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GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2009

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2009 examination in GCE MATHEMATICS . They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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Mathematics C1 May 2009

Solutions and Mark Scheme

1. (*a*) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $BC = {}^{3}/_{4}$ (or equivalent) A1

(*b*) A correct method for finding C M1

$C\ (3, 8)$ A1

(*c*) Use of $m_{AB} \times m_L = -1$ to find gradient of L M1

A correct method for finding the equation of L using candidate's coordinates for C and candidate's gradient for L. M1

Equation of L: $y - 8 = -{}^{4}/_{3}\,(x - 3)$ (or equivalent)
(f.t. candidate's coordinates for C and candidate's gradient for L) A1

Equation of L: $4x + 3y - 36 = 0$ (convincing, c.a.o.) A1

(*d*) (i) Substituting $y = 0$ in equation of L M1

$D\ (9, 0)$ A1

(ii) A correct method for finding the length of CD (AC) M1

$CD = 10$ (f.t. candidate's coordinates for C and D) A1

(iii) $AC = 5$ (f.t. candidate's coordinates for C) A1

$\tan C\hat{A}D = \dfrac{CD}{AC} = 2$ (or ${}^{10}/_{5}$ or equivalent)

(f.t. candidate's derived values for CD and AC) B1

2. (*a*) $\dfrac{8-\sqrt{7}}{\sqrt{7}-2} = \dfrac{(8-\sqrt{7})(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$ M1

Numerator: $8\sqrt{7}+16-7-2\sqrt{7}$ A1

Denominator: $7-4$ A1

$\dfrac{8-\sqrt{7}}{\sqrt{7}-2} = \dfrac{6\sqrt{7}+9}{3} = 2\sqrt{7}+3$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{7}-2$

(*b*) $\sqrt{50} = 5\sqrt{2}$ B1

$\sqrt{3}\times\sqrt{6} = 3\sqrt{2}$ B1

$-\dfrac{14}{\sqrt{2}} = -7\sqrt{2}$ B1

$\sqrt{50}+(\sqrt{3}\times\sqrt{6})-\dfrac{14}{\sqrt{2}} = \sqrt{2}$ (c.a.o.) B1

3. $\dfrac{dy}{dx} = 4x + 6$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = -1$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Gradient of tangent at $P = 2$ (c.a.o.) A1

y-coordinate at $P = 3$ B1

Equation of tangent at P: $y - 3 = 2[x - (-1)]$ (or equivalent)

(f.t. one slip provided both M1 and m1 awarded) A1

4. (*a*) (i) $a = -2{\cdot}5$ (or equivalent) B1

$b = 1{\cdot}75$ (or equivalent) B1

(ii) Greatest value $= -b$ (or equivalent) B1

(*b*) $x^2 - x - 7 = 2x + 3$ M1

An attempt to collect terms, form and solve quadratic equation m1

$x^2 - 3x - 10 = 0 \Rightarrow (x-5)(x+2) = 0 \Rightarrow x = 5, x = -2$

(both values, c.a.o.) A1

When $x = 5$, $y = 13$, when $x = -2$, $y = -1$

(both values f.t. one slip) A1

The line $y = 2x + 3$ intersects the curve $y = x^2 - x - 7$ at the points $(-2, -1)$ and $(5, 13)$ (f.t. candidate's points) E1

5. (*a*) $y + \delta y = 4(x + \delta x)^2 - 5(x + \delta x) - 3$ B1

Subtracting y from above to find δy M1

$\delta y = 8x\delta x + 4(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 8x - 5$ (c.a.o.) A1

(*b*) Required derivative $= 7 \times {}^3/_4 \times x^{-1/4} - 2 \times (-4) \times x^{-5}$ B1, B1

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = (2k)^2 - 4(k+1)(k-1)$ A1

$b^2 - 4ac = 4$ (c.a.o.) A1

candidate's value for $b^2 - 4ac > 0$ ($\Rightarrow$ two distinct real roots) A1

(*b*) Finding critical values $x = -2$, $x = {}^3/_5$ B1

$-2 \le x \le {}^3/_5$ **or** ${}^3/_5 \ge x \ge -2$ **or** $[-2, {}^3/_5]$ **or** $-2 \le x$ and $x \le {}^3/_5$
or a correctly worded statement to the effect that x lies between -2 and ${}^3/_5$ (both inclusive)
(f.t. critical values ± 2, $\pm {}^3/_5$) B2

Note: $-2 < x < {}^3/_5$,
$-2 \le x, x \le {}^3/_5$
$-2 \le x \;\; x \le {}^3/_5$
$-2 \le x$ or $x \le {}^3/_5$
all earn B1

7. (*a*) $\left(x + \frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$

(three terms correct) B1
(all terms correct) B2

$\left(x + \frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (three terms correct) B1

(all terms correct) B2

(-1 for incorrect further 'simplification')

(*b*) A correct equation in n, including ${}^nC_2 = 55$ M1

$n = 11, \; -10$ (c.a.o.) A1

$n = 11$ (f.t. $n = 10$ from $n = -11, 10$) A1

8. (*a*) Use of $f(-1) = -3$ M1

$-a - 1 + 6 + 5 = -3 \Rightarrow a = 2$

A1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1

$f(2) = 0 \Rightarrow x - 2$ is a factor A1

$f(x) = (x - 2)(8x^2 + ax + b)$ with one of a, b correct M1

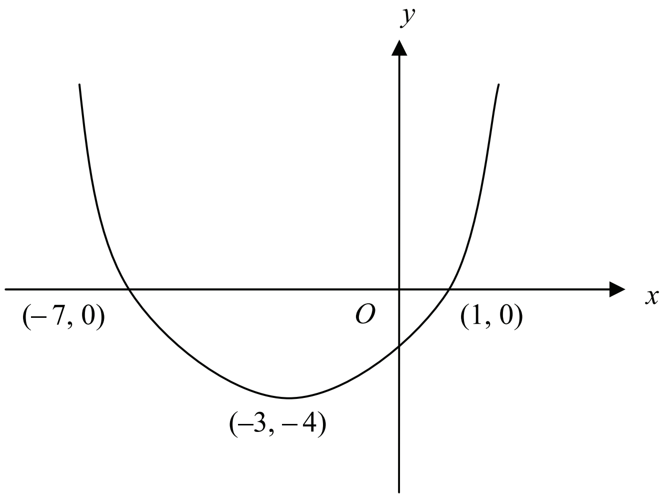
$f(x) = (x - 2)(8x^2 + 2x - 3)$ A1

$f(x) = (x - 2)(4x + 3)(2x - 1)$ (f.t. only $8x^2 - 2x - 3$ in above line) A1

Special case

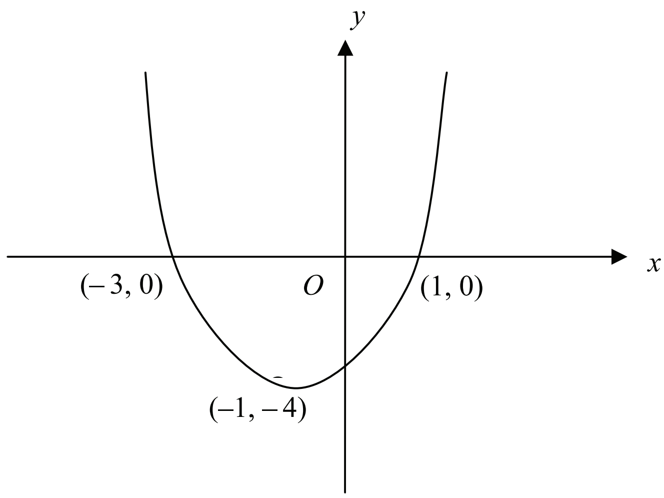
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 3 marks

9. (*a*)



Concave up curve and y-coordinate of minimum = – 4	B1
x-coordinate of minimum = – 3	B1
Both points of intersection with x-axis	B1

(*b*)



Concave up curve and y-coordinate of minimum = – 4	B1
x-coordinate of minimum = – 1	B1
Both points of intersection with x-axis	B1

10. (*a*) $\frac{dy}{dx} = 3x^2 - 6x + 3$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$3(x - 1)^2 = 0 \Rightarrow x = 1$ A1

$x = 1 \Rightarrow y = 6 \Rightarrow$ stationary point is at (1, 6) (c.a.o) A1

(*b*) **Either:**

An attempt to consider value of $\frac{dy}{dx}$ at $x = 1^-$ and $x = 1^+$ M1

$\frac{dy}{dx}$ has same sign at $x = 1^-$ and $x = 1^+ \Rightarrow$ (1, 6) is a point of inflection A1

Or:

An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = 1$, $x = 1^-$ and $x = 1^+$ M1

$\frac{d^2y}{dx^2} = 0$ at $x = 1$ and $\frac{d^2y}{dx^2}$ has different signs at $x = 1^-$ and $x = 1^+$

$\Rightarrow$ (1, 6) is a point of inflection A1

Or:

An attempt to find the value of y at $x = 1^-$ and $x = 1^+$ M1

Value of y at $x = 1^- < 6$ and value of y at $x = 1^+ > 6 \Rightarrow$ (1, 6) is a point of inflection A1

Or:

An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 1$ M1

$\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = 1 \Rightarrow$ (1, 6) is a point of inflection A1

Mathematics C2 May 2009

Solutions and Mark Scheme

1.

0	0·5		
0·1	0·43173647		
0·2	0·408628001		
0·3	0·392507416	(3 values correct)	B1
0·4	0·379873463	(5 values correct)	B1

Correct formula with $h = 0{\cdot}1$ M1

$I \approx \frac{0{\cdot}1}{2} \times \{0{\cdot}5 + 0{\cdot}379873463 + 2(0{\cdot}43173647 + 0{\cdot}408628001 + 0{\cdot}392507416)\}$

$I \approx 0{\cdot}167280861$

$I \approx 0{\cdot}167$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}08$

0	0·5		
0·08	0·438050328		
0·16	0·416666666		
0·24	0·401622886		
0·32	0·389759395		
0·4	0·379873463	(all values correct)	B1

Correct formula with $h = 0{\cdot}08$ M1

$I \approx \frac{0{\cdot}08}{2} \times \{0{\cdot}5 + 0{\cdot}379873463 + 2(0{\cdot}438050328 + 0{\cdot}416666666 + 0{\cdot}401622886 + 0{\cdot}389759395)\}$

$I \approx 0{\cdot}16688288$

$I \approx 0{\cdot}167$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) $5\cos^2\theta + 2 = 3(1 - \cos^2\theta) - 2\cos\theta.$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$8\cos^2\theta + 2\cos\theta - 1 = 0 \Rightarrow (4\cos\theta - 1)(2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{4}, -\frac{1}{2}$ A1

$\theta = 75{\cdot}5(225)°, 284{\cdot}4(775)°, 120°, 240°$ (75·5°, 284·5°) B1

(120°) B1

(240°) B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks
$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $2x + 12° = -32, 212°, 328°$ (one value) M1

$x = 100°, 158°$ (one value) A1

(two values) A1

Note: Subtract 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $\frac{\sin A\hat{C}B}{16} = \frac{\sin 23°}{9}$

(substituting the correct values in the correct places in the sin rule) M1

$A\hat{C}B = 44°, 136°$ (both values) A1

(*b*) (i) Use of angle sum of a triangle = 180° M1

$B\hat{A}C = 21°$

(f.t. candidate's values for $A\hat{C}B$ provided acute value < 67°) A1

(ii) Area of triangle $ABC = \frac{1}{2} \times 16 \times 9 \times \sin 21°$

(substituting 16, 9 and candidate's value for $B\hat{A}C$ in the correct places in the area formula) M1

Area of triangle $ABC = 25{\cdot}8\ \text{cm}^2$.

