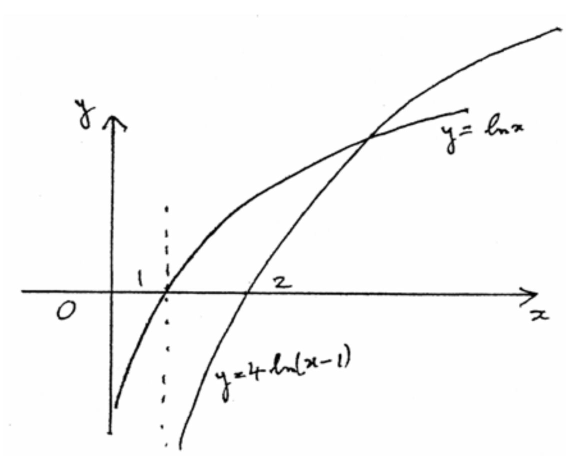
$\left(k\cos\frac{5}{6} - k\cos\frac{\pi}{6}, \text{allowable } ks\right)$

$= \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{1}{4}\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4}$ (≈ 0.433, correct to 3 decimal places) A1 (C.A.O)

[10]

8.



<u>ln$x$</u>
B1 (y asymptote)
B1 (1,0)

<u>4ln(x–1)</u>
B1 (translation to right, necessary to see (2,0))
B1 (intersection of asymptote at x=1)
B1 (intersection in first quadrant)

[5]

9.	Let	$y = \ln(x - 2) + 3$	
		$y - 3 = \ln(x - 2)$	B1 ($y - 3 = \ln(x - 2)$, o.e.)
		$e^{y-3} = x - 2$	M1 (attempt to exponentiate and isolate x)
		$x = e^{y-3} + 2$	A1
		$f^{-1}(x) = e^{x-3} + 2$	A1 (F.T one slip)
			[4]

10.	(a)	Range of f, g $(1, \infty)$, $(7,\infty)$ respectively (o.e.)	B1, B1 (penalise equality once)
	(b)	f(1) = 2 and 2 is not in the domain of g.	B1
	(c)	$fg(x) = (2x - 3)^2 + 1$	M1 (correct composition)
		$(2x - 3)^2 + 1 = 3x^2 - 6x + 17$	
		$x^2 - 6x - 7 = 0$	A1
		$x = -1, 7$	A1 (F.T. one slip)
		$x = 7$ (-1 not in domain of g)	A1 (F.T. one slip)
			[7]

<u>Special Case</u>	$gf(x) = 3x^2 - 6x + 17$	
	gives $x^2 - 6x + 18 = 0$	
	and attempt to solve	B1

Mathematics C4

Solutions and Mark Scheme

1. (a) Let $\dfrac{8x^2+x-5}{(2x-1)^2(x+2)} \equiv \dfrac{A}{2x-1} + \dfrac{B}{(2x-1)^2} + \dfrac{C}{(x+2)}$ M1

$\therefore$ $8x^2 + x - 5 \equiv A(2x-1)(x+2) + B(x+2) + C(2x-1)^2$ M1

$x = \dfrac{1}{2}$ B = –1, $x = -2$, C = 1 A1

Equate coefficients of x^2: 2A + 4C = 8
A = 2 A1

(b) $\displaystyle\int \frac{2}{2x-1}\,dx - \int \frac{1}{(2x-1)^2}\,dx + \int \frac{1}{x+2}\,dx$

$= \ln|2x-1| + \dfrac{1}{2(2x-1)} + \ln|x+2|$ (+ C) B1, B1, B1

(no need for modulus)

[7]

2. $(1-2x)^{-\frac{1}{2}} = 1 + \left(-\dfrac{1}{2}\right)(-2x) + \left(-\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right)\dfrac{1}{2!}(-2x)^2 + \ldots\ldots\ldots$

$= 1 + x + \dfrac{3}{2}x^2 + \ldots\ldots\ldots$ B1 (1 + x) B1 ($\frac{3}{2}x^2$)

Expansion valid for $|x| < \dfrac{1}{2}$ B1

$\left(1-\dfrac{1}{4}\right)^{-\frac{1}{2}} \approx 1 + \dfrac{1}{8} + \dfrac{3}{128} = \dfrac{147}{128}$ B1 (F.T one slip)

$\left(\dfrac{3}{4}\right)^{\frac{1}{2}} = \dfrac{128}{147}$

$\therefore \sqrt{3} \approx \dfrac{256}{147}$ B1 (C.A.O)

[5]

3.	$8x + 3y + 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	B1 ($2y\frac{dy}{dx}$)
		B1 ($3y + 3x\frac{dy}{dx}$)
	$\frac{dy}{dx} = \frac{8x+3y}{2y-3x}$	
	$= -\frac{19}{4}$	B1 (C.A.O.)
	Equation is $y - 1 = -\frac{19}{4}(x-2)$	B1 (F.T one slip)
		[4]
4.	(a) 2sinθ cosθ = cosθ	M1
	cosθ = 0, sinθ = $\frac{1}{2}$	
	θ = 90°, 270°, 30°, 150°	A3 (–1 for each omission) (–1 for each additional value)
	(b) Using R sin(θ + α) with R = $\sqrt{17}$, α = 14.04°	M1 A1 A1
	sin (θ + 14.04°) = $\frac{2}{\sqrt{17}}$	
	θ + 14.04° = 29.02°, 150.98°	B1 (1 value)
	θ = 14.98°, 136.94°	B1, B1
		[10]
5.	Volume = $\pi\int_1^4 \left(\sqrt{x} + \frac{4}{\sqrt{x}}\right)^2 dx$	B1
	$= \pi\int_1^4 \left(x + 8 + \frac{16}{x}\right)dx$	B1
	$= \pi\left[\frac{x^2}{2} + 8x + 16\ln x\right]_1^4$	B1
	$= \pi\left(31\frac{1}{2} + 16\ln 4\right)$	M1
	≈ 168.6	A1 (C.A.O.) [5]

