MS3
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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

**General Certificate of Education**
**Advanced Subsidiary/Advanced**

**Tystysgrif Addysg Gyffredinol**
**Uwch Gyfrannol/Uwch**

## MARKING SCHEMES

## SUMMER 2006

# MATHEMATICS
# C1-C4 and FP1-FP3

WJEC
CBAC

## INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

## MATHEMATICS C1

| | | | |
|---|---|---|---|
| **1.** | (*a*) | Gradient of $AC(BD) = \dfrac{\text{increase in } y}{\text{increase in } x}$ | M1 |
| | | Gradient $AC = 2$ | A1 |
| | | Gradient $BD = -½$ | A1 |
| | | Gradient $AC \times$ Gradient $BD = -1$ | M1 |
| | | $\therefore$ $AC$ and $BD$ are perpendicular | A1 |
| | (*b*) | A correct method for finding the equation of $AC(BD)$ | M1 |
| | | Equation of $AC$ : $y - 2 = 2(x - 3)$ (or equivalent) | |
| | | (f.t. candidate's gradient of $AC$) | A1 |
| | | Equation of $AC$ : $2x - y - 4 = 0$ (convincing) | A1 |
| | | Equation of $BD$ : $y - 3 = -½(x + 4)$ (or equivalent) | |
| | | (f.t. candidate's gradient of $BD$) | A1 |
| | (*c*) | An attempt to solve equations of $AC$ and $BD$ simultaneously | M1 |
| | | $x = 2, y = 0$ (c.a.o.) | A1 |
| | (*d*) | A correct method for finding the length of $AE$ | M1 |
| | | $AE = \sqrt{5}$ | A1 |
| **2.** | (*a*) | $\dfrac{5-\sqrt{3}}{\sqrt{3}+1} = \dfrac{(5-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$ | M1 |
| | | Numerator: $5\sqrt{3} - 5 - 3 + \sqrt{3}$ | A1 |
| | | Denominator: $3 - 1$ | A1 |
| | | $\dfrac{5-\sqrt{3}}{\sqrt{3}+1} = 3\sqrt{3} - 4$ | A1 |
| | | **Special case**<br>If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $a + \sqrt{3}b$ | |
| | (*b*) | Removing brackets | M1 |
| | | $\sqrt{12} = 2 \times \sqrt{3}$ | B1 |
| | | $\sqrt{12} \times \sqrt{3} = 6$ | B1 |
| | | $(2 + \sqrt{3})(4 - \sqrt{12}) = 2$ (c.a.o.) | A1 |

| | | | | |
|---|---|---|---|---|
| **3.** | (*a*) | An attempt to find $\frac{dy}{dx}$ | | M1 |
| | | $\frac{dy}{dx} = 2x - 4$ | | A1 |
| | | Value of $\frac{dy}{dx}$ at $A = -2$ | (f.t. candidate's $\frac{dy}{dx}$) | A1 |
| | | Equation of tangent at A: $y - 4 = -2(x - 1)$ | (or equivalent)<br>(f.t. one error) | A1 |
| | (*b*) | Gradient of normal × Gradient of tangent = –1 | | M1 |
| | | Equation of normal at A: $y - 4 = \frac{1}{2}(x - 1)$ | (or equivalent)<br>(f.t. candidate's numerical value for $\frac{dy}{dx}$) | A1 |
| **4.** | (*a*) | An expression for $b^2 - 4ac$, with $b = \pm 4$, and at least one of *a* or *c* correct | | M1 |
| | | $b^2 - 4ac = 4^2 - 4k(k - 3)$ | | A1 |
| | | $b^2 - 4ac = 4(k - 4)(k + 1)$ | | A1 |
| | | Putting $b^2 - 4ac = 0$ | | m1 |
| | | $k = -1, 4$ | (f.t. one slip) | A1 |
| | (*b*) | $a = 4$ | | B1 |
| | | $b = -14$ | | B1 |
| | | Least value = –14 | (f.t. candidate's *b*) | B1 |
| **5.** | (*a*) | Use of $f(2) = -20$ | | M1 |
| | | $8p - 4 + 2q - 6 = -20$ | | A1 |
| | | Use of $f(3) = 0$ | | M1 |
| | | $27p - 9 + 3q - 6 = 0$ | | A1 |
| | | Solving simultaneous equations for *p* and *q* | | M1 |
| | | $p = 2, q = -13$ | (c.a.o.) | A1 |
| | | **Special case assuming *p* = 2** | | |
| | | Use of one of the above equations to find *q* | | M1 |
| | | $q = -13$ | | A1 |
| | | Use of other equation to verify $q = -13$ | | A1 |
| | (*b*) | Dividing $f(x)$ by $(x - 3)$ and getting coefficient of $x^2$ to be 2 | | M1 |
| | | Remaining factor = $2x^2 + ax + b$ with one of *a*, *b* correct | | A1 |
| | | $f(x) = (x - 3)(2x + 1)(x + 2)$ | (c.a.o.) | A1 |

**6.** (*a*) $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ B1

An attempt to substitute $3x$ for $a$ and $\pm\frac{1}{3}$ for $b$ in r. h. s. of above expansion M1

Required expression = $81x^4 - 36x^2 + 6 - \frac{4}{9x^2} + \frac{1}{81x^4}$

(3 terms correct) A1

(all terms correct) A2

(f.t. one slip in coefficients of $(a + b)^4$ )

(*b*) Either: $\frac{n(n-1)}{2} \times 2^k = 40 (k = 1,2)$

Or: ${}^nC_2 \times 2^2 = 40$ M1

$n = 5$ A1

**7.** (*a*) $y + \delta y = (x + \delta x)^2 - 3(x + \delta x) + 4$ B1

Subtracting $y$ from above to find $\delta y$ M1

$\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$ A1

Dividing by $\delta x$, letting $\delta x \to 0$ and referring to limiting value of $\frac{\delta y}{\delta x}$ M1

$\frac{dy}{dx} = 2x - 3$ A1

(*b*) Required derivative $= -4x^{-3} + \frac{7x^{-\frac{1}{2}}}{2}$ B1, B1

**8.** (*a*) An attempt to collect like terms across the inequality M1

$x > -\frac{7}{6}$ A1

(*b*) An attempt to remove brackets M1

$x^2 + 6x + 8 < 0$ A1

Graph crosses $x$-axis at $x = -4$, $x = -2$ B1

Either: $-4 < x < -2$

Or: $-4 < x$ and $x < -2$

Or: $(-4, -2)$ B1

| | | | |
|---|---|---|---|
| **9.** | ($a$) | Translation along $y$-axis so that stationary point is $(0, a)$, $a = 0, -8$ | M1 |
| | | Correct translation and stationary point at (0, 0) | A1 |
| | ($b$) | Translation of 2 units to left along $x$-axis | M1 |
| | | Stationary point is (–2, –4) | A1 |
| | | Points of intersection with $x$-axis are (–4, 0) and (0, 0) | A1 |
| | | **Special case** | |
| | | Translation of 2 units to right along $x$-axis with correct labelling | B1 |

| | | |
|---|---|---|
| **10.** | $\frac{dy}{dx} = 3x^2 - 6x - 9$ | B1 |
| | Putting derived $\frac{dy}{dx} = 0$ | M1 |
| | $x = 3, -1$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) | A1 |
| | Stationary points are (–1, 7) and (3, –25) (both correct) (c.a.o) | A1 |
| | A correct method for finding nature of stationary points | M1 |
| | (–1, 7) is a maximum point (f.t. candidate's derived values) | A1 |
| | (3, –25) is a minimum point (f.t. candidate's derived values) | A1 |

# MATHEMATICS C2

**1.** $h = 0{\cdot}1$

M1 (correct formula $h = 0{\cdot}1$)

Integral $\approx \frac{0\cdot 1}{2}$ [1 + 1·012719 + 2(1·0000500 + 1·0007997 — B1 (3 values)
+ 1·0040418)] — B1 (2 values)

$\approx 0{\cdot}401$ — A1 (F.T. one slip)

<u>S. Case</u> $h = 0{\cdot}08$

Integral $\approx \frac{0\cdot 08}{2}$ [1 + 1·012719 + 2 (1·0000205 + 1·0003276 + 1·0016575 + 1·0052292)] — M1 (correct formula $h = 0{\cdot}08$)

$\approx 0{\cdot}401$ — B1 (all values)
A1 (F.T. one slip)

**4**

**2.** *(a)* $x = 158{\cdot}2°, 338{\cdot}2°$ — B1, B1

*(b)* $3x = 60°, 300°, 420°, 660°$ — B1 (any value)

$x = 20°, 100°, 140°$ — B1, B1, B1

*(c)* $2(1 - \sin^2\theta) + 3\sin\theta = 0$ — M1 (correct use of $\cos^2\theta = 1 - \sin^2\theta$)

$2\sin^2\theta - 3\sin\theta - 1 = 0$ — M1 (attempt to solve quad in $\sin\theta$ correct formula or

$(2\sin\theta + 1)(\sin\theta - 2) = 0$ — $(a\cos\theta + b)(c\sin\theta + d)$ with $ac$ = coefft. of $\sin^2\theta$ $bd$ = constant term)

$\sin\theta = -\frac{1}{2}, 2$ — A1

$\theta = 210°, 330°$ — B1 (210°) B1 (330°)

**11**

**3.** *(a)* Area $= \frac{1}{2}x \times 8\sin 150°$ — B1

$\frac{1}{2}x \times 8 \times \sin 150° = 10$ — B1 (correct equation)

$x = \frac{10}{4\sin 150°} = 5$ — B1 (C.A.O.)

| | | | |
|---|---|---|---|
| *(b)* | $BC^2 = 5^2 + 8^2 + 2.5.8\cos 30°$ (o.e.) | B1 | |
| | $= 25 + 64 + 68{\cdot}29$ | B1 | |
| | $BC \approx 12{\cdot}58$ | B1 | |
| | | | **6** |

| | | | | |
|---|---|---|---|---|
| **4.** | *(a)* | $S_n = a + a + d + ... + a + (n-2)d + a(n-1)d$ | B1 (at least 3 terms one at each end) | |
| | | $S_n = a + (n-1)d + a + (n-2)d + ... + a + d + a$ | | |
| | | $2S_n = 2a + (n-1)d + 2a + (n-1)d + ...$ | M1 | |
| | | $+ 2a + (n-1)d + 2a + (n-1)d$ | | |
| | | $= n[2a + (n-1)d]$ | | |
| | | $\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$ | A1 (convincing) | |
| | *(b)* | (i) $\frac{20}{2}[2a + 19d] = 540$ | B1 | |
| | | $\frac{30}{2}[2a + 29d] = 1260$ | B1 | |
| | | $2a + 19d = 54$ (1) | | |
| | | $2a + 29d = 84$ (2) | | |
| | | Solve (1), (2), $d = 3$ | M1 (reasonable attempt to solve equations) | |
| | | $a = -\frac{3}{2}$ | A1 (both) C.A.O. | |
| | | (ii) $50^{\text{th}}$ term $= -\frac{3}{2} + (n-1)3$ ($n = 50$) | M1 (correct ) | |
| | | $= 145{\cdot}5$ | A1 (F.T. derived values) | |
| | | | | **9** |

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|---|---|---|---|
| **5.** | *(a)* | $ar = 9\,ar^3$ | M1 ($ar = kar^3$, $k = 9$, $\frac{1}{9}$) |
| | | | A1 (correct) |
| | | $1 = 9r^2$ | A1 (F.T. value of $k$) |
| | | $r = \pm\frac{1}{3}$ | A1 (F.T. value of $k$, $r = \pm 3$) |

| | | | |
|---|---|---|---|
| | *(b)* | $\dfrac{a}{1-\frac{1}{3}} = 12$ | M1 (use of correct formula) |
| | | $a = 8$ | A1 (F.T. derived *r*) |
| | | Third term = $8 \times \left(\dfrac{1}{3}\right)^2 = \dfrac{8}{9}$ | (F.T. *r*) Al |
| | | | **7** |
| **6.** | | $3x^{\frac{4}{3}} + \dfrac{3}{2}x^{-2} + 5x(+C)$ | B1, B1, B1 |
| | | | **3** |
| **7.** | *(a)* | $7 + 2x - x^2 = x + 1$ | M1 (equating *y*s) |
| | | $x^2 - x - 6 = 0$ | M1 (correct attempt to solve quad) |
| | | $(x - 3)(x + 2) = 0$ | |
| | | $x = 3, -2$ | A1 |
| | | $B\ (3, 4)$ | A1 |
| | *(b)* | Area $= \displaystyle\int_0^3 (7 + 2x - x^2)dx$ | M1 (use of integration to find areas) |
| | | $\displaystyle - \int_0^3 (x+1)dx$ | m1 |
| | | $= \displaystyle\int_0^3 (6 + x - x^2)dx$ | B1 (simplified) |
| | | $= \left[6x + \dfrac{x^2}{2} - \dfrac{x^3}{3}\right]_0^3$ | B3 (3 correct integrations) |
| | | $= 18 + \dfrac{9}{2} - 9 - (0 + 0 - 0)$ | M1 (use of limits) |
| | | $= \dfrac{27}{2}$ | A1 (C.A.O.) |
| | | | **12** |

**8.** *(a)* Let $\log_a x = p$ — B1 (props of logs)

$\therefore x = a^p$

$x^n = (a^p)^n = a^{pn}$ — B1 (laws of indices)

$\log_a x^n = pn = n \log_a x$ — B1 (convincing)

*(b)* $\ln 5^{3x+1} = ln6$ — M1 (taking logs)

$(3x + 1) \ln 5 = ln6$ — A1 (correct)

$3x \ln 5 = \ln 6 - \ln 5$

$\therefore x = \dfrac{ln6 - ln5}{3ln5}$ (o.e.) — m1 (reasonable attempt to isolate $x$)

$\approx 0{\cdot}0378$ — A1 (C.A.O.)