(f.t. candidate's acute value for $B\hat{A}C$) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$ Or:

$2S_n = [a + a + (n-1)d] + \ldots$ (n times) M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $a + 7d = 46$ B1

$\frac{9}{2} \times [2a + 8d] = 225$ B1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$a = -3$, $d = 7$ (both values) (c.a.o.) A1

(*c*) $a = 3$, $d = 4$ B1

$S_n = \frac{n}{2}[2 \times 3 + (n-1)4]$ (f.t. candidate's d) M1

$S_n = n(2n + 1)$ (c.a.o.) A1

5. (*a*) $r = \frac{108}{36} = 3$ (c.a.o.) B1

$t_7 = \frac{36}{3^2}$ (f.t. candidate's value for r) M1

$t_7 = 4$ (c.a.o.) A1

(*b*) (i) $ar = 9$ B1

$\frac{a}{1-r} = 48$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$16r^2 - 16r + 3 = 0$ (convincing) A1

(ii) $r = {}^1/_4, {}^3/_4$ B1

$a = 36, 12$ (c.a.o.) B1

6. (*a*) $5 \times \frac{x^{-2}}{-2} - 3 \times \frac{x^{5/4}}{5/4} + c$ (Deduct 1 mark if no c present) B1,B1

(*b*) (i) $6 + 4x - x^2 = x + 2$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b$ = candidate's constant m1

$(x - 4)(x + 1) = 0 \Rightarrow x = 4, y = 6$ at A (both values) A1

Note: Answer only with no working earns 0 marks

(ii) **Either:**

Total area $= \int_0^4 (6 + 4x - x^2)\,dx - \int_0^4 (x + 2)\,dx$

(use of integration) M1

$= [4x + (3/2)x^2 - (1/3)x^3]_0^4$

(correct integration) B3

$= 16 + 24 - 4^3/3$

(f.t. candidate's limits in at least one integral) m1

Correct subtraction of integrals with correct use of 0 and candidate's x_A as limits m1

$= \frac{56}{3}$ (c.a.o.) A1

Or:

Area of trapezium = 16

(f.t. candidate's coordinates for A) B1

Area under curve $= \int_0^4 (6 + 4x - x^2)\,dx$ (use of integration) M1

$= [6x + 2x^2 - (1/3)x^3]_0^4$

(correct integration) B2

$= 24 + 32 - 64/3$

(f.t. candidate's limits) m1

$= \frac{104}{3}$

Finding total area by subtracting values m1

Total area $= \frac{104}{3} - 16 = \frac{56}{3}$ (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$
Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1
$\dfrac{x}{y} = \dfrac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1
$\log_a x/y = p - q$ (the relationship between log and power)
$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) $(5 - 2x) \log 3 = \log 7$ (taking logs on both sides) M1
An attempt to isolate x (no more than 1 slip) m1
$x = 1{\cdot}614$ (c.a.o.) A1
Note: Candidates who write down $x = 1{\cdot}614$ without explanation are awarded M0 m0 A0

(*c*) $\log_a (x - 3) + \log_a (x + 3) = \log_a [(x - 3)(x + 3)]$ (addition law) B1
$2 \log_a (x - 2) = \log_a (x - 2)^2$ (power law) B1
$(x - 3)(x + 3) = (x - 2)^2$ (removing logs) M1
$x = 3{\cdot}25$ (c.a.o.) A1

8. (*a*) $A\,(3, -1)$ B1
A correct method for finding the radius M1
Radius = 5 A1

(*b*) Gradient $AP = \dfrac{\text{inc in } y}{\text{inc in } x}$ M1
Gradient $AP = \dfrac{2 - (-1)}{7 - 3} = \dfrac{3}{4}$ (f.t. candidate's coordinates for A) A1
Use of $m_{\tan} \times m_{\text{rad}} = -1$ M1
Equation of tangent is:
$y - 2 = -\dfrac{4}{3}(x - 7)$ (f.t. candidate's gradient for AP) A1

(*c*) Distance between centres of C_1 and $C_2 = 17$ B1
(f.t. candidate's coordinates for A)
Use of the fact that distance between centres =
sum of the radii + the shortest possible length of the line QR M1
Shortest possible length of the line $QR = 4$ (f.t. one slip) A1

9. (*a*) $\dfrac{1}{2} \times 13 \times 13 \times \theta = 60$ M1
$\theta = 0{\cdot}71$ A1

(*b*) $QR = 13\varphi$, $RS = 13(\pi - \varphi)$, (at least one value) B1
$13\varphi = 13(\pi - \varphi) \pm 7$ (or equivalent) M1
$\varphi = 1{\cdot}84$ (c.a.o.) A1

Mathematics C3 Summer 2009

Solutions and Mark Scheme

1. $h = 0{\cdot}2$ ($h = 0{\cdot}2$, correct formula) M1

Integral $\approx \frac{0 \cdot 2}{3}[3 + 3 \cdot 7191397 + 4(3 \cdot 1189742 + 3 \cdot 4779304) + 2(3 \cdot 2778041)]$

(3 values) B1
(2 values) B1

$\approx 2 \cdot 6442$ A1

[Special case: B1 for 6 correct values, at least 5 decimal places:
3, 3·09206, 3·20936, 3·35287, 3·52292, 3·71914]

2. *(a)* $\theta = 90°$

$\cos\theta + \cos 3\theta = 0 + 0 = 0$ (choice of θ and evaluating one side) B1

$2\cos 2\theta \cos 4\theta = 2 \times (-1) \times 1 = -2$ (other side) B1

$(\cos\theta + \cos 3\theta \not\equiv 2\cos 2\theta \cos 4\theta)$

(b) $-9 + \cot^2\theta = \operatorname{cosec}\theta - \operatorname{cosec}^2\theta$

$-9 + \operatorname{cosec}^2\theta - 1 = \operatorname{cosec}\theta - \operatorname{cosec}^2\theta$ ($\cot^2\theta = \operatorname{cosec}^2\theta - 1$) M1

$2\operatorname{cosec}^2\theta - \operatorname{cosec}\theta - 10 = 0$

$(2\operatorname{cosec}\theta - 5)(\operatorname{cosec}\theta + 2) = 0$ (reasonable attempt to solve) M1

$\sin\theta = \frac{2}{5}, -\frac{1}{2}$ A1

$\theta = 23 \cdot 6, 156 \cdot 4, 210°, 330°$ (23·6, 156·4) B1

(210°) B1
(330°) B1

[Notes: 1) Lose 1 mark for each additional value in range,
1 for each branch
2) Allow to nearest degree
3) F.T. for sinθ = + , – 3 marks
F.T. for sinθ = – , – 2 marks
F.T. for sinθ = + , + 1 mark]

3. *(a)* $3x^2 + 2y\frac{dy}{dx} + \tan 2y + 2x\sec^2 2y\frac{dy}{dx} = 0$ $\left(2y\frac{dy}{dx}\right)$ B1

$\left(\tan 2y + k\sec^2 2y\frac{dy}{dx};\quad k = 1, 2\right)$ B1

$(k = 2)$ B1

$\frac{dy}{dx} = -\frac{3x^2 + \tan 2y}{2y + 2x\sec^2 2y}$ (All correct. F.T. for last B1 if first two Bs gained) B1

(b) (i) $\frac{dy}{dt} = \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2}$ $\left(\frac{(3+2t)f(t) - (1+4t)g(t)}{(3+2t)^2}\right)$ M1

$[f(t) = 4,\ g(t) = 2]$ A1

(*Give additional mark for simplification here, see last* A1 *of part* (ii))

(ii) $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt}$ (attempt to use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$) M1

$= \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2} \cdot \frac{1}{(3+2t)}$ A1

$= \frac{10}{(3+2t)^3}$ (*This may have been given in* (i).) A1

(Penalise faulty simplification on last line)

4. *(a)* $(f'(x) = 0)$

$2(2x-3)e^{2x} + 2e^{2x} - 4$ (Attempt to find $f'(x)$, -4 present) M1

$((2x-3)f(x) + e^{2x}g(x))$ M1

$(f(x) = ke^{2x},\ k = 1, 2;\quad g(x) = 2)$ A1

$(k = 2)$ A1

$(4x-6+2)e^{2x} - 4$

$(4x-4)e^{2x} - 4 = 0$ (equate $f'(x)$ to zero) M1

$(x-1)e^{2x} - 1 = 0$ (result – convincing) A1

(b)

x	$f'(x)$	
1	– 1	(Attempt to find values or signs) M1
2	53·6	(correct values) A1

Change of sign indicates presence of root between 1 and 2.

$x_0 = 1{\cdot}1,\ x_1 = 1{\cdot}1108...,\ x_2 = 1{\cdot}1084...,\ x_3 = 1{\cdot}1089...$ (x_1) B1

$x_3 \approx 1{\cdot}1089$ (x_3 correct to 4 decimal places) B1

Check 1·10895, 1·10885

x	$f'(x)$	
1·10885	– 0·00008	(Attempt to find values) M1
1·10895	+ 0·001	(Correct signs or values) A1

Change of sign indicates presence of root
so root is 1·1089 correct to 4 decimal places. A1

Note: *(b)* must involve 'change of sign' (o.e.) at least once.

5. *(a)* $\dfrac{4x}{3+2x^2}$ $\left(\dfrac{f(x)}{3+2x^2},\ f(x) \neq 1\right)$ M1

$(f(x) = 4x)$ A1

(b) $\dfrac{x^2}{1+x^2} + 2x\tan^{-1}x$ $(x^2 f(x) + g(x)\tan^{-1}x)$ M1

$\left(f(x) = \dfrac{1}{1+x^2},\ g(x) = 2x\right)$ A1

(c) $10(5+7x^2)^9.14x$ $(10(5+7x^2)^9.f(x))$ M1

$(f(x) = 14x)$ A1

$= 140x(5+7x^2)^9$ (Simplified answer) A1

(F.T. for $f(x) = 7x$, i.e. $70x$)

6. *(a)* $9x - 7 \leq 3,\quad x \leq \dfrac{10}{9}$ B1

<u>and</u>

$9x - 7 \geq -3$ M1

$x \geq \dfrac{4}{9}$ (answer must involve the word 'and' o.e.) A1

$\dfrac{4}{9} \leq x \leq \dfrac{10}{9}$ *(Note: lose A1 for the omission of ' = ')*

Alternative Scheme: $(9x-7)^2 \leq 9$

$81x^2 - 126x + 40 \leq 0$

$(9x-10)(9x-4)$ B1

Any method M1

Correct answer A1

(b) $5|x| + 1 = 9$

$5|x| = 8$ $(a|x| = b;\ a = 5,\ b = 8)$ B1

$x = \pm\dfrac{8}{5}$ (both answers, F.T. a, b) B1

7. *(a)* **Penalise the absence of constant one mark once only in part *(a)***

(i) $-\frac{1}{5}\cos 5x\ (+C)$ $\quad (k\cos 5x;\quad k = \pm\frac{1}{5}, -5)$ M1