6.	(a)	$\frac{dy}{dx} = \frac{2p}{2} = p$	M1 ($\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$, one correct) A1
		Equation is	
		$y - (p^2 + 3) = p\,(x - 2p - 1)$	M1
		$px - y = p^2 + p - 3$	A1 (convincing)
	(b)	$2p + 3 = p^2 + p - 3$	M1
		$p^2 - p - 6 = 0$	A1
		$p = 3, -2$	A1 (FT one slip)
		Choose $p = -2$ (2^{nd} quadrant)	
		Tangent is $2x + y = 1$	A1 (FT candidate's values) [8]
7.	(a)	$u = 2x - 1$, $du = 2\,dx$	
		when $x = 0$, $u = -1$; $x = 1$, $u = 1$	
		$\int_{-1}^{1} \frac{(u+1)}{2} u^9 \frac{du}{2}$	M1 A1 (no limits required)
		$= \frac{1}{4}\int_{-1}^{1} (u^{10} + u^9)du$	m1
		$= \frac{1}{4}\left[\frac{u^{11}}{11} + \frac{u^{10}}{10}\right]_{-1}^{1}$	A1
		$= \frac{1}{22}$	A1 (F.T. one slip)
	(b)	(i) $\int x\cos 2x dx = \frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}dx$	M1 A1 A1
		$= \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}$ $(+C)$	A1 (F.T. one slip)
		(ii) $\int x\cos^2 x dx = \int x\frac{(1+\cos 2x)}{2}dx$	M1 A1
		$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}$ $(+C)$	A1 (F.T. first result) [12]

8.	(a)	$\frac{dP}{dt} = kP$		B1
	(b)	$\int \frac{dP}{P} = \int k dt$		M1
		$\ln P = kt\ (+C)$		A1
		$t = 0, P = P_0 \quad \ln P_0 = C$		M1
		$\ln P = kt + \ln P_0$		
		$\ln\left(\frac{P}{P_0}\right) = kt$		A1
		$\frac{P}{P_0} = e^{kt}$		
		$P = P_0\, e^{kt}$		A1 (convincing)
	(c)	$1.2\, P_0 = P_0\, e^{2k}$		M1 (attempt to find k)
		$2\, k = \ln 1.2$		
		$k = \frac{1}{2}\ln 1.2$		A1
		$T = \frac{\ln 2}{\frac{1}{2}\ln 1.2} \approx 7.6$		m1 A1 (F.T. one slip)
				[10]
9.	(a)	(i)	**OP** = **OA** + λ**AB**	M1
			$= 5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} + \lambda\,(-12\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})$	B1 (**AB**)
			$\mathbf{r} = 5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} + \lambda\,(-12\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})$	A1 (must have **r**, F.T. **AB**)
		(ii)	Point of intersection:	
			$5 - 12\lambda = -1 + 2\mu$	M1
			$1 + 3\lambda = 7 - 5\mu$	A1
			$\lambda = \frac{1}{3}, \mu = 1$	M1 A1 (CAO)
			Position vector = $\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$	B1 (candidate's parameters)

	(b)	$(3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}).(\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k})$	M1
		$= 3 – 8 + 5 = 0$	
		$\therefore$ Vectors are perpendicular	A1 (value and conclusion)
			[10]
10.	$x^2 – 10x + 25 < 0$		B1
	$(x – 5)^2 < 0$		B1
	$x – 5$ not real		B1
	(x not real)		
	Contradiction		B1
	so $x + \frac{25}{x} \geq 10$		
			[4]

A/AS Level Mathematics - FP1 – June 2005 - Markscheme

1	$\lvert x+1+\mathrm{i}y\rvert = 2\lvert x+\mathrm{i}(y-2)\rvert$	M1
	$(x+1)^2+y^2 = 4[x^2+(y-2)^2]$	M1A1
	$x^2+2x+1+y^2 = 4x^2+4y^2-16y+16$	A1
	$3x^2+3y^2-2x-16y+15=0$	A1
2	$S_n = 4\sum_{r=1}^{n} r^3 - 4\sum_{r=1}^{n} r$	M1
	$= \frac{4n^2(n+1)^2}{4} - \frac{4n(n+1)}{2}$	A1A1
	$= n(n+1)(n^2+n-2)$	M1A1
	$= (n-1)n(n+1)(n+2)$	A1
3	$\mathrm{f}(x+h) - \mathrm{f}(x) = \frac{1}{(x+h)^2+x+h} - \frac{1}{x^2+x}$	M1A1
	$= \frac{x^2+x-[(x+h)^2+x+h]}{[(x+h)^2+x+h](x^2+x)}$	m1
	$= \frac{-h(2x+h+1)}{[(x+h)^2+x+h](x^2+x)}$	A1
	$\mathrm{f}'(x) = \lim_{h\to 0} \frac{-(2x+h+1)}{[(x+h)^2+x+h](x^2+x)}$	M1
	$= -\frac{2x+1}{(x^2+x)^2}$	A1
4	(a) Matrix of translation $= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	B1
	Matrix of reflection $= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1
	T - Matrix $= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1
	$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	A1