**7**

**9.** *(a)* Centre $(-1, 4)$ — B1

Radius = $\sqrt{1^2 + 4^2 - 8} = 3$ — B1 (use of formula or std form)

B1 (answer)

*(b)*


$DP^2 = 29$ (o.e.) — B1 (F.T. coords of centre)

$PT^2 = DP^2 - (\text{radius})^2$ — M1 (use of Pythagoras)

$= 29 - 9$

$= 20$

$PT = \sqrt{20}$ — A1 (convincing)

*(c)* Equation of circle is — M1 (use of $x^2 + y^2 + 2gx + 2fy + c = 0$ or $(x - 4)^2 + (y - 6)^2$ = any +ve no)

$(x - 4)^2 + (y - 6)^2 = 20$

or $x^2 + y^2 - 8x - 12y + 32 = 0$ — A1 (either)

**8**

**10.** *(a)* $x = 2 \times 4 + 4\theta = 8 + 4\theta$ — B1

$A = \frac{1}{2} \times 4^2\theta = 8\theta$ — B1

$8 + 4\theta = 3 \times 8\theta$ — B1 (correct equation)

$20\theta = 8,\ \theta = 0{\cdot}4$ — B1 (convincing)

*(b)* Area = $\frac{1}{2} \times 4^2 \times 0{\cdot}4 - \frac{1}{2} \times 4^2 \times \sin 0{\cdot}4$ — B1 (sector)

B1 ($\Delta$)

M1 (sector – $\Delta$)

$\approx 0{\cdot}085$ — A1 (C.A.O.)

**8**

# MATHEMATICS C3

| | | | | |
|---|---|---|---|---|
| **1.** | | $h = 0{\cdot}25$ | M1 ($h = 0{\cdot}25$ correct formula) | |
| | | Integral $\approx \frac{0.25}{3}$ [0 + 0·8325546 + 4(0·4723807 + 0·7480747) | B1 (3 values) | |
| | | + 2(0·6367614)] | B1 (2 values) | |
| | | $\approx 0{\cdot}582$ | A1 (F.T. one slip) | |
| | | | | **4** |
| **2.** | *(a)* | $a = b = 45^{\circ}$, for example | B1 (choice of values) | |
| | | $\cos(a+b) = 0$ | | |
| | | $\cos a + \cos b = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \approx 1\cdot 41$ | B1 (for correct demonstration) | |
| | | $(\therefore \cos(a+b) \neq \cos a + \cos b)$ | | |
| | | [other cases, $\cos 1^{\circ} + \cos 2^{\circ} = 0{\cdot}999848 + 0{\cdot}999391$ $\approx 1{\cdot}999$ | | |
| | | $\cos 3^{\circ} \approx 0{\cdot}999$ | | |
| | | $\cos 2^{\circ} + \cos 3^{\circ} \approx 0{\cdot}9994 + 0{\cdot}9986 = 1{\cdot}998$ | | |
| | | $\cos 5^{\circ} = 0{\cdot}9962$] | | |
| | *(b)* | $7 - (1 + \tan^2\theta) = \tan^2\theta + \tan\theta$ | M1 (substitution of $\sec^2\theta = 1 + \tan^2\theta$) | |
| | | $2\tan^2\theta + \tan\theta \;\; 6 = 0$ | M1 (attempt to solve quad | |
| | | | $(a\tan\theta + b)(c\tan\theta + d)$ | |
| | | $(2\tan\theta - 3)(\tan\theta + 2) = 0$ | with $ac$ = coefficient of $\tan^2\theta$ | |
| | | | $bd$ = constant term, | |
| | | $\tan\theta = \frac{3}{2}, -2$ | or formula) | |
| | | | A1 | |
| | | $\theta = 56{\cdot}3^{\circ}, 236{\cdot}3^{\circ}, 116{\cdot}6^{\circ}, 296{\cdot}6^{\circ}$ | B1 ($56{\cdot}3^{\circ}, 236{\cdot}3^{\circ}$) | |
| | | | B1 ($116{\cdot}6^{\circ}, 296{\cdot}6^{\circ}$) | |
| | | Full F.T. for $\tan\theta = t$, | | |
| | | 2 marks for $\tan\theta = -, -$ | | |
| | | 1 mark for $\tan\theta = +, +$ | | |
| | | | | **8** |

| | | | |
|---|---|---|---|
| **3.** | *(a)* | $\frac{dy}{dx} = \frac{2\cos 2t}{-\sin t}$ | M1 (attempt to use $\frac{dy}{dx} = \frac{\dot{y}}{x}$), |
| | | | B1 ($-\sin t$) |
| | | | B1 ($k\cos 2t$, $k = 1, 2, -2, \frac{1}{2}$) |
| | | | A1 $\left(\frac{2\cos 2t}{-\sin t}, \text{C.A.O.}\right)$ |
| | *(b)* | $4x^3 + 2x^2\frac{dy}{dx} + 4xy + 2y\frac{dy}{dx} = 0$ | B1 ($2x^2 + 4xy$) |
| | | | B1 ($2y\frac{dy}{dx}$) |
| | | | B1 ($4x^3$, 0) |
| | | $\frac{dy}{dx} = \frac{(4x^3 + 4xy)}{2x^2 + 2y}$ | B1 (C.A.O.) |

**8**

| | | | |
|---|---|---|---|
| **4.** | *(a)* | (i) $\left[\frac{e^{2x}}{2} - x\right]_0^a = \frac{e^{2a}}{2} - a - \frac{1}{2}$ | M1 ($ke^{2x}$, $k = \frac{1}{2}, 1$) |
| | | | A1 $\left(\frac{e^{2x}}{2} - x\right)$ |
| | | | A1 (F.T. one slip (in $k$)) |
| | | (ii) $\frac{e^{2a}}{2} - a - \frac{1}{2} = \frac{1}{2}(9 - a)$ | |
| | | $e^{2a} - 2a - 1 = 9 - a$ | |
| | | $e^{2a} - a - 10 = 0$ | B1 (convincing) |

*(b)*

| a | f(a) | | |
|---|---|---|---|
| 1 | –3·61 | change of sign | M1 (attempt to find values or signs) |
| 2 | 42·6 | indicates presence | |
| | | of root (between 1 and 2) | A1 (correct values or signs and conclusion) |

$a_0 = 1{\cdot}2$, $a_1 = 1{\cdot}2079569$, $a_2 = 1{\cdot}20831198$, $a_3 = 1{\cdot}2083278$ — B1 ($a_1$)
$a_4 \approx 1{\cdot}20833$ (1·2083285) — B1 ($a_4$ to 5 places, C.A.O.)

Try 1·208324, 1·208335

| a | f(a) | |
|---|---|---|
| 1·208325 | –0·00008 | M1 (attempt to find values or signs) |
| 1·208335 | 0·00014 | A1 (correct values or signs) |

Change of sign indicates root is 1·20833
(correct to 5 decimal places) — A1

**11**

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|---|---|---|---|---|
| **5.** | *(a)* | (i) | $\frac{1}{1+(4x)^2}\times 4\left(=\frac{4}{1+16x^2}\right)$ | M1 $\left(\frac{k}{1+(4x)^2}\ k=1,4\right)$ |
| | | | (Allow M1 for $\frac{4}{1+4x^2}$ ) | A1 ($k = 4$) |
| | | (ii) | $\frac{1}{1+x^2}\times 2x=\frac{2x}{1+x^2}$ | M1 $\left(\frac{f(x)}{1+x^2},\ f(x)\neq 1\right)$<br>A1 ($f(x) = 2x$) |
| | | (iii) | $3x^2 e^{3x} + 2x e^{3x}$ | M1 ($x^2f(x) + e^{3x} g(x)$)<br>A1 ($f(x) = ke^{3x}$, $g(x) = 2x$)<br>A1 (all correct) |
| | *(b)* | | $\frac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{\sin^2 x}$ | M1 $\left(\frac{\sin x f(x)-\cos x g(x)}{\sin^2 x}\right)$<br>A1 ($f(x) = -\sin x$, $g(x) = \cos x$) |
| | | = | $\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$ | |
| | | = | $\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}$ | |
| | | = | $-\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$ | A1 (convincing) |
| | | | | **10** |
| **6.** | *(a)* | $5\mid x\mid = 2$ | | B1 |
| | | $x = \pm\ \frac{2}{5}$ | | B1 (both)<br>(F.T. $a\mid x\mid = b$) |
| | *(b)* | $7x - 5 \geq 3$ | | |
| | | $x \geq \frac{8}{7}$ | | B1 |
| | | $7x - 5 \leq -3$ | | M1 ($7x - 5 \leq -3$) |
| | | $x \leq \frac{2}{7}$ | | A1 |
| | | | | **5** |
| **7.** | *(a)* | (i) | $-\frac{7}{15(5x+2)^3}$ (+C) (o.e.) | M1 $\left(\frac{k}{(5x+2)^3}\right)$ A1 $\left(k=\frac{7}{15}\right)$ |
| | | (ii) | $\frac{1}{4}\ ln\ \mid 8x+7\mid$ (+C) (o.e.) | M1 ($k \ln\mid 8x + 7\mid$)<br>A1 ($k = \frac{1}{4}$ (o.e.)) |

(b) $\left[\frac{1}{3}\sin 3x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ — M1 ($k \sin 3x$, $k = \frac{1}{3}, -\frac{1}{3}, 3$)

A1 ($k = \frac{1}{3}$)

$= \frac{1}{3}\sin\left(\frac{\pi \times 3}{3}\right) - \frac{1}{3}\sin\left(\frac{\pi}{6} \times 3\right)$ — M1 (use of limits, F.T. allowable $k$)

$= -\frac{1}{3}$ — A1 (C.A.O.)

**8**

**8.** (a) $f'(x) = 1 + \frac{1}{x^2}$ — B1

$f'(x) > 0$ since ($1 > 0$ and) $\frac{1}{x^2} > 0$ — B1

Least value when $x = 1$ and is 0 — B1

(b) Range of $f$ is $[o, \infty)$ — B1

(c) $3(x - \frac{1}{x})^2 + 2 = \frac{3}{x^2} + 8$ — B1 (correct composition)

M1 (writing equations and correct expansion of binomial)

$3(x^2 - 2 + \frac{1}{x^2}) = \frac{3}{x^2} + 8$

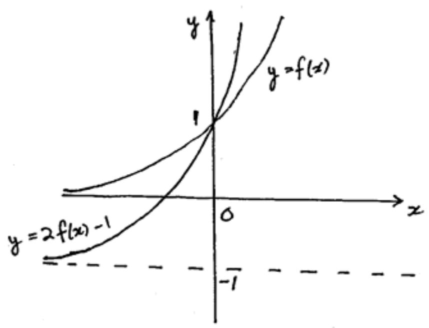
$3x^2 = 12$

$x = \pm 2$ (accept 2) — A1 (C.A.O.)