$\left(k = -\frac{1}{5}\right)$ A1

(ii) $-\frac{3}{4(2x+7)^2}\ (+C)$ $\quad \left(\frac{k}{(2x+7)^2};\quad k = -\frac{3}{2}, \pm\frac{3}{4}\right)$ M1

$\left(k = -\frac{3}{4}\right)$ A1

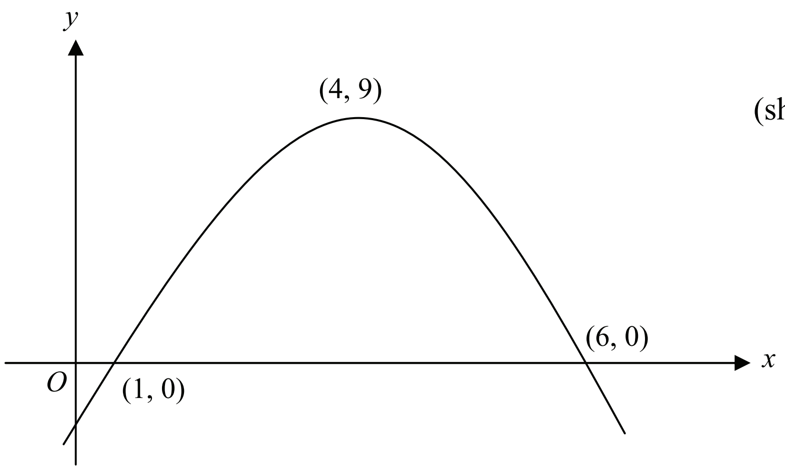
(b) $\left[\frac{2}{5}\ln|5x+3|\right]_0^3$ $\quad \left(k\ln|5x+3|;\quad |\ |\ \text{may be omitted};\quad k = 2, \frac{2}{5}\right)$ M1

$\left(k = \frac{2}{5}\right)$ A1

$= \frac{2}{5}[\ln 18 - \ln 3]$ $\quad (k(\ln 18 - \ln 3);$ F.T. for any k except $k = 1)$ M1

$\approx 0\cdot 717...$ A1

8.



(shape and coordinates of one point) B1

(another point) B1

(another point) B1

Note: 3 correct points, no graph: B2
2 or less correct points, no graph: B0

Special Case: All correct with left *x* translation: B1

9. *(a)* Domain of $fg = (0, \infty)$ B1

Range $= (0, \infty)$ B1

(b) $3e^{2\ln 4x} = 12$ ($fg(x)$, correct order and equating) M1

$3e^{\ln 16x^2} = 12$ (correct attempt to use laws of logs) m1

$48x^2 = 12$ A1

$x = \pm\frac{1}{2}$ (F.T. $12x^2 = 12$) A1

$x = \frac{1}{2}$ (since domain $(0, \infty)$) A1

[**Alternatively:**

$3e^{2\ln 4x} = 12$ (correct order and equating) M1

$e^{2\ln 4x} = 4$ A1

$2\ln 4x = \ln 4$ (taking logs) m1

$\ln 4x = \ln 2$ (o.e.) A1

$x = \frac{1}{2}$ A1 **]**

10. *(a)* $f'(x) = \dfrac{-2\times -1}{(3x^2+2)^2}\times 6x$ $\left(\dfrac{f(x)}{(3x^2+2)^2};\quad f(x)\neq \pm 1, \pm 2\right)$ M1

$= \dfrac{12x}{(3x^2+2)^2}$ $(f(x)=12x)$ A1

Since $12x>0$, $\dfrac{1}{(3x^2+2)^2}>0$, $f'(x)>0$ $(12x>0)$ B1

$\left(\dfrac{1}{(3x^2+2)^2}>0\right)$ B1

(b) Range is (0, 1) B1

(c) Let $y = 1-\dfrac{2}{3x^2+2}$ $\left(y-1=\dfrac{-2}{(\quad)}\right)$ M1

$y-1=\dfrac{-2}{3x^2+2}$

$\dfrac{2}{1-y}=3x^2+2$

$\dfrac{2}{1-y}-2=3x^2$ A1

$x=\pm\sqrt{\dfrac{2}{3}\left(\dfrac{1}{1-y}-1\right)}$ o.e. (must have $\pm\sqrt{}$) A1

$x=\sqrt{\dfrac{2}{3}\left(\dfrac{y}{1-y}\right)}$ (simplified form not required) o.e. (domain $x>0$) B1

$f^{-1}(x)=\sqrt{\dfrac{2}{3}\left(\dfrac{x}{1-x}\right)}$ (simplified form not required)

(F.T. candiate's expression) B1

Domain of f^{-1} is (0, 1) o.e. (F.T. from *(a)*)
) B1
Range of f^{-1} is $(0, \infty)$)

Mathematics C4 Summer 2009

Solutions and Mark Scheme

1. *(a)* $\dfrac{3x}{(1+x)^2(2+x)} \equiv \dfrac{A}{1+x} + \dfrac{B}{(1+x)^2} + \dfrac{C}{2+x}$ (correct form) M1

$3x \equiv A(1+x)(2+x) + B(2+x) + C(1+x)^2$ (correct attempt to clear fractions and substitute for x) M1

$x = -1$ $\quad -3 = B(1)$

$B = -3$

$x = -2$ $\quad -6 = C(-1)^2$ (2 constants) A1

$C = -6$

x^2 $\quad 0 = A + C$

$A = 6$ (3rd constant) A1

(F.T. one slip)

(b) $\displaystyle\int_0^1 \left(\frac{6}{1+x} - \frac{3}{(1+x)^2} - \frac{6}{2+x} \right) \mathrm{d}x$

$= \left[6\ln(1+x) + \dfrac{3}{1+x} - 6\ln(2+x) \right]_0^1$ $\left(\dfrac{3}{1+x}\right)$ B1

(F.T. candidate's equivalent work) (logs) B1, B1

$= 6\ln 2 + \dfrac{3}{2} - 6\ln 3 - 6\ln 1 - 3 + 6\ln 2$

$\approx 0\cdot 226$ (must be at least 3 decimal places) C.A.O. B1

2. $3\times 2\sin\theta\cos\theta = 2\sin\theta$ (Use of $\sin 2\theta = 2\sin\theta\cos\theta$) M1

$\sin\theta = 0$ A1

or $3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$ A1

$\theta = 0°, 180°, 360°$) (F.T. one slip) B1

$70\cdot5°, 289\cdot5°$) B1

No workings shown – no marks

3. *(a)* $R = 2$ B1

$\tan\alpha = \sqrt{3},\ \alpha = 60°$ (any method) M1

A1

(b) $2\cos(\theta - 60°) = 1$

$\cos(\theta - 60°) = \frac{1}{2}$ (F.T. R and α) M1

$\theta - 60° = -60°, 60°, 300°$ (one value) A1

$\theta = 0°,\ 120°, 360°$ (A2 for 3 answers, A1 for 2 answers A2
A0 for 1 answer, lose 1 for more than 3 answers)

4. Volume $= \pi\int_0^{\frac{\pi}{8}} \cos^2 2x\ \mathrm{d}x$ (must contain limits) B1

$= (\pi)\int_0^{\frac{\pi}{8}} \frac{1+\cos 4x}{2}\ \mathrm{d}x$ ($\cos^2 2x = a + b\cos 4x;\quad a, b \neq 0$) M1

$\left(a = \frac{1}{2}, b = \frac{1}{2}\right)$ A1

$= (\pi)\left[\frac{x}{2} + \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{8}}$ A1

$= (\pi)\left(\frac{\pi}{16} + \frac{1}{8} - 0 - 0\right)$ (correct use of limits) m1

$= \frac{\pi}{2}\left(\frac{\pi}{8} + \frac{1}{4}\right)$ or $1\cdot0095$ (C.A.O.) A1

[If substitution used, marks are gained after

$\frac{1}{2}\cos^2 u = a + b\cos 2u$ M1]

5. *(a)* $\frac{dy}{dx} = \frac{3t^2}{2t}$ $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$ M1

$= \frac{3t}{2}$ (simplified form) A1

Equation of tangent is

$y - p^3 = \frac{3}{2}p(x - p^2)$ (use of any method) M1

$2y - 2p^3 = 3px - 3p^3$

$3px - 2y = p^3$ (convincing) A1

(b) Substitute $x = q^2$, $y = q^3$ (substitution of $x = q^2$, $y = q^3$ and $p = 2$) M1

$3pq^2 - 2q^3 = p^3$

When $p = 2$,

$6q^2 - 2q^3 = 8$

$q^3 - 3q^2 + 4 = 0$ (convincing) A1

$(q+1)(q^2 - 4q + 4) = 0$ (attempt to solve) M1

$q = -1$ or $q = 2$ A1

Disregard $q = 2$ (as this relates to point *P*) A1

[Alternatively:

$\frac{y - q^3}{x - q^2} = 3$ (must have gradient 3) M1

$q^3 - 3q^2 + 4 = 0$ (convincing) A1 **]**

6. *(a)* $\int (x+3)e^{2x}\,dx = (x+3)\frac{e^{2x}}{2} - \int 1.e^{2x}\,dx$

$((x+3)f(x) - \int Af(x)dx;\ f(x) \neq k,\ A = 1, 3)$ M1

$(f(x) = ke^{2x})$ A1

$\left(k = \frac{1}{2},\ A = 1\right)$ A1

$= (x+3)\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$ C.A.O. (must contain *C*) A1 *(b)* $\int_3^2 -\frac{1}{2u^{\frac{1}{2}}}\,\mathrm{d}u$ — $\left(\frac{k}{u^{\frac{1}{2}}}\right)$ M1

$\left(k = -\frac{1}{2}\right)$ A1

$= \left[-u^{\frac{1}{2}}\right]_3^2$ — (integration, any k, no limits) A1

(correct use of limits) m1

$= \sqrt{2} + \sqrt{3} \approx 0\cdot318$ — C.A.O. (either answer) A1

Answer only gains 0 marks

7. *(a)* $\frac{\mathrm{d}P}{\mathrm{d}t} = -kP^3$ — (allow $\pm k$) B1

(b) $\int \frac{\mathrm{d}P}{p^3} = -\int k\,\mathrm{d}t$ — (separation of variables & attempt to integrate $\frac{1}{p^n}$, any n) M1

$-\frac{1}{2p^2} = -kt + C$ — (C may be omitted, $n \neq 1$) A1

$t = 0,\ P = 20$ — (attempt to find C) M1

$\therefore\ -\frac{1}{800} = C$ — (F.T. similar work) A1

$\therefore\ -\frac{1}{2p^2} = -kt - \frac{1}{800}$

$\therefore\ \frac{1}{p^2} = 2kt + \frac{1}{400}$

$\frac{1}{p^2} = At + \frac{1}{400}$ ($A = 2k$) — (convincing) A1

(c) $t = 1,\ P = 10$

$\frac{1}{100} = A + \frac{1}{400}$ — (attempt to find A) M1

$\therefore\ A = \frac{3}{400}$ — A1

$\frac{1}{25} = \frac{3}{400} + \frac{1}{400}$ — (substitute $p = 5$) m1

$\frac{15}{400} = \frac{3}{400}t$

$t = 5$ — (F.T. one slip) A1

8. *(a)* (i) $\mathbf{AB} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ B1

Equation of *AB* is

$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ ($\mathbf{r} = \mathbf{a} + \lambda\mathbf{AB}$, o.e.) M1
A1

[Alternative:
$(1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ ($\mathbf{a}$, $\mathbf{b}$ substituted) M1
$\mathbf{r} = ...$ A1
(all correct) A1 **]**

(ii) Assume *AB* and *L* intersect. Equate coefficients of **i**, **j** (o.e.).