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

giving $y + 1 = x$ and $x + 2 = y$ A1

These equations are inconsistent so no fixed points. A1

5 The proposition is true by inspection for $n = 1$. B1

Assume the proposition is true for $n = k$. M1

Consider

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2^k - 1 \\ 0 & 2^k \end{bmatrix}$$ m1

$$= \begin{bmatrix} 1 & 2^k - 1 + 2^k \\ 0 & 2.2^k \end{bmatrix}$$ A1

$$= \begin{bmatrix} 1 & 2^{k+1} - 1 \\ 0 & 2^{k+1} \end{bmatrix}$$ A1

So, if the proposition is true for $n = k$, it is also true for $n = k + 1$. A1

Extra mark for goods presentation A1

6 METHOD 1

If 1 + i is a root, so is 1 - i. B1

So, $x^2 - 2x + 2$ is a factor of the cubic. B1

Using long division,

$$x + 4$$ M1

$$x^2 - 2x + 2 \;)\; x^3 + 2x^2 + \lambda x + \mu\,($$

$$x^3 - 2x^2 + 2x$$ A1

$$4x^2 + (\lambda - 2)x \;+ \mu$$

$$4x^2 - \;\; 8x \;\;\; + \;\; 8$$ A1

It follows that $\lambda - 2 = -8$ so $\lambda = -6$ M1A1

And $\mu = 8$. A1

Since the sum of the roots is –2, the other root is –4. M1A1

METHOD 2

Substituting 1 + i, M1

$$(1+\text{i})^3 + 2(1+\text{i})^2 + \lambda(1+\text{i}) + \mu = 0$$ A1

$$(1 + 3\text{i} + 3\text{i}^2 + \text{i}^3) + 2(1 + 2\text{i} + \text{i}^2) + \lambda(1+\text{i}) + \mu = 0$$ M1A1

$-2 + \lambda + \mu = 0$ and $\lambda + 6 = 0$ A1

$\lambda = -6$ and $\mu = 8$ M1A1

Since 1 + i is a root, so is 1 – i. B1

Since the sum of the roots is –2, the other root is –4. M1A1

5 7 (a) Consider $z = \frac{1}{w}$ giving $x + iy = \frac{1}{u + iv}$

M1

$$= \frac{u - iv}{u^2 + v^2}$$ M1A1

Taking real and imaginary parts, M1

$$x = \frac{u}{u^2 + v^2} \text{ and } y = \frac{-v}{u^2 + v^2}$$ A1

(b) Substituting,

$$\frac{u^2}{(u^2 + v^2)^2} + \frac{v^2}{(u^2 + v^2)^2} = 2$$ M1A1

giving $u^2 + v^2 = 1/2$ A1

6 8 We note that $\alpha + \beta + \gamma = 2$

$\beta\gamma + \gamma\alpha + \alpha\beta = 3$

and $\alpha\beta\gamma = -3$. B1

Consider $\frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$ M1A1

$= -2/3$ A1

Consider $\frac{1}{\gamma\alpha} \cdot \frac{1}{\alpha\beta} + \frac{1}{\alpha\beta} \cdot \frac{1}{\beta\gamma} + \frac{1}{\beta\gamma} \cdot \frac{1}{\gamma\alpha}$ M1

$= \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha^2\beta^2\gamma^2}$ A1

$= 1/3$ A1

Consider $\frac{1}{\beta\gamma} \cdot \frac{1}{\gamma\alpha} \cdot \frac{1}{\alpha\beta} = 1/9$ M1A1

The required equation is

$$x^3 + \frac{2}{3}x^2 + \frac{1}{3}x - \frac{1}{9} = 0$$ M1A1

9 (a) Det = $1(\lambda - 15) - 2(\lambda - 9) + (5 - 3) = -\lambda + 5$ M1A1

Matrix is singular when $\lambda = 5$. A1

(b)(i) Using reduction to echelon form,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ M1A1

Put $z = \alpha$. M1

$y = 2\alpha - 1$ A1

$x = 3 - 5\alpha$ A1

(ii) Det(**A**) = 2 B1

Cofactor matrix $= \begin{bmatrix} -12 & 6 & 2 \\ -1 & 0 & 1 \\ 5 & -2 & -1 \end{bmatrix}$ M1A1

Inverse matrix $= \frac{1}{2}\begin{bmatrix} -12 & -1 & 5 \\ 6 & 0 & -2 \\ 2 & 1 & -1 \end{bmatrix}$ M1A1

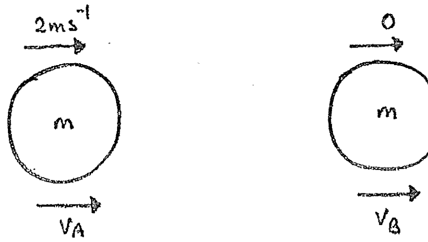
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -12 & -1 & 5 \\ 6 & 0 & -2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$ A1

Mathematics M1 (June 2005) FINAL

Marks Scheme

1.