$x = 2$ since domain of $f$ is $x \geq 1$ — B1 (F.T. removal of –ve root)

**8**

**9.**




$y = f(x)$ B1 (0,1)

B1 (correct behaviour for large +ve, –ve $x$)

$y = 2f(x) - 1$

B1 (0, 1)

B1 (behaviour for –ve $x$, must approach –ve value of $y$)

B1 (greater slope even only if in 1st quadrant)

**5**

**10.** *(a)* Let $y = \sqrt{x+1}$ — M1 (attempt to isolate $x$, $y^2 = x + 1$)

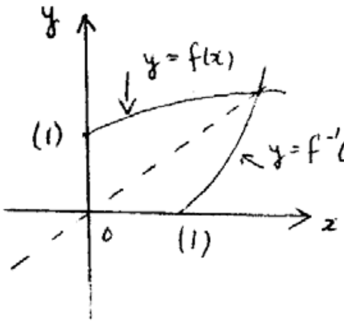
$y^2 = x + 1$

$x = y^2 - 1$ — A1

$f^{-1}(x) = x^2 - 1$ — A1 (F.T. one slip)

*(b)* domain $[1, \infty)$, range $[0, \infty)$ — B1, B1

*(c)*




B1 (parabola $y = x^2 - 1$)

B1 (relevant part of parabola)

B1 ($y = f(x)$, F.T. graph of $y = f^{-1}(x)$)

**8**

## MATHEMATICS C4

| | | |
|---|---|---|
| **1.** *(a)* | Let $\dfrac{2x^2+4}{(x-2)^2(x+4)} \equiv \dfrac{A}{x+4} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$ | M1 (Correct form) |
| | $2x^2 + 4 \equiv A(x-2)^2 + B(x-2) + C(x+4)$ | M1 (correct clearing of fractions and attempt to substitute) |
| | $\underline{x = 2}$ $\quad 12 = C(6), \quad C = 2$ | |
| | $\underline{x = -4}$ $\quad 36 = A(36), \quad A = 1$ | A1 (two constants, C.A.O.) |
| | Coefft of $x^2$ $\quad 2 = A + B, \quad B = 1$ | A1 (third constant, F.T. one slip) |
| *(b)* | $f'(x) = \dfrac{-1}{(x+4)^2} - \dfrac{1}{(x-2)^2}$ | B1 (first two terms) |
| | $-\dfrac{4}{(x-2)^3}$ | B1 (third term) |
| | $f'(0) = -\dfrac{1}{16} - \dfrac{1}{4} + \dfrac{4}{8} = \dfrac{3}{16}$ (o.e.) | B1 (C.A.O.) |
| | | **7** |

| | | |
|---|---|---|
| **2.** | $6x^2 + 6y^2 + 12xy\dfrac{dy}{dx} - 4y^2\dfrac{dy}{dx} = 0$ | B1 ($4y^3\dfrac{dy}{dx}$) |
| | | B1 ($6y^2 + 12xy\dfrac{dy}{dx}$) |
| | $\dfrac{dy}{dx} = -\dfrac{3}{2}$ | B1 (C.A.O.) |
| | Gradient of normal = $\dfrac{3}{2}$ | M1 (F.T. candidate's $\dfrac{dy}{dx}$) |
| | Equation is $y - 1 = \dfrac{2}{3}(x-2)$ | A1 (F.T. candidate's gradient of normal) |
| | | **5** |

| | | |
|---|---|---|
| **3.** | $2 + 3\,(2\cos^2\theta - 1) = \cos\theta$ | M1 (correct substitution for $\cos 2\theta$) |
| | $6\cos^2\theta - \cos\theta - 1 = 0$ | M1 (correct method of solution, $(a\cos\theta + b)(\cos\theta + d)$ with $ac$ = coefft of $\cos^2\theta$, $bd$ = constant term, or correct formula) |
| | $(3\cos\theta + 1)(2\cos\theta - 1) = 0 \quad \cos\theta = -\frac{1}{3}, \frac{1}{2}$ | A1 |
| | $\theta = 109{\cdot}5^\circ, 250{\cdot}5^\circ, 60^\circ, 300^\circ$ | B1 ($109{\cdot}5^\circ$)<br>B1 ($250{\cdot}5^\circ$)<br>B1 ($60^\circ$, $300^\circ$) |
| | | **6** |

Full F.T. for $\cos\theta = +, -$
2 marks for $\cos\theta = -, -$
1 mark for $\cos\theta = +, +$
Subtract 1 mark for each additional value in range, for each branch.

| | | |
|---|---|---|
| **4.** *(a)* | $R\cos\alpha = 4,\ R\sin\alpha = 3$ | |
| | $R = 5,\ \alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36{\cdot}87^\circ$ | M1 (reasonable approach) |
| | (or $36{\cdot}9^\circ$, $37{\cdot}0^\circ$) | A1 ($\alpha$)<br>B1 ($R = 5$) |
| *(b)* | Write as $\dfrac{1}{5\sin(x^\circ + 36\cdot87^\circ) + 7}$ | M1 (attempt to use $\sin(x + \alpha) = \pm 1$) |
| | Greatest value = $\dfrac{1}{-5+7} = \dfrac{1}{2}$ | A1 (F.T. one slip) |
| | | **5** |

| | | |
|---|---|---|
| **5.** | Volume $= \pi \displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ | B1 |
| | $= (\pi)\displaystyle\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}dx$ | M1 ($a + b\cos 2x$) |
| | | A1 ($a = \frac{1}{2}$, $b = \frac{1}{2}$) |
| | $= (\pi)\left[\dfrac{x}{2} - \dfrac{\sin 2x}{4}\right]$ | A1 $\left(\dfrac{x}{2} - \dfrac{\sin 2x}{4}\right)$ (F.T. $a$, $b$)) |
| | $= \dfrac{\pi^2}{4}$ ($\approx 2{\cdot}467$, accept $2{\cdot}47$, 3 sig. figures) | A1 (C.A.O.) |
| | | **5** |

| | | | |
|---|---|---|---|
| **6.** | *(a)* | $\frac{dy}{dx} = \frac{2t}{-\frac{1}{t^2}} - 2t^3$ | M1 $\left(\frac{\dot{y}}{\dot{x}}\right)$ |
| | | | A1 (simplified) |
| | | Equation of tangent is | |
| | | $y - p^2 = -2p^3\left(x - \frac{1}{p}\right)$ | M1 ($y - y_1 = m(x - x_1)$) o.e. |
| | | $y - p^2 = -2p^3x + 2p^2$ | |
| | | $2p^3x + y - 3p^2 = 0$ | A1 (convincing) |
| | *(b)* | $y = 0,\ x = \frac{3}{2p}$ (o.e.) | B1 |
| | | $x = 0,\ y = 3p^2$ | B1 |
| | | $PA^2 = \left(\frac{3}{2p} - \frac{1}{p}\right)^2 + \left(p^2 - 0\right)^2 = \frac{1}{4p^2} + p^4$ (o.e.) | M1 (correct use of distance formula in context) |
| | | | A1 (one correct simplified distance C.A.O.) |
| | | $PB^2 = 4PA^2,\ PB = 2PA$ | A1 (convincing) |
| | | | **9** |
| **7.** | *(a)* | $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\frac{1}{x}dx$ | M1 $(f(x)\ln x - \int g(x).\frac{1}{x}dx)$ |
| | | | A1($f(x)$=$g(x)$) A1 ($f(x) = g(x) = \frac{x^2}{2}$) |
| | | $= \frac{x^2}{2}\ln x - \int \frac{x}{2}dx$ | A1 |
| | | $= \frac{x^2}{2}\ln x - \frac{x^2}{4}(+C)$ | A1 (C.A.O.) |
| | *(b)* | $u = 2\sin x + 3,\ du = 2\cos x dx$ (limits are 3, 4) | |
| | | $\int_3^4 \frac{1}{2}\frac{du}{u^2}$ | M1 $\left(\int \frac{a}{u^2}du \text{ with } a = \pm\frac{1}{2}, 1, 2\right)$ |
| | | | A1 ($a = \frac{1}{2}$) |
| | | $= \left[-\frac{1}{2u}\right]_3^4$ | A1 ($-\frac{a}{u}$, allowable $a$) |
| | | $= \frac{1}{24}$ (o.e.) | A1 (F.T. allowable $a$ or one slip) |
| | | | **9** |

| | | | |
|---|---|---|---|
| **8.** | *(a)* | $\frac{dx}{dt} = -k\sqrt{x}$ | B1 |
| | *(b)* | $\int \frac{dx}{\sqrt{x}} = \int -k\,dt$ | M1 (attempt to separate variables, allow similar work) |
| | | $2x^{\frac{1}{2}} = -kt + C$ | A1 (unsimplified version, allow absence of C) |
| | | $t = 0,\ x = 9,\quad 2\sqrt{9} = C$ | M1 (correct attempt to find C) |
| | | $C = 6$ | |
| | | $kt = 6 - 2\sqrt{x}$ | A1 (convincing) |
| | *(c)* | $t = 20,\ x = 4$ gives $k = \frac{1}{10}$ (o.e.) | M1 (attempt to find *k*) |
| | | | A1 |
| | | Tank is empty when $6 = \frac{1}{10}t$, $t = 60$ (mins.) | A1 |
| | | | **8** |
| **9.** | *(a)* | **OP** = **OA** + λ **AB** | M1 (correct formula for r.h.s. and method for finding **AB**) |
| | | = **i** + 3**j** + **k** + λ (**i** + 5**j** – 3**k**) | B1 (**AB**) |
| | | **r** = **i** + 3**j** + **k** + λ (**i** + 5**j** – 3**k**) | A1 (must contain **r**, F.T. **AB**) |
| | *(b)* | (Point of intersection is on both lines)<br>Equate coefficients of **i** and **j** | |
| | | $1 + \lambda = 2 + \mu$ | M1 (attempt to write equations using candidate's equations, one correct) |
| | | $3 + 5\lambda = -1 + 2\mu$ | A1 (two correct, using candidate's equations) |
| | | $\lambda = -2,\ \mu = -3$ | M1 (attempt to solve equations)<br>A1 (C.A.O.) |
| | | (Consider coefficients in **k**) | |
| | | $p - \mu = 1 - 3\lambda$ | M1 (use of equation in **k** to find *p*) |
| | | $p = \mu + 1 - 3\lambda = 4$ | A1 (F.T. candidate's λ, μ) |
| | *(c)* | **b** . **c** = (2**i** + 8**j** – 2**k**).(3**i** – **j** – **k**) | M1 (correct method) |
| | | = 6 – 8 + 2 = 0 | A1 (correct) |
| | | **b** and **c** are $\perp^r$ vectors | A1 (C.A.O.) |
| | | | **12** |

**10.** $\left(1+\frac{x}{8}\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(\frac{x}{8}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1.2}\left(\frac{x}{8}\right)^2+\ldots$

$= 1+\frac{x}{16}-\frac{x^2}{512}+\ldots$ B1 $\left(1+\frac{x}{16}\right)$

B1 $\left(-\frac{x^2}{512}\right)$

Expansion is valid for $|x| < 8$ B1 (only for conditions on $|x|$)

$\left(1+\frac{1}{8}\right)^{\frac{1}{2}} = 1+\frac{1}{6}-\frac{1}{512}$

$\frac{3}{2\sqrt{2}} = \frac{543}{512}$ (o.e.) B1 (expression must involve $\sqrt{2}$ )

$\sqrt{2} = \frac{3}{2}\times\frac{512}{543} = \frac{256}{181}$ B1 (convincing)

**5**

**11.** $4k^2 = 2b^2$ B1

$b^2 = 2k^2$
$b^2$ has a factor 2 } one or other of these statements B1

$\therefore b$ has a factor 2 B1 (if and only if previous B gained)

*a* and *b* have a common factor
Contradiction B1 (depends upon previous B being gained)

($\therefore$ $\sqrt{2}$ is irrational)

**4**

## MATHEMATICS FP1

1. Sum = $\sum_{r=1}^{n} r(r+1)(r+5) = \sum_{r=1}^{n} r^3 + 6\sum_{r=1}^{n} r^2 + 5\sum_{r=1}^{n} r$ M1A1

$= \frac{n^2(n+1)^2}{4} + \frac{6n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}$ A1

$= \frac{n(n+1)}{4}(n^2 + n + 8n + 4 + 10)$ A1

$= \frac{n(n+1)}{4}(n^2 + 9n + 14)$ A1

$= \frac{n(n+1)(n+2)(n+7)}{4}$ A1

[Award 4/6 for a correct unfactorised answer]

2. $f(x+h) - f(x) = \frac{1}{2(x+h)-3} - \frac{1}{2x-3}$ M1A1

$= \frac{2x-3-2(x+h)+3}{[2(x+h)-3](2x-3)}$ A1

$= \frac{-2h}{[2(x+h)-3](2x-3)}$ A1

$f'(x) = \lim_{h \to 0} \frac{-2h}{h[2(x+h)-3](2x-3)}$ A1

$= \frac{-2}{(2x-3)^2}$ A1

3. Cross multiplying,

$z = (z+1)(2+3i)$ M1

$= 2z + 3iz + 2 + 3i$ A1

$z(1+3i) = -(2+3i)$ A1

$z = -\frac{2+3i}{1+3i}$ A1

$= \frac{-(2+3i)(1-3i)}{(1+3i)(1-3i)}$ M1

$= -\frac{11}{10} + \frac{3}{10}i$ A1

Alternative solutions:

Candidates who replace z by $x + iy$ and multiply the LHS by $x + 1 - iy$ to obtain

$\frac{x(x+1)+y^2}{(x+1)^2+y^2} = 2; \frac{y}{(x+1)^2+y^2} = 3$ Award M1A1

No further progress is possible at this level.
Candidates who cross multiply and then replace $z$ by $x + iy$ to obtain

| | |
|---|---|
| $x + iy = 2x + 2 - 3y + i(2y + 3x + 3)$ | Award M1A1 |
| $x - 3y = -2,\ 3x + y = -3$ | Award M1A1 |
| Solving these equations | Award M1A1 |

**4.** *(a)* Let the roots be $a$, $2a$ and $4a$. — M1

Then $14a^2 = 56$ — m1

$a = 2$ — A1

The roots are 2, 4 and 8. — A1

*(b)* $7a = -p$ gives $p = -14$ — B1

$8a^3 = -q$ gives $q = -64$ — B1

**5.** *(a)*

$$\mathbf{A} + \lambda\mathbf{I} = \begin{bmatrix} -4+\lambda & -4 & 4 \\ -1 & \lambda & 1 \\ -7 & -6 & 7+\lambda \end{bmatrix}$$ — M1A1

*(b)* $\text{Det} = (\lambda - 4)(\lambda(7 + \lambda) + 6) + 4(-7 - \lambda + 7) + 4(6 + 7\lambda)$ — M1A1