$(3+\lambda) = 5 + 3\mu$ (F.T. candidate's values) M1
$4 - 2\lambda = 6 - 2\mu$ A1

Solve for λ, μ, (attempt to solve for λ, μ) m1
$\lambda = -\frac{5}{2}$, $\mu = -\frac{3}{2}$ (one value; F.T. one slip) A1

Check **k** coefficient (o.e.)
L.H.S. $= 7 + 3\lambda = -\frac{1}{2}$ (attempt to check) m1
R.H.S. $= 1 + \mu = -\frac{1}{2}$
(Terms check so lines intersect)
Point of intersection is $\mathbf{i} + 9\mathbf{j} - \frac{1}{2}\mathbf{k}$. C.A.O. A1
(dependent on M1, m1 earlier)

(b) $(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}).(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 0$ (correct method of finding scalar product) M1
$6 - 2 - 4 = 0$ A1
(therefore vectors are perpendicular)

9. $(1+4x)^{\frac{1}{2}} = 1+\frac{1}{2}(4x)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(4x)^2+...$ (first line with possibly $4x^2$) M1

$= 1+2x-2x^2+....$ $(1+2x)$ A1

$(-2x^2)$ A1

Valid for $|x| < \frac{1}{4}$ B1

$(1+4k+16k^2) = 1+2(k+4k^2)-2(k+4k^2)+...$ (correct substitution for x and attempt to evaluate) M1

$= 1+2k+8k^2-2k^2+....$

$= 1+2k+6k^2+....$ (F.T. quadratic in x) A1

[**Alternative:**
First principles with three terms M1
Answer A1 **]**

10. $9k^2 = 3b^2$ B1
$b^2 = 3k^2$ B1
(b^2 has a factor 3)
b has a factor 3 B1
a and b have a common factor – contradiction (must mention contradiction) B1
($\sqrt{3}$ is irrational)

A/AS Level Maths – FP1 – June 2009 – Markscheme

1. $S_n = \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$ M1

$= \frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$ A1A1A1

$= \frac{n(n+1)(3n^2+3n+8n+4+6)}{12}$ m1

$= \frac{n(n+1)(n+2)(3n+5)}{12}$ A1

2. $\alpha + \beta = -3$

$\alpha\beta = 4$ B1

Consider

$\alpha + \beta + \alpha\beta = 1$ cao M1A1

$\alpha\beta + \alpha^2\beta + \alpha\beta^2 = \alpha\beta + \alpha\beta(\alpha + \beta)$ M1

$= -8$ cao A1

$\alpha^2\beta^2 = 16$ B1

The required equation is

$x^3 - x^2 - 8x - 16 = 0$ M1A1

[FT from above]

3. (a) det**A** = 1(6 – 5) + 2(3 – 4) + 3(10 – 9) = 2 M1A1

Cofactor matrix $= \begin{bmatrix} 1 & -1 & 1 \\ 11 & -7 & 1 \\ -7 & 5 & -1 \end{bmatrix}$ M1

Adjugate matrix $= \begin{bmatrix} 1 & 11 & -7 \\ -1 & -7 & 5 \\ 1 & 1 & -1 \end{bmatrix}$ A2

[A1 for 1,2 or 3 errors]

Inverse matrix $= \frac{1}{2}\begin{bmatrix} 1 & 11 & -7 \\ -1 & -7 & 5 \\ 1 & 1 & -1 \end{bmatrix}$ A1

[Award 0 marks for answer only : FT from their adjugate]

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 11 & -7 \\ -1 & -7 & 5 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 13 \\ 13 \\ 22 \end{bmatrix}$ M1

$= \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ A1

[FT from their inverse]

4. (a) $z = \dfrac{(9+7\text{i})(3+\text{i})}{(3-\text{i})(3+\text{i})}$ M1

$= \dfrac{27-7+21\text{i}+9\text{i}}{9+1}$ A1A1

$= 2 + 3\text{i}$ A1

(b) mod(z) = $\sqrt{13}$, arg(z) = 0.983 (56.3°) B1B1

[FT on their z]

5. The statement is true for $n = 1$ since the formula gives1/2 which is correct.

B1

Let the statement be true for $n = k$, ie

$$\sum_{r=1}^{k} \frac{1}{r(r+1)} = \frac{k}{k+1}$$ M1

Consider

$$\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \sum_{r=1}^{k} \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+2)}$$ M1

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$ A1

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$ A1

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$ A1

$$= \frac{k+1}{k+2}$$ A1

True for $n = k \Rightarrow$ true for $n = k + 1$, hence proved by induction. A1

6. (a) det(A) = $\lambda(-4-\lambda^2)+3\lambda-8+2(2\lambda+3) = -(\lambda^3-3\lambda+2)$ M1A1

This is zero when λ = 1 (so the matrix is singular) B1

$\lambda^3-3\lambda+2 = (\lambda-1)(\lambda^2+\lambda-2) = (\lambda-1)^2(\lambda+2)$ M1

[For candidates using long division award M1 for 2 terms]

Therefore λ = 1 is the only positive value giving a zero determinant.

A1

(b)(i) $x + y + 2z = 2$ M1

$-3y - 3z = -6$ A1

$-2y - 2z = -4$ A1

[Award the M1 for 1 correct row]

The equations are consistent because the 3rd equation is a multiple of the 2nd A1

(ii) Put $z = \alpha$ M1

$y = 2 - \alpha$, $x = -\alpha$ cao

A1

7. Putting $z = x + \mathrm{i}y$, [Award for attempting to use] M1

$\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x+2)^2 + y^2}$ A1

$x^2 - 2x + 1 + y^2 = 4(x^2 + 4x + 4 + y^2)$ A1

$x^2 + y^2 + 6x + 5 = 0$ [Accept any multiple] A1

$(x+3)^2 + y^2 = 4$ M1

Centre (– 3, 0), radius = 2 A1A1

[FT from their circle]

8. (a) Reflection matrix $= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Translation matrix = $\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$ B1

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ B1

$$= \begin{bmatrix} 0 & -1 & h \\ -1 & 0 & k \\ 0 & 0 & 1 \end{bmatrix}$$

(b)(i) We are given that

$$\begin{bmatrix} 0 & -1 & h \\ -1 & 0 & k \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$ M1

giving

$h = 4$ A1

$k = 2$ A1

(ii) EITHER

A general point on the line is $(\lambda, 3\lambda + 2)$. M1

Consider

$$\begin{bmatrix} 0 & -1 & 4 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \lambda \\ 3\lambda + 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2-3\lambda \\ 2-\lambda \\ 1 \end{bmatrix}$$ M1

$x = 2 - 3\lambda; y = 2 - \lambda$ A1

Eliminating λ, $3y - x = 4$ cao M1A1

OR

Let $(x, y) \to (x', y')$ M1

$$\begin{bmatrix} 0 & -1 & 4 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$ M1

$-y + 4 = x'$ or $y = 4 - x'$

$-x + 2 = y'$ or $x = 2 - y'$ A1

$y = 3x + 2 \to 4 - x' = 6 - 3y' + 2$ M1

equation of image is $3y - x = 4$ cao A1

9. (a)(i) $\ln f(x) = \ln(x^x) + \ln(e^{-2x}) = x\ln x - 2x$ B1

(ii) $\dfrac{f'(x)}{f(x)} = \ln x + \dfrac{x}{x} - 2 = \ln x - 1$ B1B1

$f'(x) = f(x)(\ln x - 1)$ (so $a = 1$, $b = -1$) B1

(b) $f''(x) = f'(x)(\ln x - 1) + \dfrac{f(x)}{x}$ B1

(c) At a stationary point,

$\ln x = 1$ B1

$x = \mathrm{e}$ (2.72) B1

$y = \mathrm{e}^{-\mathrm{e}}$ (0.066) B1

From (b), $f''(\mathrm{e}) > 0$ B1

so it is a minimum. B1

[Award 0 marks for answer only]

A/AS Maths – FP2 – June 2009 – Markscheme

1. (a) h is not continuous — B1

Any valid reason, eg f is not defined for $x = 0$, $f(x)$ jumps from large negative to large positive going through 0. — B1

(b(i) Attempting to compare $g(x)$ with $g(-x)$. — M1

g is even. — A1

(ii) Attempting to compare $h(x)$ with $h(-x)$. — M1

h is odd. — A1

2. $u = \tan x \Rightarrow \mathrm{d}u = \sec^2 x\mathrm{d}x$ — B1

and $[0,\pi/6] \to [0,\ 1/\sqrt{3}]$ — B1

$$I = \int_0^{1/\sqrt{3}} \frac{\mathrm{d}u}{\sqrt{3-(1+u^2)}}$$ — M1

[Must be correct here]

$$= \int_0^{1/\sqrt{3}} \frac{\mathrm{d}u}{\sqrt{2-u^2}}$$ — A1

$$= \left[\sin^{-1}\left(\frac{u}{\sqrt{2}}\right)\right]_0^{1/\sqrt{3}}$$ — A1

$= 0.421$ — A1

Any valid reason, eg the denominator would be the square root of a negative number towards the upper limit. — B1

3. Modulus = 16 — B1

Argument = $2\pi/3$ — B1

$-8+8\sqrt{3}\mathrm{i} = 16[\cos(2\pi/3) + \mathrm{i}\sin(2\pi/3)]$ — B1

$(-8+8\sqrt{3}\mathrm{i})^{1/4} = 16^{1/4}[\cos(2\pi/3\times 1/4) + \mathrm{i}\sin(2\pi/3\times 1/4)]$ — M1

$= 2(\cos(\pi/6) + \mathrm{i}\sin(\pi/6))$ — A1

Second root = $2(\cos(2\pi/3) + \mathrm{i}\sin(2\pi/3))$ — A1

Third root = $2(\cos(7\pi/6) + \mathrm{i}\sin(7\pi/6))$ — A1

Fourth root = $2(\cos(5\pi/3) + \mathrm{i}\sin(5\pi/3))$ — A1

[FT from their modulus and argument]

4. The equation can be rewritten

$\sin 2\theta + 2\sin 2\theta\cos\theta = 0$ — M1A1

$\sin 2\theta(1 + 2\cos\theta) = 0$ — A1

$\sin 2\theta = 0$ gives $\theta = \dfrac{n\pi}{2}$ — M1A1

$\cos\theta = -\dfrac{1}{2}$ gives $\theta = 2n\pi \pm 2\pi/3$ — M1A1

5. (a) Let $\frac{1}{(x+1)(x+2)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$

$$= \frac{A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)}{(x+1)(x+2)(x+3)} \quad \text{M1}$$

x = -1 gives A = 1/2 ; x = -2 gives B = -1 ; x = -3 gives C = 1/2 A1A1A1

(b) $I = \left[1/2\ln(x+1) - \ln(x+2) + 1/2\ln(x+3)\right]_0^5$ B1B1

[B1 for 1 error]

$$= \frac{1}{2}\left(\ln 6 - \ln 49 + \ln 8 + \ln 4 - \ln 3\right) \quad \text{M1}$$

$$= \frac{1}{2}\ln\left(\frac{6\times 8\times 4}{49\times 3}\right) \quad \text{A1}$$

$$= \ln\left(\frac{8}{7}\right) \quad \text{A1}$$

6. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta} = -\frac{b\cos\theta}{a\sin\theta}$ M1A1

Equation of tangent is

$$y - b\sin\theta = -\frac{b\cos\theta}{a\sin\theta}(x - a\cos\theta) \quad \text{M1A1}$$

$$bx\cos\theta + ay\sin\theta = ab(\cos^2\theta + \sin^2\theta) \quad \text{A1}$$

whence the printed result.