$e = 0.6$

Conservation of momentum M1

$$2m + 0.m = mV_A + mV_B$$ A1

$$V_A + V_B = 2$$

Restitution M1

$$V_B - V_A = -0.6(0-2)$$ A1

$$-V_A + V_B = 1.2$$

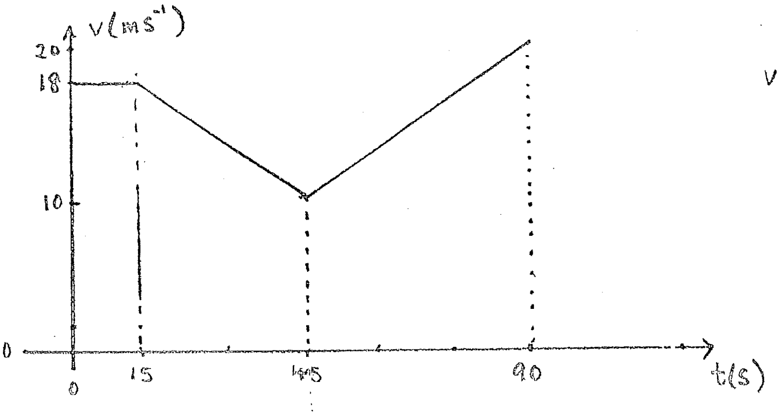
Subtract $2V_A = 0.8$ m1 (dep on both M's)

Speed of first ball $= V_A = \underline{0.4\ ms^{-1}}$

Speed of 2nd ball $= V_B = \underline{1.6\ ms^{-1}}$ both A1

2.

(a)



v-t graph M1

A3

(b) $a = \dfrac{20 - 10}{45}$ M1

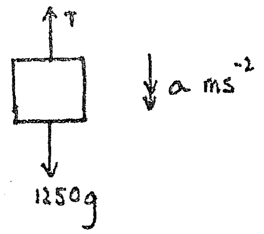
$= \dfrac{2}{9}\ ms^{-2}$ A1

(c) Distance AB $= (18 \times 15) + \frac{1}{2} \times 30\,(18 + 10) + \frac{1}{2} \times 45\,(10 + 20)$ M1 A1

$= 270 + 420 + 675$ B1

$= 1365$ m. cao A1

3.

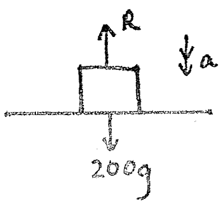


(a) N2L $\quad mg - T = ma$ $\quad$ M1

$1250 \times 9.8 - 11625 = 1250a$ $\quad$ A1

$a = \underline{0.5 \text{ ms}^{-2}}$ $\quad$ cao $\quad$ A1

(b)

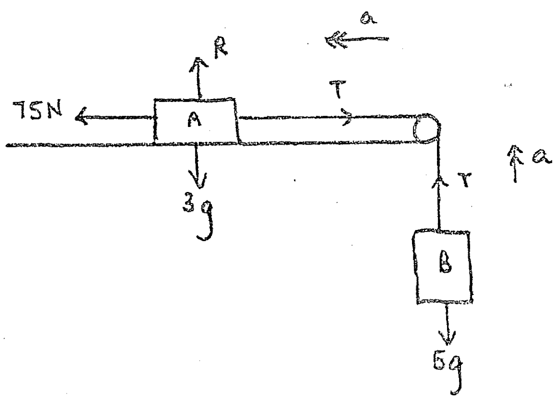


$200g - R = 200a$ $\quad$ B1

$R = 200(9.8 - 0.5)$

$= \underline{1860 \text{ N}}$ $\quad$ ft c's a $\quad$ B1

4.



(a) For A $\quad 75 - T = 3a \quad$ M1 A1

For B $\quad T - 5g = 5a \quad$ M1 A1

(b)

$$75 - T = 3a$$
$$-5g + T = 5a$$

attempt to solve same T, a $\quad$ m1

$$8a = 75 - 5 \times 9.8$$

$$a = \underline{3.25 \text{ ms}^{-2}}$$ A1

$$T = 5 \times 9.8 + 5 \times 3.25$$

$$= 65.25 \text{ N}$$ A1

5.

(a) Using $v^2 = u^2 + 2as$ with $v = 10$, $a = (\pm)9.8$, $s = (\pm)\ 0.4$ M1

$10^2 = u^2 + 2 \times 9.8 \times 0.4$ A1

$u = \underline{9.6\ ms^{-1}}$ cao A1

(b) $10 \times e = 3.5$

$e = \underline{0.35}$ B1

(c) Impulse $|I| = 0.7\,(10 + 3.5)$ M1

$= \underline{9.45\ Ns}$ cao A1

Direction of impulse is upwards B1

(d) Using $v = u + at$ with $u = 3.5$, $a = (\pm)9.8$, $t = 0.5$ M1

$v = 3.5 - 9.8 \times 0.5$ A1

$= -1.4$

Speed is $\underline{1.4\ ms^{-1}}$, moving downwards A1

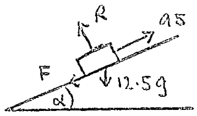
(e) Using $s = ut + \frac{1}{2}at^2$ with $s = 0$, $u = 3.5$, $a = (\pm)9.8$ M1

$0 = 3.5t - 4.9t^2$ A1

$t = \underline{\frac{5}{7}\ s}$ A1

$(= 0.7143)$

6.