$= \lambda^3 + 3\lambda^2 + 2\lambda$ — A1

The matrix is singular when this equals zero, — M1

ie, when $\lambda = 0$ or — A1

$\lambda^2 + 3\lambda + 2 = 0$ — m1

giving $\lambda = -1, -2$. — A1

6. *(a)*

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$ — B1

$$\mathbf{T}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ — B1

$$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$ — M1

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$ — A1

*(b)* Fixed points satisfy

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ — M1

giving $y + 1 = x$ and $x - 1 = y$ — A1A1

The required equation is $y = x - 1$. — A1

| | | |
|---|---|---|
| **7.** | The statement is true for $n = 1$ since $9 - 5 = 4$ is divisible by 4. | B1 |
| | Let $T_n = 9^n - 5^n$ and assume that $T_k$ is divisible by 4. | M1 |
| | Consider $T_{k+1} = 9^{k+1} - 5^{k+1}$ | M1 |
| | $= 9^k.9 - 5^k.5$ | m1 |
| | $= 9^k.8 - 5^k.4 + (9^k - 5^k)$ | A1 |
| | It follows that $T_{k+1}$ is divisible by 4. | A1 |
| | It follows by induction that $9^n - 5^n$ is divisible by 4 for all positive integers $n$. | |
| **8.** | Using reduction to echelon form, | |
| | $\begin{bmatrix}1 & 3 & 2\\0 & 5 & 3\\0 & 7 & 7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}13\\19\\35\end{bmatrix}$ | M1A1A1 |
| | $\begin{bmatrix}1 & 3 & 2\\0 & 5 & 3\\0 & 0 & 14\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}13\\19\\42\end{bmatrix}$ | A1 |
| | It follows that $z = 3$, $y = 2$ and $x = 1$. | B1B1B1 |
| **9.** | *(a)* $\ln y = -x\ln(x)$ | M1 |
| | $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = -\ln(x) - 1$ | m1A1 |
| | At the stationary point, | |
| | $\ln(x) = -1$ | M1 |
| | so $x = \mathrm{e}^{-1}$ (0.368) and $y = (\mathrm{e}^{-1})^{-\mathrm{e}^{-1}}$ (1.44) | A1A1 |
| | *(b)* Differentiating again, | |
| | $\frac{1}{y}\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{1}{y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = -\frac{1}{x}$ | M1A1A1 |
| | whence the printed result. | AG |
| | At the stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ so | M1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{y}{x} < 0$ | A1 |
| | It is therefore a maximum. | A1 |
| **10.** | $u + \mathrm{i}v = (x + \mathrm{i}y)^2 = x^2 - y^2 + 2xy\mathrm{i}$ | M1 |
| | $u = x^2 - y^2; v = 2xy$ | A1A1 |
| | Substituting $y = x - 1$, M1 | |
| | $u = x^2 - (x-1)^2 = 2x - 1$ | A1 |
| | $v = 2x(x-1)$ A1 | |
| | Substituting from first into second, | M1 |
| | $v = \frac{(u^2 - 1)}{2}$ | A1 |

# MATHEMATICS FP2

**1.** (i) $f$ is continuous because $f(x)$ passes through zero from both sides around $x = 0$. B1B1

(ii) For $x \ge 0$, $f'(x) = \cos x = 1$ when $x = 0$ and for $x < 0$, $f'(x) = 1$. M1A1

So $f'$ is continuous. A1

**2.** Converting to trigonometric form,

$i = \cos(\pi/2) + i\sin(\pi/2)$ B2

Cube roots $= \cos(\pi/6 + 2n\pi/3) + i\sin(\pi/6 + 2n\pi/3)$ (si) M1A1

$n = 0$ gives $\cos(\pi/6) + i\sin(\pi/6) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ M1A1

$n = 1$ gives $\cos(5\pi/6) + i\sin(5\pi/6)$ M1

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$ A1

$n = 2$ gives $\cos(3\pi/2) + i\sin(3\pi/2) = -i$ A1

[FT on candidate's first line but award a max of 4 marks for the cube roots of 1]

**3.** *(a)* $f'(x) = -\frac{1}{x^2(1+x^2)^2} \times (3x^2 + 1)$ M1A1

$< 0$ for all $x > 0$ (cso) A1

*(b)* $f$ is odd because $f(-x) = -f(x)$ B2

*(c)* The asymptotes are $x = 0$ and $y = 0$. B1B1

*(d)* Graph G2

**4.** *(a)* Completing the square,

$2\{(x-1)^2-1\}-(y+2)^2+4=4$ M1A1

$\dfrac{(x-1)^2}{1}-\dfrac{(y+2)^2}{2}=1$ A1

The centre is (1,–2) A1

*(b)* In the usual notation, $a = 1$, $b = \sqrt{2}$. M1

$2 = 1(e^2-1)$ A1

$e=\sqrt{3}$ A1

Foci are $(1 \pm \sqrt{3}, -2)$, Directrices are $x=1\pm\dfrac{1}{\sqrt{3}}$ (FT) B1B1

**5.** Putting $t = \tan(\theta/2)$ and substituting, M1

$\dfrac{3(1-t^2)}{1+t^2}+\dfrac{4.2t}{1+t^2}=3-t$ A1

$3-3t^2+8t=3+3t^2-t-t^3$ A1

$t^3-6t^2+9t=0$ A1

Either $t = 0$ B1

giving $\theta/2 = n\pi$ or $\theta = 2n\pi$ B1

Or $t = 3$ B1

giving $\theta/2 = 1.25 + n\pi$ B1

$\theta = 2.50 + 2n\pi$ (Accept degrees) B1

**6.** *(a)* Putting $n = 1$,

LHS = $\cos\theta + i\sin\theta$ = RHS so true for $n = 1$. B1

Let the result be true for $n = k$, ie

$(\cos\theta+i\sin\theta)^k=\cos k\theta+i\sin k\theta$ M1

Consider

$(\cos\theta+i\sin\theta)^{k+1}=(\cos k\theta+i\sin k\theta)(\cos\theta+i\sin\theta)$ M1

$=\cos k\theta\cos\theta-\sin k\theta\sin\theta+i(\sin k\theta\cos\theta+\sin\theta\cos k\theta)$ A1

$=\cos(k+1)\theta+i\sin(k+1)\theta$ A1

Thus, true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$,
proof by induction follows. A1

*(b)* $\cos 5\theta+i\sin 5\theta=(\cos\theta+i\sin\theta)^5$ M1

$=\cos^5\theta+5\cos^4\theta(i\sin\theta)+10\cos^3\theta(i\sin\theta)^2$
$+10\cos^2\theta(i\sin\theta)^3+5\cos\theta(i\sin\theta)^4+(i\sin\theta)^5$ A1

$=\cos^5\theta+5i\cos^4\theta\sin\theta-10\cos^3\theta\sin^2\theta$
$-10i\cos^2\theta\sin^3\theta+5\cos\theta\sin^4\theta+i\sin^5\theta$ A1

Equating imaginary parts,

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ M1

$= 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$ A1

$= 5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta$ A1

$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ A1

**7.** *(a)* Let $\dfrac{x}{(x+2)(x^2+4)} \equiv \dfrac{A}{x+2} + \dfrac{Bx+C}{(x^2+4)}$ M1

$x \equiv A(x^2+4) + (x+2)(Bx+C)$

$x = -2$ gives $A = -1/4$ A1

Coeff of $x^2$ gives $B = 1/4$ A1

Constant term gives $C = 1/2$ A1

*(b)* Integral = $\displaystyle\int_2^3 \left(-\frac{1}{4(x+2)} + \frac{x}{4(x^2+4)} + \frac{1}{2(x^2+4)}\right)dx$ M1

$= \left[-\dfrac{1}{4}\ln(x+2) + \dfrac{1}{8}\ln(x^2+4) + \dfrac{1}{4}\arctan\left(\dfrac{x}{2}\right)\right]_2^3$ A1A1A1

$= -\dfrac{1}{4}\ln 5 + \dfrac{1}{8}\ln 13 + \dfrac{1}{4}\arctan\left(\dfrac{3}{2}\right) + \dfrac{1}{4}\ln 4 - \dfrac{1}{8}\ln 8 - \dfrac{1}{4}\arctan 1$ A1

= 0.054 cao A1

**8.** *(a)* The line meets the circle where

$x^2 + m^2(x-2)^2 = 1$ M1

$(1+m^2)x^2 - 4m^2x + 4m^2 - 1 = 0$ A1

$x$ coordinate of M = $\dfrac{\text{Sum of roots}}{2}$ M1

$= \dfrac{2m^2}{1+m^2}$ AG

Substitute in the equation of the line.

$y = m\left(\dfrac{2m^2}{1+m^2} - 2\right)$ M1A1

$= -\dfrac{2m}{1+m^2}$ AG

*(b)* Dividing,

$\dfrac{x}{y} = -m$ or $m = -\dfrac{x}{y}$ M1A1

Substituting,

$y = \dfrac{2x/y}{1+x^2/y^2}$ M1A1

$= \dfrac{2xy}{x^2+y^2}$ A1

whence $x^2 + y^2 - 2x = 0$ A1

[Accept alternative forms, e.g.

$y = \sqrt{x(2-x)}$ ]

# MATHEMATICS FP3

| | | | |
|---|---|---|---|
| **1.** | *(a)* | $2\sinh^2 x + 1 = 2\left(\frac{e^x - e^{-x}}{2}\right)^2 + 1$ | M1 |
| | | $= \frac{(e^{2x} - 2 + e^{-2x} + 2)}{2}$ | A1 |
| | | $= \frac{(e^{2x} + e^{-2x})}{2} = \cosh 2x$ | A1 |
| | *(b)* | Substitute to obtain the quadratic equation | M1 |
| | | $2\sinh^2 x - 3\sinh x + 1 = 0$ | A1 |
| | | $\sinh x = 1, 1/2$ | M1A1 |
| | | $x = 0{\cdot}881,\ 0{\cdot}481$ cao | A1A1 |
| **2.** | | $t = \tan\left(\frac{x}{2}\right) \Rightarrow dx = \frac{2dt}{1+t^2}$ | B1 |
| | | $(0,\pi/2) \rightarrow (0,1)$ | B1 |
| | | $I = \int_0^1 \frac{2dt/(1+t^2)}{1 + 3(1-t^2)/(1+t^2)}$ | M1A1 |
| | | $= \int_0^1 \frac{dt}{2-t^2}$ | A1 |
| | | $= \frac{1}{2\sqrt{2}}\left[\ln\left(\frac{\sqrt{2}+t}{\sqrt{2}-t}\right)\right]_0^1$ or $\left[\frac{1}{\sqrt{2}}\tanh^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_0^1$ | M1A1 |
| | | $= \frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ or $\frac{1}{\sqrt{2}}\tanh^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (0.623) cao | A1 |
| **3.** | *(a)* | $f(0) = 0$ si | B1 |
| | | $f'(x) = \frac{1}{\sec x} \times \sec x \tan x = \tan x,\ f'(0) = 0$ si | B1B1 |
| | | $f''(x) = \sec^2 x,\ f''(0) = 1$ si | B1 |
| | | $f'''(x) = 2\sec^2 x \tan x,\ f'''(0) = 0$ si | B1B1 |
| | | $f''''(x) = 4\sec^2 x \tan^2 x + 2\sec^4 x,\ f''''(0) = 2$ si | B1B1 |

[This expression has several similar looking forms, eg $6\sec^2 x \tan^2 x + 2\sec^2 x$ ]

The series is

$$f(x) = \frac{x^2}{2} + \frac{x^4}{12} + \ldots \qquad \text{B1}$$

*(b)* Substituting the series gives

$$x^4 + 126x^2 - 12 = 0 \qquad \text{M1}$$

Solving,

$$x^2 = 0.0952, \qquad \text{M1A1}$$

$$x = 0.3085 \qquad \text{A1}$$

**4.** *(a)* $\frac{dx}{d\theta} = 1+\cos\theta, \frac{dy}{d\theta} = -\sin\theta$ B1B1

$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 1 + 2\cos\theta + \cos^2\theta + \sin^2\theta$ M1A1

$= 2(1 + \cos$ A1

$= 4\cos^2\left(\frac{\theta}{2}\right)$ AG

*(b)* Arc length $= \int_0^{\pi} 2\cos\left(\frac{\theta}{2}\right)d\theta$ M1A1

$= 4\left[\sin\left(\frac{\theta}{2}\right)\right]_0^{\pi}$ A1

$= 4$ A1

*(c)* CSA $= 2\pi\int_0^{\pi}(1+\cos\theta).2\cos\left(\frac{\theta}{2}\right)d\theta$ M1A1

$= 2\pi\int_0^{\pi} 2\cos^2\left(\frac{\theta}{2}\right).2\cos\left(\frac{\theta}{2}\right)d\theta$ or $4\pi\int_0^{\pi}\cos\left(\frac{\theta}{2}\right)d\theta + 4\pi\int_0^{\pi}\cos\theta\cos\left(\frac{\theta}{2}\right)d\theta$ A1

$= 8\pi\int_0^{\pi}\cos^3\left(\frac{\theta}{2}\right)d\theta$ or $4\pi\int_0^{\pi}\cos\left(\frac{\theta}{2}\right) + 2\pi\int_0^{\pi}(\cos\left(\frac{3\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right))d\theta$ A1