(b) Putting y = 0, P is $\left(\frac{a}{\cos\theta}, 0\right)$ M1A1

Putting x = 0, Q is $\left(0, \frac{b}{\sin\theta}\right)$ A1

Therefore R is $\left(\frac{a}{2\cos\theta}, \frac{b}{2\sin\theta}\right)$ A1

Eliminating θ,

$$\cos\theta = \frac{a}{2x}; \sin\theta = \frac{b}{2y} \quad \text{M1}$$

$$\frac{a^2}{4x^2} + \frac{b^2}{4y^2} = \cos^2\theta + \sin^2\theta = 1 \quad \text{M1A1}$$

7. (a) $z^{-n} = \cos(-n\theta) + \text{i}\sin(-n\theta)$ M1

$= \cos n\theta - \text{i}\sin n\theta$ A1

$z^n + z^{-n} = \cos n\theta + \text{i}\sin n\theta + (\cos n\theta - \text{i}\sin n\theta)$ M1

$= 2\cos n\theta$ AG

(b) Using the result in (a),

$2\cos 2\theta - 4\cos\theta + 3 = 0$ M1

$2(2\cos^2\theta - 1) - 4\cos\theta + 3 = 0$ M1

$4\cos^2\theta - 4\cos\theta + 1 = 0$ A1

$(2\cos\theta - 1)^2 = 0$ M1

$\cos\theta = \frac{1}{2}$ A1

$\sin\theta = \pm\frac{\sqrt{3}}{2}$ A1

The roots are $\frac{1}{2} \pm \frac{\sqrt{3}}{2}\text{i}$ A1

[The ± must be there for the last 2 marks; accept cos(π/3) ± isin(π/3)]

8. (a) By long division,

$$\begin{array}{r} x+4 \\ x-1\overline{)x^2+3x} \\ x^2-x \\ 4x \\ 4x-4 \\ 4 \end{array}$$

$4x$ (Must be $2x$ or $4x$ for M1) M1A1

Therefore,

$f(x) = x + 4 + \frac{4}{x-1}$ A1

[FT 2 from above]

(b) $f'(x) = 1 - \frac{4}{(x-1)^2}$ B1

[FT their expression from (a)]

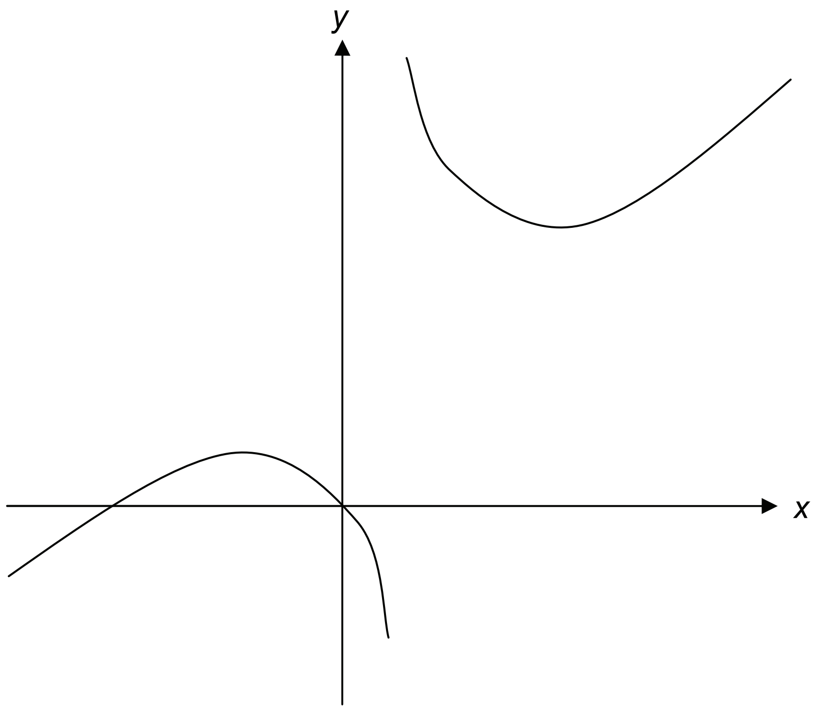
Stationary points occur when

$1 = \frac{4}{(x-1)^2}$ M1

$x = -1, 3$ A1

$y = 1, 9$ A1

(c) The asymptotes are $x = 1$ and $y = x + 4$. B1B1



G2

(d) Since $f(-3) = f(0) = 0$, part of the answer is [-3,0]. B1

Consider

$$\frac{x(x+3)}{(x-1)} = 10$$ M1

$$x^2 - 7x + 10 = 0$$ A1

$$x = 2,\ 5$$ A1

$$f^{-1}(A) = [-3,0] \cup [2,5]$$ A1

A/AS Level Mathematics – FP3 – June 2009 – Mark Scheme

1. The equation can be rewritten

$1+2\sinh^2\theta = 6\sinh\theta - 3$	M1A1
$\sinh^2\theta - 3\sinh\theta + 2 = 0$	A1
$(\sinh\theta - 1)(\sinh\theta - 2) = 0$	M1
$\sinh\theta = 1, 2$	A1
$\theta = \ln(1+\sqrt{2}), \ln(2+\sqrt{5})$	B1B1

2.

$f(0) = 0$	B1
$f'(x) = -\dfrac{e^x}{(2-e^x)}; f'(0) = -1$	B1B1
$f''(x) = -\dfrac{2e^x}{(2-e^x)^2}; f''(0) = -2$	B1B1
$f'''(x) = -\dfrac{2e^x(2-e^x)^2 + 2e^x 2(2-e^x)e^x}{(2-e^x)^4}; f'''(0) = -6$	B1B1
The Maclaurin series is	
$0 - x - \dfrac{2x^2}{2} - \dfrac{6x^3}{6} + ... = -x - x^2 - x^3 + ...$	M1A1
[FT on their derivatives]	

3.

$dx = 2\cosh u du$; $[0,2] \to [0, \sinh^{-1} 1]$	B1B1
$I = \displaystyle\int_0^{\sinh^{-1}1} \frac{2\cosh u du}{(4\sinh^2 u + 4)^{3/2}}$	M1
$= \displaystyle\int_0^{\sinh^{-1}1} \frac{2\cosh u du}{8\cosh^3 u}$	A1
$= \dfrac{1}{4}\displaystyle\int_0^{\sinh^{-1}1} \mathrm{sec\,h}^2 u \, d\theta$	A1
$= \dfrac{1}{4}[\tanh u]_0^{\sinh^{-1}1}$	A1
$= \dfrac{1}{4}\tanh\sinh^{-1}(1)$	A1
$= 0.18$	A1
[Award 0 marks for answer only]	

4. EITHER

$$2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$$ B1

$$\text{CSA} = 2\pi\int_0^a y\sqrt{1+\frac{4a^2}{y^2}}dx$$ M1A1

$$= 2\pi\int_0^a \sqrt{4ax}\sqrt{1+\frac{a}{x}}\,dx$$ A1

$$= 4\sqrt{a}\pi\int_0^a \sqrt{x+a}\,dx$$ A1

$$= 4\sqrt{a}\pi.\frac{2}{3}\left[(x+a)^{3/2}\right]_0^a$$ A1

$$= \frac{8}{3}\sqrt{a}\pi\left[2^{3/2}a^{3/2} - a^{3/2}\right]$$ A1

= Answer given

OR

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$$ B1B1

$$\text{CSA} = 2\pi\int y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$ M1

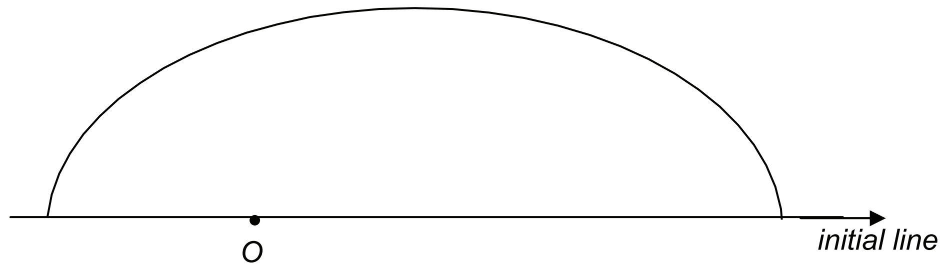
$$= 2\pi\int_0^1 2at\sqrt{4a^2t^2+4a^2}\,dt$$ A1

$$= 8\pi a^2\int_0^1 t\sqrt{t^2+1}\,dt$$ A1

$$= 8\pi a^2\left[\frac{(t^2+1)^{3/2}}{3}\right]_0^1$$ M1A1

= Answer given

5. (a)



G1

(b) $$\text{Area} = \frac{1}{2}\int_0^{\pi/2}(2+\cos\theta)^2 \mathrm{d}\theta$$ M1

$$= \frac{1}{2}\int_0^{\pi/2}(4+4\cos\theta+\cos^2\theta)\mathrm{d}\theta$$ A1

$$= \frac{1}{2}\int_0^{\pi/2}\left(4+4\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$$ M1A1

$$= \frac{1}{2}\left[\frac{9\theta}{2}+4\sin\theta+\frac{1}{4}\sin 2\theta\right]_0^{\pi/2}$$ A1

[Limits not required until here]

$$= \frac{9}{8}\pi+2 \ (5.53)$$ A1

(c) Consider

$y = r\sin\theta = (2 + \cos\theta)\sin\theta$ M1

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -\sin^2\theta + \cos\theta(2+\cos\theta)$$ A1

At a stationary point, $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$ giving M1

$$2\cos^2\theta + 2\cos\theta - 1 = 0$$ A1

$$\cos\theta = \frac{\sqrt{3}-1}{2}$$ A1

$\theta = 1.20$, $r = 2.37$ A1A1

6. (a)

$$I_n = \int_0^{\pi/4} \tan^{n-2} x \tan^2 x \,dx$$ M1

$$= \int_0^{\pi/4} \tan^{n-2} x(\sec^2 x - 1)\,dx$$ m1A1

$$= \frac{1}{n-1}\left[\tan^{n-1} x\right]_0^{\pi/4} - I_{n-2}$$ A1A1

$$= \frac{1}{n-1} - I_{n-2}$$

(b)

$$I_0 = \int_0^{\pi/4} dx = [x]_0^{\pi/4} = \frac{\pi}{4}$$ M1A1

$$I_4 = \frac{1}{3} - I_2$$ M1

$$= \frac{1}{3} - (1 - I_0)$$ A1

$$= \frac{\pi}{4} - \frac{2}{3} \quad (0.119)$$ A1

7. (a)(i) $f'(x) = \sinh x - x\cosh x$, $f'(0) = 0$ M1A1

$f''(x) = -x\sinh x, f''(0) = 0$ A1

(ii) Any valid method, eg looking at the behaviour of $f'(x)$ or $f''(x)$ around $x = 0$ or noting that f is an even function. M1

Concluding that it is not a stationary point of inflection A1

b)(i) $2\cosh\alpha - \alpha\sinh\alpha = 0$ M1

$$\frac{\alpha\sinh\alpha}{\cosh\alpha} = 2$$ A1

whence $\alpha\tanh\alpha = 2$

(ii) Let $f(\alpha) = \alpha\tanh\alpha - 2$

$f(2) = -0.0719\ldots$ M1

$f(2.1) = 0.0379\ldots$

The change of sign shows that the value of α lies between 2 and 2.1.