$\alpha = 20°$.

(a) $R = 12.5g\cos\alpha$ M1 A1

$F = 0.4 \times 12.5 \times 9.8\cos 20°$ B1

$= \underline{46.0\ (45)\ \text{N}}$ ft R B1√

(b) N2L dim correct, all forces M1

$95 - F - 12.5g\sin\alpha = 12.5a$ A2 (-1 each error)

$a = \underline{0.56\ (4.6)\ \text{m s}^{-2}}$ cao A1

7. Resolve to the right

$X = 56 - 60\sin\alpha$ M1 A1

$(= 20\text{ N})$

Resolve up the page

$Y = 78 + 60\cos\alpha - 27$ M1 A1

$(= 99\text{ N})$

Magnitude $= \sqrt{20^2 + 99^2}$ M1

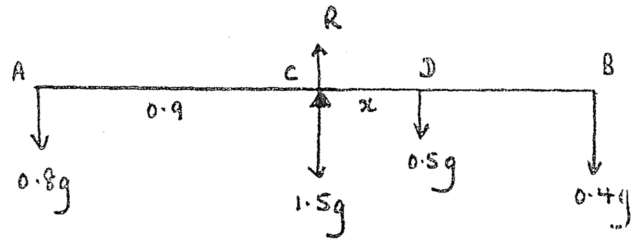
$= \underline{101\text{ N}}$ ft X, Y A1√

required angle $= \tan^{-1}\left(\frac{Y}{X}\right)$ M1

$= \tan^{-1}\left(\frac{99}{20}\right)$

$= \underline{78.58^\circ}$ ft X, Y A1√

8.



(a) $R = 0.8g + 1.5g + 0.5g + 0.4g$ M1

$= \underline{3.2g \text{ N}}$

$= (31.36 \text{ N})$

(b) Moments about C — dim. correct, all forces eqn M1

correct moment B1

$0.8(g) \times 0.9 = 0.5(g)x + 0.4(g) \times 0.9$ A1

$x = \underline{0.72 \text{ m}}$ cao A1

9(a)

	Area	Dist. of c. of m. from AB	Dist. of c. of m. from AD		
ABCD	8×12	4	6		B1
Circle	$\pi(2)^2$	6	3		B1
lamina	$96 - 4\pi$	x	y	areas	B1

Moments about AB — M1

$(96 - 4\pi)x + (4\pi)6 = 96 \times 4$ — ft c.of m. — A1√

$x = 3.69877$

$= \underline{3.7 \text{ cm}}$ — cao — A1

Moments about AD — M1

$(96 - 4\pi)y + (4\pi).3 = 96 \times 6$ — ft c.of m — A1√

$y = 6.45185$

$= \underline{6.5 \text{ cm}}$ — cao — A1

(b)

$\tan\theta = \dfrac{8 - x}{y}$ — M1 m

$= 0.6\dot{6}$

$\theta = \underline{33.69^\circ}$ — ft x, y — A1√

(c) BP $= \underline{3.7\text{cm}}$ — ft x — B1√

New Syllabus

Mathematics M2 (June 2005) FINAL

Marks Scheme

1(a).

$$T = 5g \quad \text{B1}$$

$$T = \frac{\lambda(1.3 - 0.8)}{0.8} \quad \text{M1}$$

$$5 \times 9.8 = \frac{\lambda \times 0.5}{0.8}$$

$$\lambda = \underline{78.4\text{ N}} \quad \text{A1}$$

(b)

$$\text{Elastic Energy} = \frac{1}{2} \times \frac{78.4 \times 0.5^2}{0.8} \quad \text{M1}$$

$$= \underline{12.25\text{ J}} \quad \text{ft } \lambda \quad \text{A1}\checkmark$$

2(a) $v = \int a \, dt$ M1

$= \int 4 - 6t \, dt$

$v = 4t - 3t^2 + c$ A1

$t = 0, \; v = 4, \quad c = 4$ A1

$v = 4t - 3t^2 + 4$

(b) $s = \int v \, dt$ M1

$= \int 4t - 3t^2 + 4 \, dt$

$= 2t^2 - t^3 + 4t + c$ A1

$t = 0, \; s = 0, \quad c = 0$ A1

$s = 2t^2 - t^3 + 4t$

(c) Particle at rest $\Rightarrow v = 0$ M1

$3t^2 - 4t - 4 = 0$

$(3t + 2)(t - 2) = 0$

$t = 2$ A1

At $t = 2$, $\quad s = 2(2)^2 - 2^3 + 4(2)$

$= 8$. A1

(d) When $t = 3$, $\quad v = 4(3) - 3(3)^2 + 4 = -11$

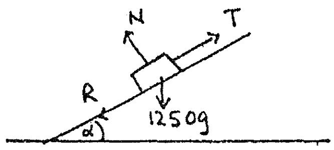
Speed $= 11$ B1

$a = 4 - 6(3)$ B1

$= -14$

a is negative, so velocity is decreasing, ie speed increasing B1

3.