$= 16\pi\int_0^{\pi}\left\{1 - \sin^2\left(\frac{\theta}{2}\right)\right\}d\sin\left(\frac{\theta}{2}\right)$ or $6\pi\int_0^{\pi}\cos\left(\frac{\theta}{2}\right)d\theta + 2\pi\int_0^{\pi}\cos\left(\frac{3\theta}{2}\right)d\theta$ M1A1

$= 16\pi\left[\sin\left(\frac{\theta}{2}\right) - \frac{1}{3}\sin^3\left(\frac{\theta}{2}\right)\right]_0^{\pi}$ or $12\pi\left[\sin\left(\frac{\theta}{2}\right)d\theta\right]_0^{\pi} + \frac{4\pi}{3}\left[\sin\left(\frac{3\theta}{2}\right)\right]_0^{\pi}$ A1

$= \frac{32}{3}\pi$ (33·5) A1

**5.** *(a)* $I_n = -\int_0^{\pi}\theta^n d\cos\theta$ M1

$= \left[-\theta^n\cos\theta\right]_0^{\pi} + \int_0^{\pi}\cos\theta.n\theta^{n-1}d\theta$ A1A1

$= \pi^n + n\int_0^{\pi}\theta^{n-1}\cos\theta d\theta$ A1

$= \pi^n + n\int_0^{\pi}\theta^{n-1}d\sin\theta$ M1

$= \pi^n + n\left[\theta^{n-1}\sin\theta\right]_0^{\pi} - n(n-1)\int_0^{\pi}\theta^{n-2}\sin\theta d\theta$ A1A1

$= \pi^n - n(n-1)I_{n-2}$ A1

(b) $I_4 = \pi^4 - 12I_2$ B1

$= \pi^4 - 12(\pi^2 - 2I_0)$ B1

$= \pi^4 - 12\pi^2 + 24\int_0^{\pi} \sin\theta \mathrm{d}\theta$ M1

$= \pi^4 - 12\pi^2 + 24[-\cos\theta]_0^{\pi}$ A1

$= \pi^4 - 12\pi^2 + 48$ A1

**6.** (a) Area $= \frac{1}{2}\int_0^{\pi/2} \sinh^2\theta \mathrm{d}\theta$ M1A1

$= \frac{1}{4}\int_0^{\pi/2} (\cosh 2\theta - 1)\mathrm{d}\theta$ A1

$= \frac{1}{4}\left[\frac{\sinh 2\theta}{2} - \theta\right]_0^{\pi/2}$ A1

$= \frac{1}{4}\left(\frac{\sinh\pi}{2} - \frac{\pi}{2}\right)$ (1·05) A1

(b) (i) Consider

$x = r\cos\theta = \sinh\theta\cos\theta$ M1A1

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \cosh\theta\cos\theta - \sinh\theta\sin\theta$ M1A1

At $P$,

$\cosh\theta\cos\theta = \sinh\theta\sin\theta$ M1

so $\tanh\theta = \cot\theta$ A1

(ii) The Newton-Raphson iteration is

$x \to x - \frac{(\tanh\theta - \cot\theta)}{(\mathrm{sech}^2\theta + \mathrm{cosec}^2\theta)}$ M1A1

Starting with $x_0 = 1$, we obtain M1

$x_1 = 0\cdot 9348$ A1

MS3
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**MARKING SCHEMES**

**SUMMER 2006**

# MATHEMATICS
# M1-M3 and S1-S3

**INTRODUCTION**

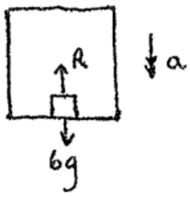
The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

# MATHEMATICS M1

**1.**

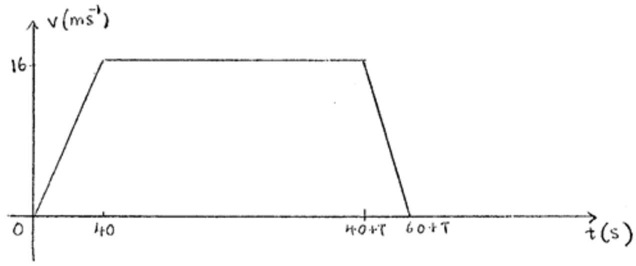



| | | | |
|---|---|---|---|
| | N2L applied to object | | M1 |
| | $6g - R = 6a$ | o.e. | A1 |
| Accelerating $a = 3$ | $R = 6g - 6 \times 3$ | | |
| | $= \underline{40{\cdot}8\text{N}}$ | c.a.o. | A1 |
| Constant speed $a = 0$ | $R = 6g$ | | |
| | $= \underline{58{\cdot}8\text{N}}$ | | B1 |
| Decelerating $a = -2$ | $R = 6 + 6 \times 2$ | | |
| | $= \underline{70{\cdot}8\text{N}}$ | c.a.o. | A1 |

**2.** *(a)* Use of $v = u + at$ with $u = 0$, $a = 0{\cdot}4$, $v = 1{\cdot}5$ — M1

$t = \frac{16}{0 \cdot 4} = \underline{40\text{s}}$ — A1

*(b)*



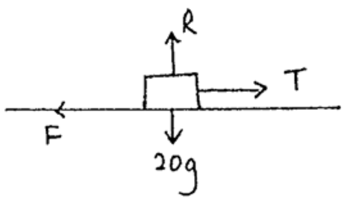

B1
B1
B1
B1

*(c)* $\frac{1}{2} \times 40 \times 16 + 16\text{T} + \frac{1}{2} \times 20 \times 16 = 2400$ — M1, B1

$16\text{T} + 4800 = 2400$ — f.t.(a) — A1

$\text{T} = \underline{120s}$ — c.ao. — A1

**3.**



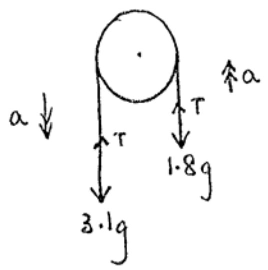

| | | | |
|---|---|---|---|
| *(a)* $\text{R} = 20g$ | | | B1 |
| F = limiting friction | $= \mu \text{R}$ | used | M1 |
| | $= 0{\cdot}3 \times 20g$ | | |
| | $= 58{\cdot}8\text{N}$ | | |
| N2L | | dim. correct | M1 A1 |
| $\text{T} - \text{F}$ | $= 20\text{a}$ | | |
| $a$ | $= \frac{65 - 58 \cdot 8}{20}$ | | |
| | $= \underline{0{\cdot}31 \text{ ms}^{-2}}$ | c.a.o. | A1 |

*(b)* T < limiting friction

$\therefore \text{F} = \text{T}$

$= 45\text{N}$ — B1

**4.**




N2L applied to A and B — M1

$3.1g - \mathrm{T} = 3{\cdot}1a$ — B1

N2L applied to B

$\mathrm{T} - 1.8g = 1{\cdot}8a$ — A1

Adding $3.1g = 1.8g = 4.9a$ — m1

$a = \underline{2{\cdot}6\text{ms}^{-1}}$ — c.a.o. — A1

$\mathrm{T} = 1{\cdot}8\ (2.6 + 9.8)$

$= \underline{22{\cdot}32\text{N}}$ — c.a.o. — A1

**5.** *(a)* Using $s = ut + \frac{1}{2}at^2$ with $s = 0$, $u = 22{\cdot}05$, $a = (\pm)\ 9{\cdot}8$ — o.e. — M1

$0 = 22{\cdot}05t - \frac{1}{2} \times 9{\cdot}8\ t^2$ — A1

$t = \underline{4.5\text{s}}$ — A1

$v = \underline{22{\cdot}05\text{ms}^{-1}}$ — B1

*(b)* Using $v^2 = u^2 + 2as$ with $v = 0$, $u = 22{\cdot}05$, $a = (\pm)\ 9{\cdot}8$ — o.e. — M1

$0 = 22{\cdot}05^2 - 2 \times 9{\cdot}8s$ — A1

$s = \underline{24{\cdot}8\ (0625)\text{m}}$ — c.a.o. — A1

*(c)* Using $v = u + at$ with $u = 22{\cdot}05$, $a = (\pm)\ 9{\cdot}8$, $t = 3$ — o.e. — M1

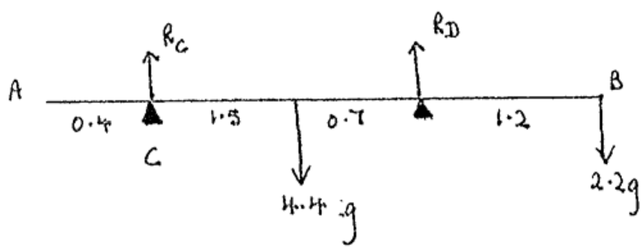
$v = 22{\cdot}05 - 9{\cdot}8 \times 3$ — f.t. (a) if used — A1

$v = -\ 7{\cdot}35$

Speed $= \underline{7{\cdot}35\text{ms}^{-1}}$ — A1

Direction is <u>downwards</u> — B1

**6.**




Moments about C — all forces dim. correct — M1

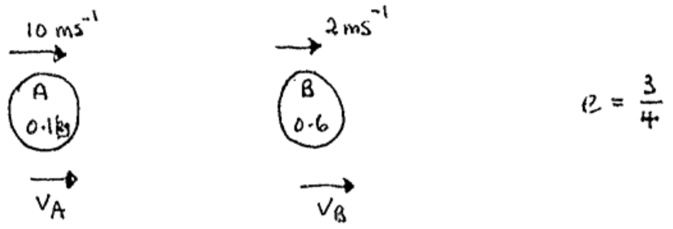
$4{\cdot}4g \times 1{\cdot}5 + 2{\cdot}2g \times 3{\cdot}4 = \mathrm{R_D} \times 2{\cdot}2$ — A2 B1

$\mathrm{R_D} = \underline{62{\cdot}72\text{N}}$ $(6.4g)$ — c.a.o. — A1

Resolve ↑ $\mathrm{R_C} + \mathrm{R_D} = 4{\cdot}4g \times 2{\cdot}2g$ — M1

$\mathrm{R_C} = 0{\cdot}2g = \underline{1{\cdot}96\text{N}}$ — f.t. $\mathrm{R_D}$ — A1

**7.** *(a)*




| | | |
|---|---|---|
| Conservation of momentum | used | M1 |
| $0{\cdot}1 \times 10 + 0{\cdot}6 \times 2 = 0{\cdot}1\, V_A + 0{\cdot}6\, V_B$ <br> $V_A + 6V_B = 22$ | | A1 |
| Restitution | used | M1 |
| $V_B - V_A = -\frac{3}{4}\,(2 - 10)$ <br> $-V_A + V_B = 6$ | | A1 |
| Adding $7V_B = 28$ | dep. on both Ms | M1 |
| $V_B = \underline{4}\ \text{ms}^{-1}$ | c.a.o. | A1 |
| $V_A = \underline{-2}\ \text{ms}^{-1}$ | c.a.o. | A1 |

*(b)* After B collide with wall, it is moving with speed $V_B'$ towards A

| | | |
|---|---|---|
| $V_B' = \frac{1}{4} \times 4 = 1\ \text{ms}^{-1}$ | f.t. | B1 |
| Since $\lvert V_B' \rvert = 1$ and $\lvert V_A' \rvert = 2$, B will not catch up with A | | B1 |

*(c)*

| | | |
|---|---|---|
| $I = 0{\cdot}6\,(4 + 1)$ | | M1 |
| $= 3$ NS | f.t. $V_B, V_B'$ | A1 |

**8.** *(a)*

| | Area | dist. from AB | dist. from AE | |
|---|---|---|---|---|
| ABDE | 60 | 3 | 5 | B1 |
| BCD | 18 | 4 | 12 | B1 B1 |
| Lamina | 78 | *x* | *y* | B1 (areas) |

Moments about AB

| | | |
|---|---|---|
| $60 \times 3 + 18 \times 4 = 78x$ | f.t. cand's values | M1 A1 |
| $x = \frac{43}{13}$ | c.a.o. | A1 |

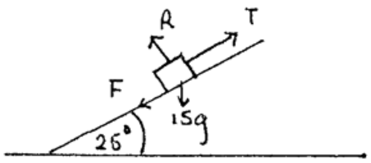
Moments about AE

| | | |
|---|---|---|
| $60 \times 5 + 18 \times 12 = 78\, xy$ | f.t. cand's values | M1 A1 |
| $y = \frac{86}{13}$ | c.a.o. | A1 |

*(b)*

| | | |
|---|---|---|
| $AX = \frac{43}{13}$ cm | f.t. *x* | B1 |

**9.** $\mu = 0{\cdot}4$




| | | | | | |
|---|---|---|---|---|---|
| $\perp$ to plane | R | = | $15g \cos 25°$ | | B1 |
| Limiting friction | F | = | $\mu R$ | used | M1 |
| | | = | $0{\cdot}4 \times 15g \cos 25°$ | si | M1 |
| | | = | $53{\cdot}2909$ N | | |