A1

[Accept consideration of $f(\alpha) = 2\cosh\alpha - \alpha\sinh\alpha$]

(iii) Let

$$y = \frac{2}{\tanh x}$$ M1

$$\frac{dy}{dx} = -\frac{2}{\tanh^2 x} \times \operatorname{sech}^2 x$$ A1

$= 0.137 < 1$ in modulus when $x = 2.05$ A1

therefore convergent.

(iv) Taking the initial value as 2.05, successive values are

2.05
2.067407830 B1
2.065063848
2.065374578
2.065333300
2.065338782

Thus α = 2.0653 correct to 4dps. B1

(c) Area = $\int_0^{2.0653} (2\cosh x - x\sinh x)\mathrm{d}x$ M1

$= \left[2\sinh x\right]_0^{2.0653} - \left[x\cosh x\right]_0^{2.0653} + \int_0^{2.0653} \cosh x\mathrm{d}x$ A1A1A1

$= \left[3\sinh x - x\cosh x\right]_0^{2.0653}$ A1

= 3.365 A1

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GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3
AS/Advanced

SUMMER 2009

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2009 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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M3	8
S1	12
S2	16
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Mathematics M1 (June 2009)
Final Markscheme

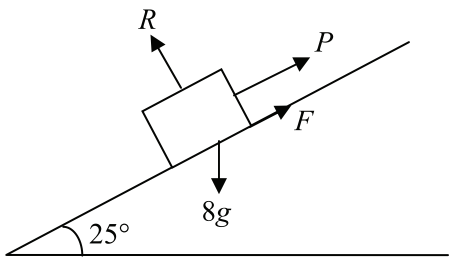
1.(a)	Using $v = u + at$ with $u = 14.7$, $a = (-)9.8$, $t = 2$.		M1
	$v = 14.7 - 9.8 \times 2$		A1
	$v = -4.9$		
	Speed = $\underline{4.9\text{ms}^{-1}}$		A1
1.(b)	Using $v^2 = u^2 + 2as$ with $u = 14.7$, $a = (-)9.8$, $s = (-)70.2$.		M1
	$v^2 = 14.7^2 + 2\times(-9.8)\times(-70.2)$		A1
	$v = \underline{39.9\text{ ms}^{-1}}$	cao	A1
1.(c)	Using $s = ut + \frac{1}{2}at^2$ with $u = 14.7$, $a = (-)9.8$, $s = 3.969$.		M1
	$3.969 = 14.7t - \frac{1}{2}\times 9\cdot 8\times t^2$		A1
	$t^2 - 3t + 0.81 = 0$	attempt to solve	m1
	$(t - 0.3)(t - 2.7) = 0$		
	$t = 0.3, 2.7$		
	Therefore required length of time $= 2.7 - 0.3$		
	$= \underline{2.4\text{ s}}$	cao	A1

2.(a)	N2L	$5g - T = 5a$	dim. correct	M1 A1
		$T - 2g = 2a$	dim. correct	M1 A1
	Adding	$3g = 7a$		m1
		$a = \underline{3g/7} = \underline{(4.2)\text{ms}^{-2}}$	cao	A1
		$T = 2\times 9.8 + 2\times 4.2$		
		$= \underline{28\text{ N}}$	cao	A1
2.(b)	Magnitude of acceleration of objects *A* and *B* are equal.			B1

3.(a)	N2L applied to lift and person	$900g - T = 900a$	dim corr.	M1
		$900\times 9.8 - 8550 = 900a$		A1
		$a = \underline{0.3\text{ ms}^{-2}}$	cao	A1
3.(b)	N2L applied to person	$65g - R = 65a$		M1
		$R = 65(9.8 - 0.3)$		A1
		$R = \underline{617.5\text{ N}}$	ft c's a	A1

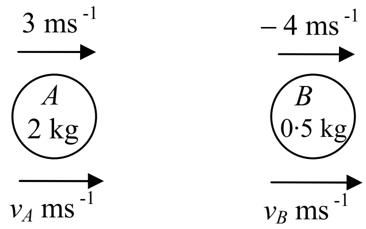
4.(a)	At $t = 10$, acceleration = $\frac{20-5}{30}$		M1
	$= \underline{0.5\text{ ms}^{-2}}$	cao	A1
	At t =420,acceleration = $\underline{0}$		B1
4.(b)	Using $v = u + at$ with $u = 5$ $t = 20$, $a = 0.5$(c).		M1
	$v = 5 + 0.5 \times 20$		
	$v = \underline{15\text{ ms}^{-1}}$	ft acce if > 0	A1
4.(c)	Distance = $\frac{1}{2}(5+20)\times 30 + 20\times 400 + \frac{1}{2}\times 20\times 50$	method for distance	M1
		any correct area	B1
		correct expression	A1
	Distance = $\underline{8875\text{ m}}$	cao	A1

5.



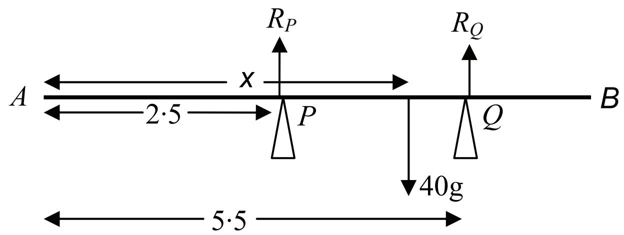
(a)	$R = 8g\cos 25°$ (=71.05)	si	M1 A1
	$F = 0.3 \times 8g\cos 25°$ (=21.32)	si	m1
	N2L up slope	dim correct, all forces, $a = 0$	M1
	$P + F = 8g\sin 25°$		A1
	$P = 8 \times 9.8\sin 25° - 2.4 \times 9.8\cos 25°$		
	$P = \underline{11.82\text{ N}}$	cao	A1
(b)	N2Ldown slope	dim correct, all forces	M1
	$P - F - 8g\sin 25° = 8a$		A1 A1
	$P = 8g\sin 25° + 2.4g\cos 25° + 8 \times 0.6$		
	$P = \underline{59.25\text{ N}}$	cao	A1

6.



(a)	conservation of Momentum		M1
	$2 \times 3 - 0.5 \times 4 = 2v_A + 0.5v_B$		A1
	$4v_A + v_B = 8$		
	Restitution		M1
	$v_B - v_A = -e(-4 - 3)$		A1
	$= \frac{2}{7} \times 7$		
	$v_B - v_A = 2$		
	Subtracting $5v_A = 6$	dep. on both M's	m1
	$v_A = \underline{1.2\text{ms}^{-1}}$	cao	A1
	$v_B = \underline{3.2\text{ms}^{-1}}$	cao	A1
(b)	Impulse on B = Change in momentum of B.	used	M1
	$I = 0.5(3.2 - (-4))$		
	$I = \underline{3.6\text{Ns}}$	ft v_A, v_B	A1 B1

7.



(a)	Resolve vertically	$R_P + R_Q = 40g$	oe	M1
	$R_P = R_Q = R$	$R + R = 40g$		
		$R = 20g = (196)\text{N}$		A1
(b)	Moments about A		moments both sides, dim correct	M1
		$2.5 R_P + 5.5 R_Q = x \times 40(g)$		A1 B1
		$8 \times 20g = 40g \times x$		
		$x = \underline{4\text{m}}$	cao	A1
	OR If $R_P = R_Q$, C must be the midpoint of PQ.			M1
	Therefore $x = 2.5 + 0.5(5.5 - 2.5)$			B1 A1
	$= \underline{4\text{ m}}$			A1

8. Resolve in one direction to obtain component of resultant — M1

$X = 7\cos30° - 2\cos60° - 5\cos50°$ — A1

$X = 1.8482$

Resolve in perpendicular direction — M1

$Y = 5\cos40° + 7\cos60° - 2\cos30°$ — A1

$Y = 5.5982$

$\text{Resultant}^2 = 1.8482^2 + 5.5982^2$ — m1

Resultant = <u>5.9 N</u> — cao — A1

9.(a)

	Area	from *AE*	from *AB*
Square	36	3	3
Triangle	12	3	$6+\frac{4}{3}=\frac{22}{3}$
The sign	48	x	y

B1 B1 B1

Distance of centre of mass from $AE = x = \underline{3}$ — B1

Moments about *AB* — M1

$48y = 12\times\frac{22}{3} + 36\times3$ — ft areas, y's — A1

$y = \frac{49}{12} = \underline{4.083\text{ cm}}$ — cao — A1

(b) $\tan\theta = \dfrac{3}{\frac{49}{12}}$ — ft x, y — M1 A1

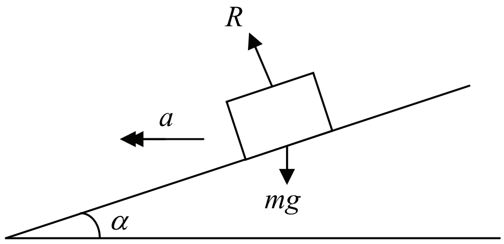
$\theta = \underline{36.3°}$ — ft x, y — A1

Mathematics M2 (June 2009)
Final Markscheme

1.(a)	$a = \frac{\mathrm{d}}{\mathrm{d}t}(\cos 2t - 3\sin 2t)$	attempted	M1
	$= -2\sin 2t - 3\cos t$		A1 A1
	When $t = \pi$		
	$a = -2\sin 2\pi - 3\cos\pi$		
	$a = 3 \text{ ms}^{-2}$		A1
1.(b)	$x = \int \cos 2t - 3\sin t \, \mathrm{d}t$	attempted	M1
	$x = 0.5\sin 2t + 3\cos t + C$		A1 A1
	When $t = 0$, $x = 4$	used	m1
	$4 = 3 + C$		
	$C = 1$	ft	A1
	$x = 0.5\sin 2t + 3\cos t + 1$		
	$x = \underline{3.62 \text{ m}}$	cao	A1
2.(a)	Using Hooke's Law $80 = \frac{\lambda(0\cdot 25 - 0\cdot 2)}{0\cdot 25}$		M1 A1
	$\lambda = \underline{400 \text{ N}}$	cao	A1
2.(b)	Energy at start $= \frac{1}{2} \times 400 \times \frac{0\cdot 05^2}{0\cdot 25}$ (= 2)	si	M1 A1
	Energy at end $= \frac{1}{2} \times 0\cdot 36\, v^2$ ($=0.18v^2$)		M1
	Conservation of energy	used	M1
	$0.18\, v^2 = 2$		
	$v = 3\frac{1}{3} \text{ ms}^{-1}$	cao	A1
3.(a)	Resolve perpendicular to plane	attempted	M1
	$R = mg\cos\alpha$		A1
	$R = 3.5 \times 9.8 \times 0.8 = 27.44$		
	$F = \mu R$	used	m1
	$F = 0.25 \times 27.44 = 6.86$		
	Work done against friction $= 6.86 \times 2$		M1
	$= \underline{13.72 \text{ J}}$	ft c's F	A1
3.(b)	K. E. at start $= 0.5mu^2$	used	M1
	$= 1.75\, u^2$		
	P. E. at end $= mgh = 3.5 \times 9.8 \times 2\sin\alpha$		M1
	$= 3.5 \times 9.8 \times 2 \times 0.6$		A1
	$= 41.16$		
	Work-energy Principle $1.75u^2 = 41.16 + 13.72$		M1 A1
	$u^2 = 31.36$		
	$u = \underline{5.6\text{ms}^{-1}}$	cao	A1