(a) $T = \frac{P}{v}$ used M1

$= \frac{30 \times 1000}{7.5}$ A1

$= 4000$ N

N2L with $a = 0$ dim. correct, all forces M1

$T - R = 1250g \sin\alpha$ A1 A1

$4000 - 1550 = 1250 \times 9.8 \sin\alpha$

$\sin\alpha = 0.2$

$\alpha \approx 11.5^0$ A1

4. K.E at A $= \frac{1}{2} \times 240 \times 2^2$ M1 A1

$= 480\text{J}$

P.E. = mgh used M1

change in P.E $= 240 \times 9.8(30 - 22) = (18816\text{J})$ A1

W D against resistance $= 132 \times 88$ M1 A1

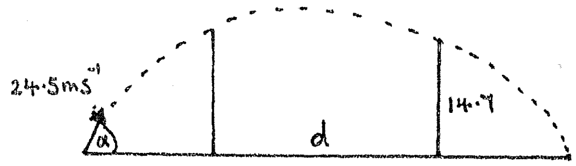
$= 11616\text{ J}$

work-energy principle

$480 + 18816 - 11616 = \frac{1}{2} \times 240\, v^2$ M1

$v = \underline{8 \text{ ms}^{-1}}$ A1

5.



$\sin\alpha = 0.8$

(a)(i) $u_V = 24.5 \sin\alpha$ B1

$= 19.6$

Using $s = ut + \frac{1}{2}at^2$ with $s = 14.7$, $u = 19.6$ (c), $a = \pm 9.8$ M1

$14.7 = 19.6t - 4.9t^2$ A1

$t^2 - 4t + 3 = 0$ m1

$(t-1)(t-3) = 0$

$t = 1, 3$ A1

(ii) $u_H = 24.5 \cos\alpha$ B1

$= 14.7$

$d = 14.7(3-1)$ m1

$= \underline{29.4 \text{ m}}$ ft c's t A1√

(b) Using $v = u + at$ with $u = 19.6$, $a = \pm 9.8$, $t = 0.75$ M1

$v = 19.6 - 9.8 \times 0.75$

$= 12.25$ A1

Speed $= \sqrt{12.25^2 + 14.7^2}$ M1

$= \underline{19.14 \text{ ms}^{-1}}$ ft v A1√

Direction $\theta = \tan^{-1}\left(\frac{12.25}{14.7}\right)$ M1

$= \underline{39.8^\circ}$ ftv A1√

6(a). $\vec{OP} = \underline{r} = (2t-5)\underline{i} + (t-3)\underline{j} + (7-2t)\underline{k}$

$OP^2 = (2t-5)^2 + (t-3)^2 + (7-2t)^2$ M1

$= 4t^2 - 20t + 25$
$+ t^2 - 6t + 9$
$+ 4t^2 - 28t + 49$

$= 9t^2 - 54t + 83$ convincing A1

P is closest to O when OP^2 is minimum M1

$\frac{d}{dt}(OP^2) = 0$ + differentiation m1

$18t - 54 = 0$

$t = \underline{3}$ A1

(b) $\underline{r} = (-5\underline{i} - 3\underline{j} + 7\underline{k}) + (2\underline{i} + \underline{j} - 2\underline{k})t$ any method M1

$\therefore \underline{v} = 2\underline{i} + \underline{j} - 2\underline{k}$ is constant A1

$|v| = \sqrt{2^2+1^2+2^2} = 3$ A1

(c) When P is closest to O,

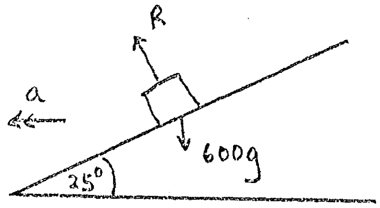
$\vec{OP} = \underline{i} + \underline{k}$ B1

$\vec{OP} \cdot \underline{v} = 2\times1 - 2\times1$ M1

$= 0$

$\therefore$ Direction of velocity of P is $\perp$ to OP A1

7.



(a) Resolve vertically — M1

$R \cos 25^\circ = 600g$ — A1

$R = \dfrac{600 \times 9.8}{\cos 25^\circ}$

$= \underline{6487.86 \text{ N}}$ — A1

(b) N2L horizontally — M1

$R \sin 25^\circ = 600a$ — A1

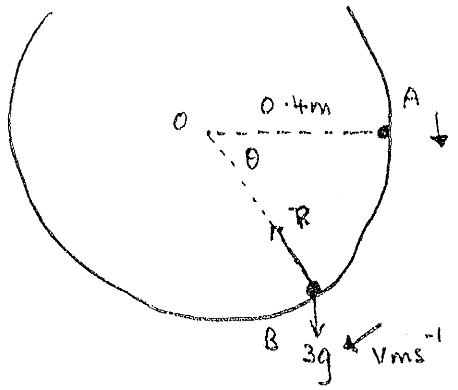
$a = \dfrac{42^2}{r}$ — B1

$\dfrac{600 \times 9.8}{\cos 25^\circ} \times \sin 25^\circ = 600 \times \dfrac{42^2}{r}$

$r = \dfrac{42^2 \cos 25^\circ}{9.8 \sin 25^\circ}$

$= \underline{386.01 \text{ m}}$ cao A1

8.