Max T when body on point of moving up plane

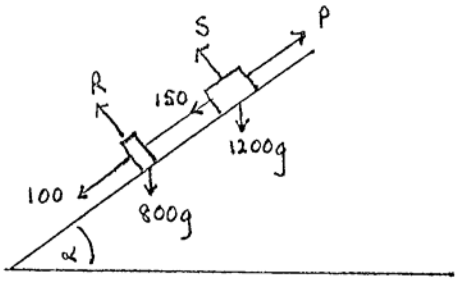
| | | | | |
|---|---|---|---|---|
| T | = | $15g \sin \alpha + F$ | f.t. F | M1 A1 |
| T | = | $115{\cdot}42$ N | c.a.o. | A1 |

Min T when body on point of moving down plane

| | | | | |
|---|---|---|---|---|
| T | = | $15g \sin \alpha - F$ | f.t. F | M1 A1 |
| T | = | $8{\cdot}83$ N | c.a.o. | A1 |

# MATHEMATICS M2

**1.**




$\sin\alpha = \dfrac{1}{28}$

*(a)* $P = \dfrac{45 \times 1000}{v}$ M1 A1

N2L to whole system M1

$\dfrac{45000}{25} - 2000g \sin\alpha - 150 - 100 = 2000a$ A2

$1800 - 700 - 250 = 2000a$

$a = \underline{0{\cdot}425 \text{ ms}^{-2}}$ A1

*(b)* N2L applied to trailer

$T - 800 \times 9{\cdot}8 \times \dfrac{1}{28} - 100 = 800 \times 0{\cdot}425$ A2

$T = \underline{720\text{N}}$ A1

**2.** *(a)* $\mathbf{r}_A = (\mathbf{i} - 10\mathbf{k}) + t(-2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k})$ B1

$= (1 - 2t)\mathbf{i} + (-2t)\mathbf{j} + (-10 - 5t)\mathbf{k}$

$\mathbf{r}_B = (7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}) + t(\mathbf{i} - 8\mathbf{j} - 5\mathbf{k})$ B1

$= (7 + t)\mathbf{i} + (9 - 8t)\mathbf{j} + (-6 - 5t)\mathbf{k}$

*(b)* When t = 2

$\mathbf{r}_A - \mathbf{r}_B = (-3 - 9)\mathbf{i} + (-4 + 7)\mathbf{j} + (-20 + 16)\mathbf{k}$ M1

$= -12\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

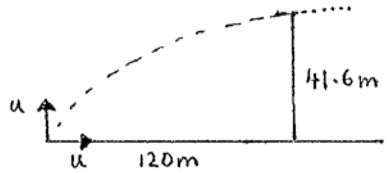
Distance between A and B $= \sqrt{12^2 + 3^2 + 4^2}$ m1

$= 13$ A1

| | | | | | |
|---|---|---|---|---|---|
| **3.** | *(a)* | $s = \int (12t - 3t^2)\,dt$ | | | M1 |
| | | $= 6t^2 - t^3\ (+C)$ m | | | A1 |
| | | When $t = 1$, s = 0 | | | |
| | | $\therefore\ 0 = 6 - 1 + C$ | | o.e. | M1 |
| | | $C = -\ 5$ | | | |
| | | $\therefore\ s = 6t^2 - t^3 - 5$ m | | | A1 |
| | *(b)* | $a = \dfrac{dv}{dt}$ | | attempted | M1 |
| | | $a = \underline{12 - 6t}$ | | | A1 |
| | *(c)* | Power = F.v. used | | M1 | |
| | | F = ma | | | |
| | | = 3 (12 – 9) | | | |
| | | = 9 | | | B1 |
| | | Power = $9 \times (12 \times 1{\cdot}5 - 3 \times 1{\cdot}5^2)$ | | | |
| | | = $9 \times 11{\cdot}25$ | | | |
| | | = $\underline{101{\cdot}25\text{W}}$ | | f.t. F | A1 |

| | | | | |
|---|---|---|---|---|
| **4.** | Initial energy = PE = $mgh$ | | used | M1 |
| | = $3 \times 9.8 \times 1.2$ | | | |
| | = 35·28J | | si | A1 |
| | Final EE = $\dfrac{1}{2} \times \dfrac{\lambda(1\cdot 2 - 0\cdot 8)^2}{0\cdot 8}$ | | | M1 |
| | = $\dfrac{1}{2} \times \dfrac{35\cdot 4 \times 0\cdot 4^2}{0\cdot 8}$ | | | |
| | = 3·54 J | | | A1 |
| | Conservation of energy | | | |
| | $1{\cdot}5v^2 + 3.54 = 35{\cdot}28$ | | | M1 A1 |
| | $v = \underline{4{\cdot}6\ \text{ms}^{-1}}$ | | | A1 |

**5.** *(a)*




Let *u* be initial horizontal and vertical velocities (*v* sin/cos 45) o.e. B1

*t* be time to hit target.

Horizontal motion $ut = 120$ M1 A1

Vertical motion

Using $s = ut + \frac{1}{2}at^2$ with $s = 41{\cdot}6$, $u = u$ (c), $a = (\pm)\ 9{\cdot}8$ M1

$$41{\cdot}6 = 120 + \frac{1}{2} \times (-9{\cdot}8)\left(\frac{120}{u}\right)^2$$ m1 (subst.)

$$u = 30 \text{ ms}^{-1}$$ A1

Speed of projection $= \sqrt{2u^2}$

$= 30\sqrt{2} = \underline{42{\cdot}4264 \text{ ms}^{-1}}$ B1

$t = \frac{120}{u}$

$= \underline{4s}$ convincing A1

*(b)* Using $v = u + at$ with $u = 30$ (c), $a = (\pm)\ 9{\cdot}8$, $t = 4$ M1

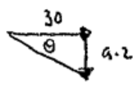
$v = 30 - 9{\cdot}8 \times 4$ f.t. *u* A1

$= -9{\cdot}2$ f.t. *u* A1

Resultant speed $= \sqrt{(-9\cdot 2)^2 + 30^2(c)}$ M1

$= \underline{31{\cdot}38 \text{ ms}^{-1}}$ f.t. *u* A1

Direction of motion $= \tan^{-1}\left(\frac{9\cdot 2}{30(c)}\right)$ M1

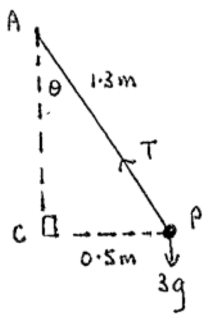



$= \underline{17{\cdot}05^{\circ}}$ [No if +9.2 used] A1

i.e. angle above horizontal

| | | | |
|---|---|---|---|
| **6.** *(a)* $\mathbf{r} = \cos 3t\,\mathbf{i} + \sin 3t\,\mathbf{j}$ | | | |
| $\mathbf{v} = \frac{d}{dt}(\mathbf{r})$ | used | M1 | |
| $= -3\sin 3t\,\mathbf{i} + 3\cos 3t\,\mathbf{j}$ | | A2 | |
| *(b)* Consider $\mathbf{v.r} = (-3\sin 3t\,\mathbf{i} + 3\cos 3t\,\mathbf{j}).(\cos 3t\,\mathbf{i} + \sin 3t\,\mathbf{j})$ | | M1 | |
| $= -3\sin 3t\cos 3t + 3\sin 3t\cos 3t$ | dot product | B1 | |
| $= 0$ | | | |
| $\therefore$ **v** is perpendicular to **r** for all values of $t$ | | A1 | |
| *(c)* Speed of P $= \lvert\mathbf{v}\rvert$ | si | M1 | |
| $= \sqrt{(-3\sin 3t)^2 + (3\cos 3t)^2}$ | | M1 | |
| $= \sqrt{9(\sin^2 3t + \cos^2 3t)}$ | | | |
| $= \underline{3}$ | c.a.o. | A1 | |

**7.** *(a)*

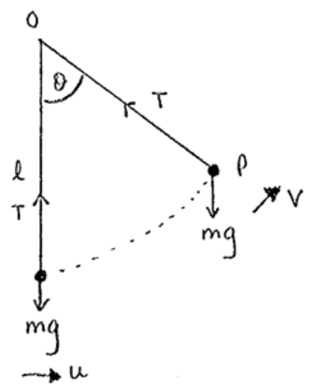



| | | |
|---|---|---|
| Resolve ↑ | | M1 |
| $T\cos\theta = mg$ | | A1 |
| $T = 3 \times 9{\cdot}8 \times \frac{1{\cdot}3}{1{\cdot}2}$ | | |
| $= \underline{31{\cdot}85\text{N}}$ | c.a.o. | A1 |

*(b)* N2L towards C

| | | |
|---|---|---|
| $T\sin\theta = mr\omega^2$ | | M1<br>A1 B1 ($a = r\omega^2$) |
| $\omega^2 = 31{\cdot}85 \times \frac{0{\cdot}5}{1{\cdot}3} \times \frac{1}{3 \times 0{\cdot}5}$ | | |
| $\omega = \underline{2{\cdot}86}$ rads$^{-1}$ | f.t. T | A1 |

**8.**




*(a)* At lowest point

$$T - mg = \frac{mv^2}{r}$$ M1 (ma) A1

T = 2 *mg*, *v* = *u*, *r* = *l* $mg = \frac{mu^2}{l}$ m1

$$u^2 = gl$$

$$u = \sqrt{gl}$$ convincing A1

*(b)* Conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 = mgl\,(1 - \cos\theta)$$ M1

A1 (KE)
A1 (all correct)

$$v^2 = gl - 2gl\,(1 - \cos\theta)$$

$$= 2gl\cos\theta - gl$$

*(c)* At max θ, $v^2 = 0$ used M1

$$\cos\theta = \frac{1}{2}$$

$$\theta = \underline{\frac{\pi^c}{3} = 60^\circ}$$ c.a.o. A1

*(d)* At general θ

$$T - mg\cos\theta = \frac{mv^2}{l}$$ M1(ma)

M1 $\left(\frac{mv^2}{r}\right)$

Subst. for $v^2$ and $r = l$ $T = \frac{m}{l}2gl\cos\theta + mg\cos\theta - \frac{m}{l}\,gl$ m1

$$= 3\,mg\cos\theta - mg$$

T = *mg* $\cos = \frac{2}{3}$

$$\theta = \underline{48{\cdot}2^\circ,\ 0{\cdot}84^\circ}$$ A1

# MATHEMATICS M3

| | | | | |
|---|---|---|---|---|
| **1.** | *(a)* | $a = v\dfrac{dv}{dx}$ | used | M1 |
| | | $= \left(\dfrac{B}{x+A}\right)\left[-B(x+A)^{-2}\right]$ | | A1 |
| | | $= \dfrac{-B^2}{(x+A)^3}$ | | A1 |
| | *(b)* | $t = 0,\ v = 12,\ x = 0$ | | M1 |
| | | $\therefore\ B = 12A$ | | |
| | | $t = 0,\ a = -16,\ x = 0$ | | |
| | | $-B^2 = -16A^3$ | | A1 |
| | | $144A^2 = 16A^3$ | | |
| | | $A = 9$ | | |
| | | $B = 108$ as required | convincing | A1 |
| | *(c)* | $v = \dfrac{dx}{dt} = \dfrac{108}{x+9}$ | | M1 |
| | | $\int(x+9)dx = 108\int dt$ | | |
| | | $\dfrac{x^2}{2} + 9x = 108t + C$ | | A1 |
| | | $t = 0,\ x = 0 \Rightarrow C = 0$ | f.t. minor error | A1 |
| | | $\therefore\ 216t = x^2 + 18x$ | | |
| | | $t = \dfrac{1}{216}x(x+18)$ | | A1 |
| **2.** | | Auxiliary equation $m^2 + 2m + 10 = 0$ | | B1 |
| | | $m = \dfrac{-2 \pm \sqrt{4-40}}{2}$ | | |
| | | $= -1 \pm 3i$ | | B1 |
| | | $\therefore$ C.F. - is $x = e^{-t}(A\sin 3t + B\cos 3t)$ | | B1 |
| | | For P.I. try $x = at + b$ | | M1 |
| | | $\dfrac{dx}{dt} = a$ | | |
| | | $\therefore\ 2a + 10(at + b) = 5t - 14$ | | A1 |
| | | $10a = 5$ | comp. coeff. | M1 |
| | | $a = \dfrac{1}{2}$ | | |

$2a + 10b = -14$

$b = -\frac{3}{2}$ both c.a.o. A1

$\therefore$ General solution is $x = e^{-t}(A \sin 3t + B \cos 3t) + \frac{1}{2}t - \frac{3}{2}$ B1

When $t = 0$, $x = 4\frac{1}{2}$, $\frac{dx}{dt} = 3\frac{1}{2}$ used M1

$4\frac{1}{2} = B - \frac{3}{2}$

$B = 6$ f.t. a.b. A1

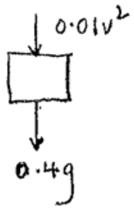
$\frac{dx}{dt} = -e^{-t}(A \sin 3t + B \cos 3t) + e^{-t}(3A \cos 3t - 3B \sin 3t) + \frac{1}{2}$ B1