4.(a)	N2L applied to particle	3 terms	M1
	$F - 1500 = 5000a$		A1
	$F = 2500$ N		
	$F = \frac{P}{v} = \frac{P}{12}$		M1
	$P =$ <u>30000 W</u>	cao	A1
4.(b)	Since maximum velocity, $a = 0$	si	M1
	$F = 1500$		A1
	$F = \frac{45 \times 1000}{v}$		M1
	$\frac{45000}{v} = 1500$		
	$v =$ <u>30 ms^{-1}</u>	cao	A1
5.(a)	Initial horizontal velocity $= 17.5\cos\alpha = 17.5 \times 0.6$ $= 10.5$		B1
	Time to reach wall $= \frac{25\cdot 2}{10\cdot 5}$		M1
	= <u>2.4 s</u>	ft c's 10.5	A1
5.(b)	Initial vertical velocity $= 17.5\sin\alpha = 17.5 \times 0.8$ $= 14$		B1
	Using $s = ut + \frac{1}{2}at^2$ with $u = 14$(c), $a = (-)9.8$, $t = 2.4$		M1
	$= 14 \times 2.4 - 4.9 \times 2.4^2$	ft if M1 in (a) awarded	A1
	= <u>5.376 m</u>		A1
5.(c)	Using $v = u + at$ with $u = 14$(c), $a = (-)9.8$, $v = 0$		M1
	$0 = 14 - 9.8t$	ft if M1 in (a) awarded	A1
	$t = \frac{10}{7}$ s		A1
6.(a)	velocity $= \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	used	M1
	$= -8t\mathbf{i} + (6t - 5)\mathbf{j}$		A1
	Momentum $= m\mathbf{v} = -16t\mathbf{i} + 2(6t - 5)\mathbf{j}$	ft c's **v**	A1
6.(b)	acceleration $= \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	used	M1
	$= -8\mathbf{i} + 6\mathbf{j}$	constant since independent of t	A1
	Magnitude $= \sqrt{8^2 + 6^2}$		M1
	= <u>10 ms^{-2}</u>	cao	A1
6.(c)	Velocity is perpendicular to acceleration when **v.a** = 0		M1
	v.a $= (-8t\mathbf{i} + (6t - 5)\mathbf{j}).(-8\mathbf{i} + 6\mathbf{j})$		
	$= 64t + 6(6t - 5)$	ft **v**, **a**	M1 A1
	$100t - 30$		
	$t =$ <u>0.3 s</u>	cao	A1

7.



Resolve vertically			M1
	$R\cos\alpha = mg$		A1
	$= 1000 \times 9.8$		
	$= 9800$		
Using N2L	$R\sin\alpha = ma$		M1
	$a = \frac{v^2}{r}$		M1
	$R\sin\alpha = \frac{1000 \times 28^2}{250}$	si	A1
	$= 3136$		
Solving	$\tan\alpha = \frac{3136}{9800}$		m1
	$= 0.32$		
	$\alpha = \underline{17.74°}$	cao	A1

8.(a)	Conservation of energy	used	M1
	$0.5 \times 5 \times 9^2 = 0.5 \times 5 \times v^2 + 5g(2 - 2\cos\theta)$		A1 A1
	$v^2 = 81 - 39.2(1 - \cos\theta)$		
	$v^2 = 41.8 + 39.2$		A1
8.(b)	N2L towards centre	used	M1
	$R - mg\cos\theta = \frac{mv^2}{r}$		A1
	$R = 5 \times 9.8\cos\theta + \frac{5}{2}(41.8 + 39.2\cos\theta)$		m1
	$= 147\cos\theta + 104.5$		A1
8.(c)	Particle leaves sphere when $R = 0$	oe	M1
	$147\cos\theta + 104.5 = 0$, $\theta = 135.3°$		
	Therefore particle will leave circle before reaching the top, i.e. particle will not complete circle.		A1

Mathematics M3 (June 2009)
Final Markscheme

1.(a) Using N2L — M1

$$-0.2 - 0.03v = 9\frac{dv}{dt}$$ A1

$$900\frac{dv}{dt} = -(20 + 3v)$$ A1

1.(b) $$900\int\frac{dv}{20+3v} = -\int dt$$ sep. var. — M1

$$900.\ \frac{1}{3}\ln(20 + 3v) = -t\ (+ C)$$ A1 A1

When $t = 0$, $v = 20$ — used — m1

$C = 300 \ln 80$

Therefore $t = 300\ln(80) - 300\ln(20 + 3v)$ — A1

$$t = 300\ln\left(\frac{80}{20 + 3v}\right)$$

1.(c) When body is at rest, $v=0$ — used — m1

$t = 300 \ln(80) - 300 \ln(20)$

$= 300\ln(4)$

$=$ <u>416 s</u> — cao — A1

2.(a) Amplitude = 24 cm =0.24 m — B1

Period = 2× 4 = 8 s — B1

Therefore $\frac{2\pi}{\omega} = 8$ — M1

$\omega = \frac{\pi}{4}$ — A1

Speed of projection $= a\omega$ — used o.e. — M1

$= 0.24 \times \frac{\pi}{4}$

$= 0.06\pi =$ <u>$0.188\ ms^{-1}$</u> $(= 18.8\ cms^{-1})$ — cao — A1

2.(b) $x = 0.24\sin(\frac{\pi}{4}t)$ — M1

$0.15 = 0.24 \sin(\frac{\pi}{4}t)$ — m1

$t = 0.86$ — cao — A1

Required time = 8 + 0.86

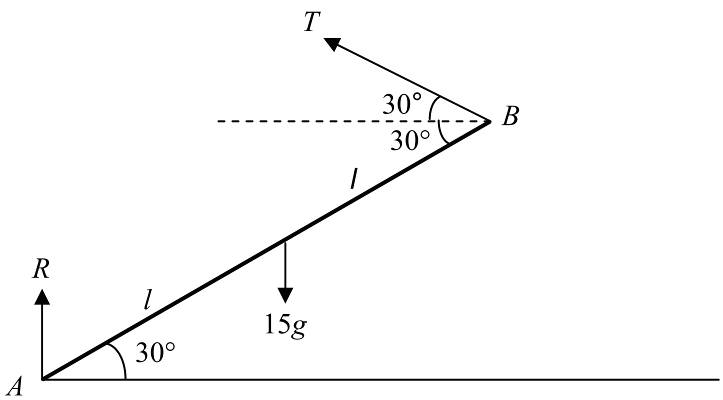
$=$ <u>8.86 s</u> — ft t and period — A1

2.(c)	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	used	M1
	$v = 0.06\pi\cos(\frac{\pi}{4}t)$	ft ω	A1
	When $t = 1.5$ $v = 0.06\pi\cos(\frac{\pi}{4} \times 1.5)$		m1
	$v = \underline{0.072 \text{ ms}^{-1}}$ (= 7.2 cms^{-1})	cao	A1
2.(d)	$v^2 = \omega^2(a^2 - x^2)$		M1
	$v^2 = \frac{\pi^2}{4^2}(0\cdot24^2 - 0\cdot2^2)$		A1
	$v = \underline{0.104 \text{ ms}^{-1}}$ (=10.4 cms^{-1})	cao	A1
3.	Apply N2L		M1
	$180 - 3v^2 = 75a$		A1
	$60 - v^2 = 25v\frac{\mathrm{d}v}{\mathrm{d}x}$		
	$25v\frac{\mathrm{d}v}{\mathrm{d}x} = 60 - v^2$		A1
	$25\int\frac{v\mathrm{d}v}{\mathrm{d}x} = \int \mathrm{d}x$	sep. var.	M1
	$-\frac{25}{2}\ln(60 - v^2) = x \quad (+C)$		A1 A1
	When $x = 0$, $v = 0$	(accept limits) used	m1
	$-\frac{25}{2}\ln(60) = C$	cao	A1
	$x = \frac{25}{2}\ln\left(\frac{60}{60 - v^2}\right)$		
	When $x = 20$		
	$\ln\left(\frac{60}{60 - v^2}\right) = 20 \times \frac{2}{25} = 1\cdot6$		
	$\frac{60}{60 - v^2} = e^{1\cdot6}$	$x = 20$ and inversion	m1
	$60 = 60e^{1.6} - e^{1.6}v^2$		
	$v^2 = \frac{60(e^{1\cdot6} - 1)}{e^{1\cdot6}}$		
	$v = \underline{6.92 \text{ ms}^{-1}}$	cao	A1

4.(a)	Impulse = change in momentum	used	M1
	$1.2 = 3v$		
	$v = \underline{0.4 \text{ ms}^{-1}}$	cao	A1
4.(b)	For Q $\quad -I = 3v - 3 \times 0.4$	attempt P or Q	M1
	$I = 3v - 1.2$		
	For P $\quad I = 5v$	attempt	m1
	Both equations correct		A1
	Solving simultaneously		m1
	$5v = 1.2 - 3v$		
	$8v = 1.2$		
	$v = \underline{0.15 \text{ ms}^{-1}}$	cao	A1
	$I = \underline{0.75 \text{ Ns}}$	cao	A1
4.(c)	Loss in energy $= 0.5 \times 3 \times 0.4^2 - 0.5 \times 8 \times 0.15^2$	ft v's	M1 A1 A1
	$= \underline{0.15 \text{ J}}$	cao	A1

5.(a)	N2L $\quad (156 - 52x) - 4v = 2a$		M1
	$2a + 4v + 52x = 156$		
	$\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 26x = 78$		A1
5.(b)	Auxiliary Equation $\quad m^2 + 2m + 26 = 0$		M1
	$m = -1 \pm 5i$	cao	A1
	Complementary function is $x = e^{-t}(A\sin 5t + B\cos 5t)$	ft m if complex	B1
	P. I. try $x = a$		
	$26a = 78$		
	$a = 3$		B1
	General solution is $x = e^{-t}(A\sin 5t + B\cos 5t) + 3$	ft CF + PI	B1
	When $t = 0$, $x = 0$	subst. into GS	m1
	$0 = B + 3$		
	$B = -3$	ft similar exp.	A1
	$\dfrac{dx}{dt} = -e^{-t}(A\sin 5t + B\cos 5t) + e^{-t}(5A\cos 5t + 5B\sin 5t)$	ft	B1
	When $t = 0$, $\dfrac{dx}{dt} = 3$	subst. into "GS"	m1
	$3 = 3 + 5A$		
	$A = 0$	cao	A1
	$x = 3 - 3e^{-0.5}\cos 5t$		
	When $t = 0.5$		m1
	$x = 3 - 3e^{-0.5}\cos(5 \times 0.5)$		
	$x = \underline{4.46 \text{ m}}$	cao	A1

6.