(a) Conservation of energy M1

$$\frac{1}{2}m.2^2 = \frac{1}{2}mv^2 - mg.0.4\sin\theta$$ A1 A1

$$v^2 = \underline{4 + 7.84\sin\theta}$$ A1

(b) $$R - mg\sin\theta = m\frac{v^2}{0.4}$$ M1 A1 A1

$$R = \frac{m}{0.4}(4 + 7.84\sin\theta) + 9.8m\sin\theta$$

$$= 30 + 58.8\sin\theta + 29.4\sin\theta$$

$$R = \underline{30 + 88.2\sin\theta}$$ A1

(c) $R = 0$ when $\theta = \sin^{-1}\left(-\frac{30}{88.2}\right)$ M1 A1

$$= 199.89^\circ$$

$\therefore$ Greatest θ is $\underline{199.89^\circ}$ A1

Marble leaves the surface of the bowl at 19.89° above the horizontal and moves under the action of gravity like a projectile B1

Mark Scheme for S1 (New Syllabus) June 2005

1. (a) Prob = $\frac{5}{11}\times\frac{4}{10}\times\frac{6}{9}\times 3=\frac{4}{11}$ (0.364) (or $\frac{\binom{5}{2}\times\binom{6}{1}}{\binom{11}{3}}$) M1A1A1

[M1 mult probs, A1 for 3, A1 ans OR M1 ratio of combs, A1 correct, A1 ans]

(b) P(3 boys) = $\frac{5}{11}\times\frac{4}{10}\times\frac{3}{9}$ $\left(\frac{2}{33}\right)$ (or $\binom{5}{3}/\binom{11}{3}$ B1

P(3 girls) = $\frac{6}{11}\times\frac{5}{10}\times\frac{4}{9}$ $\left(\frac{4}{33}\right)$ (or $\binom{6}{3}/\binom{11}{3}$ B1

P(3 same gender) = Sum = $\frac{2}{11}$ (0.182) M1A1

2. (a)(i) P(A) = $\frac{20+30+30}{150}=\frac{8}{15}$ M1A1

(ii) EITHER P($B|A$) = $\frac{20}{20+30+30}=\frac{1}{4}$ M1A1

OR P($B|A$) = $\frac{P(A\cap B)}{P(B)}=\frac{20/150}{80/150}=\frac{1}{4}$

[M0 if A,B assumed independent in numerator]

(iii) EITHER P($A\cup B$) = $\frac{15+20+30+30}{150}=\frac{19}{30}$ M1A1

OR $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{80+35-20}{150}=\frac{19}{30}$

(iv) EITHER P(B) = $\frac{35}{150}\neq P(B|A)$ M1A1

OR $P(A\cap B)(2/15)\neq P(A)P(B)(28/225)$

so not independent. A1

3. (a) ½. B1

(b) $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$ M1A1

(c) ½ , 1/8, 1/32 M1A1

(d) Prob = $\frac{1/2}{1-1/4}=\frac{2}{3}$ M1A1

4. (a)(i) P(X = 10) = $e^{-15}.\frac{15^{10}}{10!}=0.049$ (0.1185 – 0.0699 or 0.9301 – 0.8815)

M1A1

(ii) P(X < 12) = 0.1848 M1A1

(b) We require P($X\geq 21$) = 0.083 M1A1

(c) Reqd number = 25. [Award B1 for 24 or 26] B2

5. (a)(i) P(+) = $0.01 \times 0.9 + 0.99 \times 0.05 = 0.0585$ M1A1A1

[M1 Use of Law of Total Prob, A1 all correct, A1 ans]

(ii) $P(\text{Not D} \mid +) = \dfrac{0.99 \times 0.05}{0.0585} = 0.846$ (cao) B1B1B1

(b) The probability of not having the disease given a + result is too high. B1

6 (a)(i) Binomial B1

(ii) Mean = 10/6, Variance = 50/36 B1B1

(iii) $P(X \le 2) = \left(\dfrac{5}{6}\right)^{10} + \binom{10}{1}\left(\dfrac{5}{6}\right)^{9}\left(\dfrac{1}{6}\right) + \binom{10}{2}\left(\dfrac{5}{6}\right)^{8}\left(\dfrac{1}{6}\right)^{2}$ M1A1

$= 0.775$ A1

(b) Y is $B(81,1/36) \approx Po(81/36 \text{ or } 2.25)$ B1B1

$P(Y = 4) = e^{-2.25} \times \dfrac{2.25^4}{4!}$ M1

$= 0.113$ A1

7. (a) $k(2 + 3 + 4 + 5 + 6) = 1$ giving $k = 1/20$. M1A1

[Accept verification]

(b) Mean $= \dfrac{1}{20}(1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6) = 3.5$ M1A1

$E(X^2) = \dfrac{1}{20}(1 \times 2 + 4 \times 3 + 9 \times 4 + 16 \times 5 + 25 \times 6) = 14$ M1A1