$3\frac{1}{2} = -B + 3A + \frac{1}{2}$

$A = 3$ c.a.o. A1

$\therefore x = 3e^{-t}(\sin 3t + 2 \cos 3t) + \frac{1}{2}t - \frac{3}{2}$

**3.**




*(a)* N2L $-0{\cdot}01v^2 - 0{\cdot}4g = 0{\cdot}4a$ M1

$0{\cdot}4\, v\frac{dv}{dx} = -3{\cdot}92 - 0{\cdot}01\, v^2$ $a = v\frac{dv}{dx}$ A1

$\times 100$ $40\, v\frac{dv}{dx} = -(392 + v^2)$ convincing A1

*(b)* $40\int \frac{v\,dv}{(392 + v^2)} = -\int dx$ sep. var. M1

$20\, ln\,(392 + v^2) = -x + C$ A1 A1

$t = 0$, $v = 17$, $x = 0$ m1

$\therefore 20\, ln\,(392 + 17^2) = C$

$C = 20\, ln\,(681)$ f.t. minor error A1

$x = 20 ln\,(681) - 20\, ln\,(392 + v^2)$

$= 20\, ln\left(\frac{681}{392 + v^2}\right)$

At greatest height, $v = 0$ m1

$\therefore x = 20\, ln\left(\frac{681}{392}\right)$

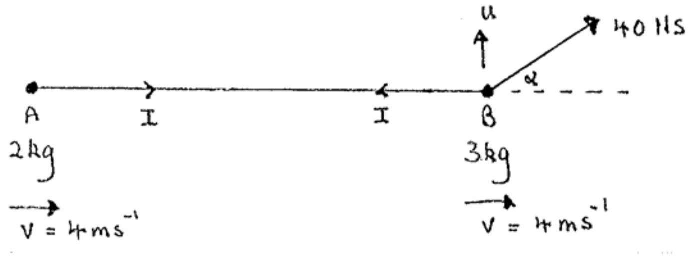
$=$ <u>11.05 m</u> c.a.o. A1

*(c)* Speed of ball when it returns to $O$ is less than 17 $ms^{-1}$ B1

because energy used (lost) in overcoming air resistance. B1

| | | | | |
|---|---|---|---|---|
| **4.** | *(a)* | Period = $\dfrac{2\pi}{\omega} = 4$ | | M1 |
| | | $\omega = \dfrac{\pi}{2}$ | | A1 |
| | | Using $v_{MAX} = a\omega$ with $v_{MAX} = 3\pi$, $\omega = \dfrac{\pi}{2}$ | | M1 |
| | | $3\pi = a \times \dfrac{\pi}{2}$ | | |
| | | $a = \underline{6\text{ m}}$ | c.a.o. | A1 |
| | *(b)* | Using $v^2 = \omega^2 (a^2 - x^2)$ with $\omega = \dfrac{\pi}{2}$ (c), $a = 6$ (c), $x = 4{\cdot}8$ | | M1 |
| | | $v^2 = \dfrac{\pi^2}{4}(36 - 4{\cdot}8^2)$ | f.t. $a$, $\omega$ | A1 |
| | | $v = \underline{1{\cdot}8\pi}$ | | |
| | | $= \underline{5{\cdot}65\text{ ms}^{-1}}$ | f.t. $a$, $\omega$ | A1 |
| | *(c)* | Let $x$ = distance from $O$, $y = 0$, $p$ is at $O$ | | |
| | | $x = 6\sin\left(\dfrac{\pi}{2}t\right)$ | f.t. $a$, $\omega$ | B1 |
| | | $4{\cdot}8 = 6\sin\left(\dfrac{\pi}{2}t\right)$ | | M1 |
| | | $\sin\left(\dfrac{\pi}{2}t\right) = \dfrac{4\cdot 8}{6} = 0\cdot 8$ | | |
| | | $t = \dfrac{2}{\pi}\sin^{-1}(0{\cdot}8)$ | | |
| | | $= \underline{0{\cdot}59s}$ | f.t. $a$, $\omega$ | A1 |
| | *(d)* | Max acceleration when $x = a$ | | M1 |
| | | $\lvert$ Max acceleration $\rvert = \omega^2 a$ | | M1 |
| | | $= \dfrac{\pi^2}{4} \times 6$ | | |
| | | $= \dfrac{3\pi^2}{2}$ | | |
| | | $= \underline{14{\cdot}8\text{ ms}^{-1}}$ | f.t. $a$, $\omega$ | A1 |
| | *(e)* | Distance travelled $= \dfrac{12}{4}$ oscillations | | |
| | | $= 3 \times (4a)$ | | M1 |
| | | $= \underline{72\text{m}}$ | f.t. $a$ | A1 |

**5.** *(a)*



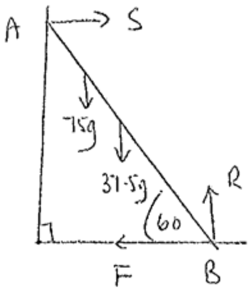

Impulse = change in momentum

| | | | | |
|---|---|---|---|---|
| Apply to A | I | = | $2v$ | M1 |
| | | = | $2 \times 4$ | A1 |
| Apply to B | – I | = | $-40\cos\alpha + 3v$ | M1 |
| | | = | $-40\cos\alpha + 3 \times 4$ | A1 |
| ∴ | –8 | = | $-40\cos\alpha + 12$ | M1 |
| | $40\cos\alpha$ | = | 20 | |
| | $\cos\alpha$ | = | $\frac{1}{2}$ | |
| | $\alpha$ | = | <u>60°</u> | A1 |

*(b)*

| | | | | |
|---|---|---|---|---|
| | $40\sin\alpha$ | = | $3u$ | |
| | $u$ | = | $40 \times \frac{\sqrt{3}}{2} \times \frac{1}{3}$ | |
| | | = | $\frac{20\sqrt{3}}{3}$ | A1 |
| Speed of *b* | | = | $\sqrt{\left(\frac{20\sqrt{3}}{3}\right)^2 + 4^2}$ | M1 |
| | | = | <u>12·22 ms⁻¹</u> | A1 |
| | $\theta$ | = | $\tan^{-1}\left(\frac{20\sqrt{3}}{3 \times 4}\right)$ | M1 |
| | | = | <u>70·89°</u> | A1 |

**6.** *(a)*




| | | |
|---|---|---|
| Moments about $B$ | dim correct, attempted equation | M1 |
| $37{\cdot}5g \times 4\cos 60° + 75g \times x\cos 60° = S \times 8\sin 60°$ | | B1 A2 |
| Resolve ↑ $R = 37{\cdot}5\,g + 75\,g$ | | M1 |
| $= 112{\cdot}5\,g$ | | |
| Resolved → $S = F$ | | M1 |
| $F = \mu R$ | | M1 |
| $\therefore S = \mu.\, 112{\cdot}5g$ | | |
| Substitute $S$ into moment equation and $\mu = 0{\cdot}25$ | | m1 |
| $x\,(75\,g\cos 60°) = 0{\cdot}25 \times 112{\cdot}5\,g \times 8\sin 60° - 37{\cdot}5\,g \times 4\cos 60°$ | | A1 |
| $x = \dfrac{112\cdot 5\sqrt{3} - 75}{37\cdot 5}$ | | |
| $= \underline{3{\cdot}196}$ | c.a.o. | A1 |

*(b)*

| | | |
|---|---|---|
| Substitute $x = 8$ and $s = \mu.112{\cdot}5g$ (c) into moment equation | | M2 |
| $\mu.112{\cdot}5g \times 8\sin 60° = 37{\cdot}5g \times 4\cos 60° + 75g \times 8\cos 60°$ | | A1 |
| $\mu. = \dfrac{300 + 75}{450\sqrt{3}}$ | | |
| $= \underline{0{\cdot}481}$ | c.a.o. | A1 |

*(c)*

| | |
|---|---|
| Person modelled as particle/<br>Ladder modelled as rod | B1 |

## MATHEMATICS S1

| | | | |
|---|---|---|---|
| **1.** | *(a)* | P(A eats red sweet) = $\frac{1}{3}$ | B1 |
| | *(b)* | P(B eats red sweet) = $\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$ | M1A1 |
| | | [Method must be shown, award M1 for either $\frac{2}{3}\times\frac{1}{2}$ or $\frac{2}{3}\times\frac{1}{3}$ ] | |
| | *(c)* | P(C eats red sweet) = $\frac{2}{3}\times\frac{1}{2}\times\frac{1}{1}$ or $1-\frac{2}{3}$ | M1 |
| | | = $\frac{1}{3}$ (no working required) | A1 |
| | | [FT on answers to (a) and (b) - Special case – award 3/5 for writing down the correct answers with no working] | |
| **2.** | *(a)* | P($A\cap B$) = P($A$) + P($B$) – P($A\cup B$) | |
| | | = 0.05 | M1A1 |
| | | P($A$).P($B$) = 0 ·12 or P(A\| B) = 1/12 or P(B\| A) = 1/4 | B1 |
| | | *A* and *B* are not independent | B1 |
| | *(b)* | P(exactly one of A,B) = $\text{P}(A\cap B')+\text{P}(A'\cap B)$ | M1 |
| | | [Award M0 if independence assumed having stated not independent in (a). If independence stated in (a), FT partially bearing in mind that the problem is now easier] | |
| | | = 0·2 – 0·05 + 0·6 – 0·05 | A1A1 |
| | | = 0·7 | A1 |
| | | OR | |
| | | P(exactly one of *A*,*B*) = P($A\cup B$) – P($A\cap B$) | M1A1 |
| | | = 075 – 0·5 | A1 |
| | | = 07 | A1 |
| **3.** | *(a)* | Var($X$) = 4 | B1 |
| | | E($Y$) = 2 × 4 + 8 = 16 | M1A1 |
| | | Var($Y$) = 4 × 4 = 16 | M1A1 |
| | *(b)* | Because *Y* can only take the values 8,10,12 etc | B1 |

| | | | | |
|---|---|---|---|---|
| **4.** | *(a)* | (i) | $P(X > 10) = 0{\cdot}6528$ (or $1 - 0{\cdot}3472$) | M1A1 |
| | | (ii) | $P(X = 15) = 0{\cdot}8444 - 0{\cdot}7720$ or $0{\cdot}2280 - 0{\cdot}1556$ | B1B1 |
| | | | $= 0{\cdot}0724$ (cao) | B1 |
| | *(b)* | (i) | $P(Y = 5) = e^{-6.3}.\dfrac{6\cdot 3^5}{5!} = 0\cdot 152$ | M1A1 |
| | | (ii) | $P(Y < 3) = e^{-6.3}\left(1 + 6\cdot 3 + \dfrac{6\cdot 3^2}{2}\right)$ | M1A1 |
| | | | $= 0{\cdot}0498$ (cao) | A1 |
| **5.** | *(a)* | | $P(DY) = 0\cdot 4\times 0\cdot 04 + 0\cdot 35\times 0\cdot 05 + 0\cdot 25\times 0\cdot 06$ | M1A1 |
| | | | $= 0{\cdot}0485$ | A1 |
| | *(b)* | (i) | $P(A\mid DY) = \dfrac{0\cdot 4\times 0\cdot 04}{0\cdot 0485}$ | B1B1 |
| | | | $= 0{\cdot}330$ | B1 |
| | | (ii) | $P(B\mid DY) = \dfrac{0\cdot 35\times 0\cdot 05}{0\cdot 0485}$ | |
| | | | $= 0{\cdot}361$ | B1 |
| | | | $P(C\mid DY) = 1 - 0{\cdot}691 = 0{\cdot}309$ (si) | B1 |
| | | | So most likely to have come from Farm B. | B1 |
| **6.** | *(a)* | (i) | B(50,0.2) | B1 |
| | | (ii) | Mean $= 50 \times 0{\cdot}2 = 10$ | M1A1 |
| | | | Var $= 50 \times 0{\cdot}2 \times 0{\cdot}8$ | M1 |
| | | | SD $= 2{\cdot}83$ $(2\sqrt{2})$ | A1 |
| | | (iii) | $P(8 \le X \le 12) = 0{\cdot}8139 - 0{\cdot}1904$ or $0{\cdot}8096 - 0{\cdot}1861$ | B1B1 |
| | | | $= 0{\cdot}6235$ (cao) | B1 |
| | | | [For candidates summing probs award M1A1 for an expression involving binomial probs and A1 for the answer] | |
| | *(b)* | | $X$ is approx Po(10). | M1A1 |
| | | | $P(X < 10) = 0{\cdot}4579$ (or $1 - 0{\cdot}5421$) | M1A1 |
| **7.** | *(a)* | | Sum of probabilities $= 15k$ | B1 |
| | | | $15k = 1$ so $k = \dfrac{1}{15}$ | B1 |
| | *(b)* | | $E(X) = 1\times\dfrac{1}{15} + 2\times\dfrac{2}{15} + ... + 5\times\dfrac{5}{15}$ | M1 |
| | | | $= \dfrac{11}{3}$ | A1 |
| | | | $E(X^2) = 1\times\dfrac{1}{15} + 4\times\dfrac{2}{15} + ... + 25\times\dfrac{5}{15}$ | M1A1 |
| | | | $(= 15)$ | |
| | | | $Var(X) = 15 - \left(\dfrac{11}{3}\right)^2$ | M1 |
| | | | $= 1.56$ | A1 |

*(c)* The possibilities are (1,5), (2,4),(3,3).

$P(Y = 6) = 2\times\frac{1}{15}\times\frac{5}{15}+2\times\frac{2}{15}\times\frac{4}{15}+\left(\frac{3}{15}\right)^2$ B1B1B1

$= \frac{7}{45}$ B1

**8.** *(a)* (i) $P(0{\cdot}25 \le X \le 0{\cdot}5) = F(0{\cdot}5) - F(0{\cdot}25)$ M1

$= \frac{1}{2}\left(0\cdot 5^2+0\cdot 5\right)-\frac{1}{2}(0\cdot 25^2+0\cdot 25)$ A1

$= \frac{7}{32}$ (0·219) A1

(ii) The median satisfies

$\frac{1}{2}\left(m^2+m\right)=\frac{1}{2}$ M1

$m^2+m-1=0$ A1

$m=\frac{-1+\sqrt{5}}{2}$ M1

$= 0{\cdot}618$ A1

[Special case for candidates integrating $F(x)$:-
For obtaining $2m^3+3m^2-6=0$ M1A1]

*(b)* (i) $f(x) = F'(x)$ M1

$= \frac{1}{2}(2x+1)$ A1

(ii) $E(X)=\frac{1}{2}\int_0^1 x(2x+1)\mathrm{d}x$ M1A1

[FT on candidate's $f(x)$ from (i). If candidates use $F(x)$ instead of $f(x)$, not having obtained an answer in (i), award M1]

$= \frac{1}{2}\left[\frac{2x^3}{3}+\frac{x^2}{2}\right]_0^1$ A1

$=\frac{7}{12}$ (0·583) A1

# MATHEMATICS S2

**1.** $\bar{x} = \dfrac{62 \cdot 6}{10}$ (= 6·26) — B1

SE of $\bar{x} = \dfrac{0 \cdot 1}{\sqrt{10}}$ (= 0·0316) — B1

95% conf limits are

6·26 ± 1.96 × 0·0316 — M1A1

[M1 correct form, A1 1·96]

giving [6·20,6·32] — A1

Yes because 6·3 is within the interval. — B1

**2.** Variance = $\dfrac{(b-a)^2}{12} = 3$ — M1A1

$(b-a)^2 = 36$ — A1

$b - a = 6$ — AG

Mean = $\dfrac{a+b}{2} = 10$ — M1

$b + a = 20$ — A1

Solving, $a = 7$, $b = 13$. — M1A1

**3.** *(a)* (i) $z_1 = \dfrac{34-30}{2} = 2\,;\, z_2 = \dfrac{28-30}{2} = -1$ — M1A1

Prob = 0·97725 – 0·15866 or 0·8413 – 0·02275 — B1B1

= 0·819 (cao) — B1

(ii) Reqd weight = 25 + 2·326 × 1·8 — M1A1

= 29·2 — A1

[M1 for 25 ± zσ]

*(b)* $X - Y$ is N(5,7·24) — B1B1

We require P($X - Y$ > 0)

$z = \dfrac{5}{\sqrt{7 \cdot 24}} = (\pm)1.86$ — M1A1

Prob = 0.969 (cao) — A1

**4.** *(a)* Prob of 1 crash on a computer $= 0 \cdot 8 \times e^{-0.8}$ — M1

= 0·3595 — A1

Prob of 1 crash on each of 5 computers = $0{\cdot}3595^5$ — M1

= 0·006 — A1

*(b)* Distribution of Total is Po(4). — B1

P(Total = 5) $= e^{-4} . \dfrac{4^5}{5!}$ — M1A1

= 0·156 (cao) — A1

**5.** *(a)* $H_0 : \mu = 2 \cdot 4$ versus $H_1 : \mu > 2 \cdot 4$ (Accept μ = 12] B1

In 5 days, number of passengers $Y$ is Poi(12) under $H_0$. si B1
[M1A0 for normal approx]

$p$-value = $P(Y \geq 18)$ = 0·0630 M1 A1
We cannot conclude that the mean has increased. B1

*(b)* Under $H_0$ the number of passengers in 100 days is Po(240) ≈ N(240,240) B1B1

$$z = \frac{279 \cdot 5 - 240}{\sqrt{240}}$$ M1A1

= 2·55 A1
Either $p$-value = 0·00539 or CV = 2·326 A1
[No cc gives $z$ = 2·58, $p$ = 0·00494, wrong cc gives $z$ = 2·61, $p$ = 0·00453]
We conclude at the 1% level that the mean has increased. B1

**6.** *(a)* (i) $X$ is B(50,$p$) (si) B1
Sig level = P($X \leq 14 \mid p = 0{\cdot}4$) M1
= 0·0540 (cao) A1
(ii) We require
P($X \geq 15 \mid p = 0{\cdot}3$) = 0·5532 M1A1

*(b)* Under $H_0$, $X$ is now B(500,0·4) ≈ N(200,120) B1B1

$$z = \frac{185 \cdot 5 - 200}{\sqrt{120}}$$ M1A1

= –1·32 A1
$p$-value = 0·0934 A1

[No cc gives $z$ = –1·37, $p$ = 0·0853, wrong cc gives $z$ = –1·41, $p$ = 0·0793]
Insufficient evidence to support the agent's belief. (oe). B1

**7.** *(a)* $H_0 : \mu_A = \mu_B$ versus $H_1 : \mu_A \neq \mu_B$ B1

*(b)* $\bar{x}_A = \frac{501}{6} = 83 \cdot 5$ B1

$\bar{x}_B = \frac{489}{6} = 81 \cdot 5$ B1

The appropriate test statistic is

$$\text{TS} = \frac{\bar{x} - \bar{y}}{\sigma\sqrt{\frac{1}{m} + \frac{1}{n}}}$$ M1

$$= \frac{83 \cdot 5 - 81 \cdot 5}{1.5\sqrt{\frac{1}{6} + \frac{1}{6}}}$$ 1A1

= 2.31 (cao) A1
Prob from tables = 0·01044 A1
$p$-value = 0·021 B1
(i) Accept $H_0$ (or the fuel consumptions are the same) at 1% SL B1
(ii) Accept $H_1$ (or the fuel consumptions are not the same) at 5% SL B1

**8.** *(a)* The **mean** of a **large** (random) sample from any distribution is (approximately) **normally** distributed. B1

*(b)* $E(\overline{X}) = 3.5, \text{Var}(\overline{X}) = \dfrac{35}{600}$ (si) B1B1

$$z = \frac{3 - 3\cdot 5}{\sqrt{35/600}} = -2\cdot 07$$ M1A1

Prob = 0·981 A1

# MATHEMATICS S3

**1.** *(a)* The possible combinations are given in the following table.

| Combination |
|---|
| 1 1 2 |
| 1 1 3 |
| 1 1 4 |
| 1 2 3 |
| 1 2 4 |
| 1 3 4 |
| 2 3 4 |

[Accept a table with 123, 124 and 134 repeated] M1A1

The sampling distribution of the mean is

| Mean | 4/3 | 5/3 | 2 | 7/3 | 8/3 | 3 |
|---|---|---|---|---|---|---|
| Prob | 0·1 | 0·1 | 0·3 | 0·2 | 0·2 | 0·1 |

(correct means) B1
(correct probs) B2

[Award B1 for 4 correct probs]

The sampling distribution of the median is

| Median | 1 | 2 | 3 |
|---|---|---|---|
| Prob | 0.3 | 0.4 | 0.3 |

(correct medians) B1
(correct probs) B2

[For each table award B1 for correct probs with replacement]

**2.** *(a)* $\hat{p} = \frac{498}{1200} = 0 \cdot 415$ B1

*(b)* $\text{SE} \approx \sqrt{\frac{0 \cdot 415 \times 0 \cdot 585}{1200}}$ M1

$= 0{\cdot}0142...$ A1

*(c)* Approx 90% confidence limits are

$0{\cdot}415 \pm 1{\cdot}645 \times 0{\cdot}0142$ M1A1

giving [0·392, 0·438]. A1

*(d)* We are using a normal approximation to a binomial situation. B1

The standard error and/or *p* are estimated and are not exact. B1

| | | | |
|---|---|---|---|
| **3**. | | $\bar{x}_A = 1\cdot034; \bar{x}_B = 1\cdot016$ | B1B1 |
| | | $s_A^2 = 3\cdot474747...\times10^{-4}$ | M1A1 |
| | | $s_B^2 = 1\cdot449664...\times10^{-4}$ | A1 |
| | | [Accept division by *n* giving 3·44.. and 1·44..] | |
| | | SE = $\sqrt{\dfrac{3\cdot474747...\times10^{-4}}{100}+\dfrac{1\cdot449664...\times10^{-4}}{150}}$ (= 0·002107) | B1 |
| | | 95% conf lims are 1·034 – 1·016 ± 1·96 × 0·002107 | M1A1 |
| | | giving [0·014,0·022] (cao) | A1 |
| **4.** | *(a)* | UE of $\mu = 8\cdot54$ | B1 |
| | | $\sum x^2 = 734\cdot2$, | B1 |
| | | UE of $\sigma^2 = \dfrac{734\cdot2}{9} - \dfrac{85\cdot4^2}{9\times10}$ | M1 |
| | | = 0·542(666..) | A1 |
| | *(b)* | $H_0 : \mu = 9$ versus $H_1 : \mu \neq 9$ | B1 |
| | | Test statistic = $\dfrac{8\cdot54-9}{\sqrt{0\cdot542666../10}}$ | M1A1 |
| | | = – 1·97 | A1 |
| | | DF = 9 | B1 |
| | | Crit value = 2·26 | B1 |
| | | Insufficient evidence to reject the farmer's claim at the 5% level. | B1 |
| | | [Award the final B1 only if *t* used] | |
| **5.** | *(a)* | Σ*x* = 75, Σ*y* = 89·2, Σ*xy* = 1270·5, $\Sigma x^2 = 1375$ | B2 |
| | | [B1 1 error] | |
| | *(b)* | $b = \dfrac{6\times1270\cdot5-75\times89\cdot2}{6\times1375-75^2}$ | M1A1 |
| | | = 0·355 | A1 |
| | | $a = \dfrac{89\cdot2-75\times0\cdot355}{6}$ | M1A1 |
| | | = 10·4 | A1 |
| | | [Note $S_{xx} = 437\cdot5$, $S_{xy} = 155\cdot5$] | |
| | *(c)* | (i) Est resist = 10·4 + 0·355 × 20 = 17·5 | M1 A1 |
| | | SE = $0.4\sqrt{\dfrac{1}{6}+\dfrac{(20-12\cdot5)^2}{1375-75^2/6}}$ | M1 |
| | | = 0·217(343…) | A1 |
| | | (ii) 95% confidence limits are | |
| | | 17·5 ± 1·96 × 0·217 | M1 |
| | | giving [17·1,18·0] | A1 |
| | | [Accept 17·9 as the upper limit] | |

| | | | |
|---|---|---|---|
| | *(d)* | Test stat = $\dfrac{b-\beta}{\sigma/\sqrt{(\Sigma x^2-(\Sigma x)^2/n)}}$ | M1 |
| | | $= \dfrac{0\cdot 355...-0\cdot 4}{0.4/\sqrt{(1375-75^2/6}}$ | A1A1 |
| | | = – 2·33 (Accept 2·34, 2·35) | A1 |
| | EITHER *p*-value = 2 × 0·0099 = 0·0198 OR Crit value = 2·576 | | A1 |
| | The value 0·4 is consistent with his prediction at the 1% level. | | B1 |
| **6.** | *(a)* | E(*U*) = *a*(μ + 2 × 2μ) | M1A1 |
| | | = μ if $a = \dfrac{1}{5}$ | A1 |
| | | E(V) = b(2 ×μ + 2μ) = μ | M1 |
| | | if $b = \dfrac{1}{4}$ | A1 |
| | *(b)* | $\text{Var}(U) = \dfrac{1}{25}\left(\sigma^2+4\times 3\sigma^2\right)=\dfrac{13}{25}\sigma^2$ | M1A1 |
| | | $\text{Var}(V) = \dfrac{1}{16}\left(4\times\sigma^2+3\sigma^2\right)=\dfrac{7}{16}\sigma^2$ | M1A1 |
| | | *V* is the better estimator (since it has the smaller variance). | B1 |
| | *(c)* | (i) $\text{E}(W) = \dfrac{\mu+k\times 2\mu}{1+2k}=\mu$ | M1A1 |
| | | [AG so must be convincing] | |
| | | (ii) $\text{Var(W)} = \dfrac{\sigma^2+k^2\times 3\sigma^2}{(1+2k)^2} = \dfrac{(1+3k^2)}{(1+2k)^2}\ \sigma^2$ | M1A1 |
| | | (iii) $\dfrac{\text{d}}{\text{d}k}\left(\text{Var}(W)\right)=\dfrac{6k(1+2k)^2-4(1+2k)(1+3k^2)}{(1+2k)^4}$ | M1A1 |
| | | For minimum variance, | |
| | | $6k(1+2k)^2 = 4(1+2k)(1+3k^2)$ | M1 |
| | | $3k(1+2k) = 2(1+3k^2)$ | A1 |
| | | $k = \dfrac{2}{3}$ | A1 |
| | | [Award full marks if correct answer given with no/partial working] | |

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