Var $= 14 - 3.5^2 = 1.75$ A1

(c) Possibilities are 2 and 2, 1 and 3 (si) B1

Prob $= \dfrac{1}{400}(3 \times 3 + 2 \times 2 \times 4)$ M1A1

$= 0.0625$ A1

(d) $E(Y) = 2 \times 3.5 + 3 = 10$ M1A1

$Var(Y) = 4 \times 1.75 = 7$ M1A1

8. (a) F(0.8) = 0.8192 B1

F(0.2) = 0.0272 B1

Prob = 0.8192 – 0.0272 = 0.792 (FT on 1 slip) B1

(b) F(0.45) = 0.241, F(0.46) = 0.255 B1B1

so the root of $F(q) = 0.25$ is between 0.45 and 0.46. B1

(c) $f(x) =$ Deriv of $4x^3 - 3x^4$ M1

$= 12x^2(1 - x)$ A1

(d) $E(X) = 12\int_0^1 x.x^2(1 - x)dx$ M1A1

$= 12\left[\dfrac{x^4}{4} - \dfrac{x^5}{5}\right]_0^1$ A1

$= 0.6$ A1

Mark Scheme for A/AS level Mathematics - S2 (New Syllabus) – June 2005

1. (a) $\bar{x} = \frac{882}{12} = 73.5$ — B1

95% confidence limits are

$73.5 \pm 1.96\sqrt{\frac{16}{12}}$ — B1B1B1

giving [71.2,75.8]. — B1

(b) Her belief is supported because 75 is inside the interval. — B1

2. (a)(i) Prob $= e^{-5}.\frac{5^6}{6!} = 0.146$ — M1A1

[Or using tables 0.7622 – 0.6160 or 0.3840 – 0.2378]

(ii) Prob $= 0.146^3 = 0.0031$ — M1A1

(iii) T is Poi(15) — M1

Prob $= e^{-15}.\frac{15^{18}}{18!} = 0.0706$ — m1A1

[Or using tables 0.8195 – 0.7489 or 0.2511 – 0.1805]

(b) T is Poi(260) ≈ N(260,260) — B1

$z = \frac{240.5 - 260}{\sqrt{260}} = -1.21$ — M1A1A1

Prob = 0.8869 — A1

3. (a) $z_1 = \frac{79-70}{6} = 1.5,\ z_2 = \frac{67-70}{6} = -0.5$ — M1A1A1

Prob = 0.93319 – (1 – 0.69146) or 1 – 0.3085 – 0.0668 — M1

= 0. 625 — A1

(b) Mean = 2 × 50 – 70 = 30 — B1

Var = 4 × 25 + 36 = 136 — B1

Prob = $P(X > 2Y) = P(2Y - X) < 0$ — M1

$z = \frac{0-30}{\sqrt{136}} = -2.57$ — A1A1

Prob = 0.00508 — A1

(c) E(Total) = 3 × 70 + 6 × 50 = 510 — B1

Var(Tot) = 3 × 36 + 6 × 25 = 258 — B1

Total is N(510,258) — M1

$z = \frac{500-510}{\sqrt{258}} = -0.62$ — A1A1

Prob = 0.7324 — A1

4 (a) $H_0 : \mu = 1.5$ versus $H_1 : \mu < 1.5$ B1

(b)(i) The critical region. B1

(ii) $SL = P(\text{Accept } H_1 | H_0)$ B1

Under H_0, X is $P(45) \approx N(45,45)$. M1A1

$$z = \frac{35.5 - 45}{\sqrt{45}} = -1.42$$ A1A1

$SL = 0.0778$ A1

5 (a)(i) $H_0 : p = 0.45$ versus $H_1 : p < 0.45$ B1

(ii) Under H_0, X (no. germinating) is $B(50,0.45)$. M1

$p\text{-value} = P(X \le 18 \mid p = 0.45)$ m1

$= 0.1273$ A1

(iii) We find that, when $p = 0.45$,

$P(X \le 14) = 0.0104$ and $P(X \le 13) = 0.0045$ M1

So Max $X = 13$ A1

(b) Under H_0, X is now $B(500,0.45) \approx N(225,123.75)$ M1A1

$$z = \frac{202.5 - 225}{\sqrt{123.75}}$$ A1A1

$= -2.02$ A1

$p\text{-value} = 0.0217$ A1

We conclude at the 5% level that the proportion germinating is less than 45%. B1

6 (a) $H_0 : \mu_G = \mu_B$ versus $H_1 : \mu_G \ne \mu_B$ B1

(b) $\Sigma g = 105.1$, $\Sigma b = 86.7$ or $\bar{g} = 13.1375, \bar{b} = 14.45$ B1

The appropriate test statistic is

$$TS = \frac{\bar{g} - \bar{b}}{\sigma\sqrt{\frac{1}{m} + \frac{1}{n}}}$$ M1

$$= \frac{105.1/8 - 86.7/6}{1.5\sqrt{\frac{1}{8} + \frac{1}{6}}}$$ A1A1

$= -1.62$ A1

tabular value = 0.0526 A1

p-value = 0.1052 B1

(c) Insufficient evidence to conclude that the means are different. B1

7 (a) The density function is

$f(\theta) = 3/\pi$ for $0 < \theta < \pi/3$ B1

$= 0$ otherwise B1

(b) $H = 2\sin\theta$ B1

(i) $P(H \le 1) = P(\sin\theta \le 1/\sqrt{2})$ M1

$= P(\theta \le \pi/4)$ A1

$= \dfrac{\pi/4}{\pi/3} = 0.75$ A1

(ii) $E(H) = \dfrac{3}{\pi}\int_0^{\pi/3} 2\sin\theta \, d\theta$ M1A1

$= \dfrac{3}{\pi}\left[-2\cos\theta\right]_0^{\pi/3}$ A1

$= \dfrac{3}{\pi}$ A1

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