Moments about A		dim correct	M1
	$15g \times l \cos 30° = T \times 2l \cos 30°$		A1
	$T = 75g$		
	$T = \underline{75.5 \text{ N}}$		A1
Resolve horizontally			M1
	$T \cos 30° = F$		A1
	$F = 73.5 \cos°30$	subst. for T	m1
	$F = 36.75\sqrt{3}$ N		
Resolve vertically			M1
	$R + T \sin 30° = 15g$		A1
	$R = 15g - 73.5 \times 0.5$	subt. for T	m1
	$R = 110.25$ N		
	$F \leq \mu R$		M1
	$\mu \geq \dfrac{36 \cdot 75\sqrt{3}}{110 \cdot 25}$	any correct expression	A1
	Therefore least value of μ is 0.577 $(\frac{1}{\sqrt{3}})$	cao	A1

A/AS Maths - S1 – June 2009 - Mark Scheme – Post Examiners' Conference

1	(a)	P(no teachers) = $\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}$ or $\frac{\binom{7}{3}}{\binom{9}{3}} = \frac{5}{12}$	M1A1
	(b)	P(1 of each) = $\frac{2}{9}\times\frac{3}{8}\times\frac{4}{7}\times 6$ or $\frac{\binom{2}{1}\times\binom{3}{1}\times\binom{4}{1}}{\binom{9}{3}}$	M1A1
		$= \frac{2}{7}$ (cao)	A1
2	(a)(i)	$P(A\cup B) = P(A) + P(B) = 0.5$	M1A1
	(ii)	$P(A\cap B) = P(A)P(B) = 0.2 \times 0.3$	M1
		$P(A\cup B) = P(A) + P(B) - P(A)P(B)$	M1
		$= 0.2 + 0.3 - 0.2 \times 0.3 = 0.44$	A1
	(b) EITHER	$P(A\cap B) = P(A) + P(B) - P(A\cup B)$	M1
		$= 0.1$	A1
		$P(A\|B) = \frac{P(A\cap B)}{P(B)}$	M1
		$= \frac{1}{3}$	A1
		[FT on $P(A\cap B)$ unless independence assumed]	
	OR	$P(A\cup B) = P(A) + P(B) - P(B)P(A\mid B)$	M1
		$0.4 = 0.2 + 0.3 - 0.3P(A\mid B)$	A1
		$P(A\|B) = \frac{0.1}{0.3}$	M1
		$= \frac{1}{3}$	A1
	(c)	Smallest value is 0.3	B1
		when A is a subset of B.	B1
3	(a)	Mean = 20, variance = 4.	B1B1
	(b)(i)	$E(Y) = 2 \times 20 - 3 = 37$ [FT from (a)]	M1A1
		$\text{Var}(Y) = 4 \times 4 = 16$	M1A1
	(ii)	$20a - b = 0$ and $4a^2 = 1$	B1B1
		The solution is $a = ½$, $b = 10$	B1B1
		[No FT on equations, treat $aX + b$ as MR and accept $b = -10$]	

4 (a) Mean = 2.4 si B1

(i) Prob = $\frac{2.4^3}{6}e^{-2.4}$ = 0.2090 M1A1

(or 0.7787 – 0.5697 or 0.4303 – 0.2213)

(ii) Prob = 0.4303 or 1 – 0.5697 = 0.4303 M1A1

(b) Prob of no fish = $e^{-0.6t}$ =0.5 M1A1

EITHER

$-0.6t\log_{10} e = \log_{10} 0.5$ so t = 1.16 M1A1

OR

-0.6t = ln(0.5) so t = 1.16 M1A1

[No marks answer only]

Special case:

Using tables,

$0.7 \approx 0.6t$ M1

so $t \approx 1.17$ A1

5 (a) P(+) = $0.05 \times 0.99 + 0.95 \times 0.02$ M1A1

= 0.0685 (cao) (137/2000) A1

(b) P(Dis | +) = $\frac{0.05 \times 0.99}{0.0685}$ B1B1

= 0.723 (99/137) B1

[FT from (a) – only award final B1 if previous 2 B marks awarded]

6 (a)(i) E(X) = $0.1 \times 1 + 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 4 + 0.1 \times 5$ M1A1

= 3.1 cao A1

(ii) $E(X^2) = 0.1 \times 1 + 0.2 \times 4 + 0.3 \times 9 + 0.3 \times 16 + 0.1 \times 25$ (10.9) M1A1

Var(X) = $10.9 - 3.1^2$ = 1.29 [FT from E(X^2) and E(X)] A1

[Accept answers with no working]

(b) The possibilities are 1,1 ; 2,2 ; 3,3 ; 4,4 ; 5,5 si B1

Prob = $0.1^2 + 0.2^2 + 0.3^2 + 0.3^2 + 0.1^2$ = 0.24 M1A1

[Award M1 if 4 or more correct probabilities seen]

7 (a)

The probability distribution for Ann is

No. of heads	0	1	2	3
Prob	1/8	3/8	3/8	1/8

B1(si)

and for Bob is

No. of heads	0	1	2
Prob	1/4	1/2	1/4

B1(si)

We now require

A = 3 OR A = 2 and B= 0 or 1 OR A = 1 and B = 0 si M1A1

(or B = 0 and A = 1,2 or 3 OR B = 1 and A = 2 or 3 OR B = 2 and A = 3)

$$P(A > B) = \frac{1}{8} + \frac{3}{8} \times \frac{3}{4} + \frac{3}{8} \times \frac{1}{4} = \frac{1}{2}$$ M1A1

$$(\text{or } P(A > B) = \frac{1}{4} \times \frac{7}{8} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{8} = \frac{1}{2}$$

(b)(i) $P(\text{M wins } 1^{\text{st}} \text{ time}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ M1A1

(ii) $P(\text{M wins } 2^{\text{nd}} \text{ time}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ A1

(iii) $P(\text{M wins}) = \frac{1}{4} + \frac{1}{16} + \ldots$ M1A1

$= \frac{1/4}{1 - 1/4} = \frac{1}{3}$ M1A1

8 (a) $E(X) = \frac{1}{2}\int_0^1 x(1+2x)\mathrm{d}x$ M1A1

$= \frac{1}{2}\left[\frac{x^2}{2} + \frac{2x^3}{3}\right]_0^1$ A1

$= \frac{7}{12}$ cao A1

(b) $F(x) = \frac{1}{2}\int_0^x (1+2u)\mathrm{d}u$ M1

[Correct limits required but accept a solution with no limits which includes a constant of integration which is then shown to be zero]

$= \frac{1}{2}\left[u + u^2\right]_0^x$ B1

$= \frac{1}{2}\left(x + x^2\right)$ A1

(c) (i) Prob = $F(0.5) - F(0.4)$ M1

$= 0.095$ A1

[FT from (b), accept with no working]

(ii) $\frac{1}{2}\left(m + m^2\right) = \frac{1}{2}$ M1

$m = \dfrac{-1 \pm \sqrt{1+4}}{2}$ m1

$= 0.618 \quad \left(\dfrac{\sqrt{5}-1}{2}\right)$ A1

[If 2 roots are given, some indication of which to accept must be given]

1 (a) Mean = 12 si — B1

p-value = P($X \ge 18$ | mean = 12) — M1

= 0.0630 — A1

(b) X is now Po(100) which is approx N(100,100) — B1

$$z = \frac{124.5 - 100}{10}$$ — M1

= 2.45 cao — A1

p-value = 0.00714 (FT from z) — A1

Very strong evidence for concluding that the mean has increased. — B1

[FT from p-value]

2 (a) (i) $z = \frac{150 - 140}{8} = 1.25$ — M1A1

Prob = 0.1056 (FT from z) — A1

(ii) Required prob = $0.1056^3 = 0.00118$ [FT from (i)] — M1A1

(b) $A - R$ is N(145-140, $8^2 + 6^2$) ie N(5, 100) — M1A1

$P(A < R) = P(A - R < 0)$ — M1

$$z = \frac{5}{\sqrt{100}} = (\pm)0.5$$ — A1

Prob = 0.3085 — A1

[No FT on mean and variance]

3 (a) $\bar{x} = \frac{66.8}{10}$ (= 6.68) si — B1

SE of $\bar{X} = \frac{0.1}{\sqrt{10}}$ (= 0.03162…) si — B1

[Accept variance of mean]

99% conf limits are

$6.68 \pm 2.576 \times 0.1/\sqrt{10}$ — M1A1

[M1 correct form, A1 2.576, allow their mean and SE for the M mark]

giving [6.60,6.76] — A1

[FT on their mean, SE and z excluding the use of 0.1 as SE]

(b) $$z = \frac{6.74 - 6.68}{0.03162}$$ — M1

[FT on their SE]

= 1.90 — A1

Conf level = 1 - 0.0287 × 2 = 0.9426 — B1B1

[M0 for trial and improvement]

4 (a) $H_0 : \mu_x = \mu_y$ versus $H_1 : \mu_x \neq \mu_y$ B1

(b) $\bar{x} = 15.8$, $\bar{y} = 16.2$ B1

SE of difference of means = $\sqrt{\frac{0.5^2}{6} + \frac{0.5^2}{5}}$ (= 0.3027…) B1

[Accept variance]

$$z = \frac{16.2 - 15.8}{0.3027}$$ M1

= 1.32 cao A1

Prob from tables = 0.0934 cao A1

p-value = 2 × 0.0934 = 0.1868 (FT from line above) B1

Mean times are equal (oe) (FT from p-value) B1

5 (a)(i) E(X) = 8 B1

(ii) Var(X) = 4.8 B1

Using Var(X) = $E(X^2) - [E(X)]^2$ M1

$E(X^2)$ = 4.8 + 64 = 68.8 A1

(b) Var(Y) = μ B1

using Var(Y) = $E(Y^2) - [E(Y)]^2$ M1

$\mu = 9.36 - \mu^2$ A1

$\mu = 2.6$ cao A1

(c) E(U)=E(X)E(Y) = 20.8 B1

$E(U^2) = E(X^2)E(Y^2) = 643.968$ B1

Var(U) = $E(X^2Y^2) - [E(XY)]^2$ M1

= 643.968 − 20.8² = 211.328 A1

[FT their values from (a) and (b) – allow a multiple of μ in first line]

6 (a)(i) $f(x) = \frac{1}{7}$ B1

(ii) $F(x) = \int_9^x \frac{1}{7}\, du$ M1

$= \left[\frac{u}{7}\right]_9^x$ A1

$= \frac{x-9}{7}$ A1

(b)(i) $E(Y) = \int_9^{16} \sqrt{x} . \frac{1}{7} dx$ — M1

[no limits required for M mark]

$= \frac{1}{7} \times \left[x^{1.5} . \frac{2}{3} \right]_9^{16}$ — A1

$= 3.52$ — A1

(ii) The median m satisfies

$P(Y \le m) = 0.5$ — M1

$P(\sqrt{X} \le m) = 0.5$ — A1

$P(X \le m^2) = 0.5$ — A1

$F(m^2) = \frac{m^2 - 9}{7} = 0.5$ — M1

$m = 3.54$ — A1

[FT their $F(x)$ from (a)]

7 (a) $H_0 : p = 0.7$ versus $H_1 : p > 0.7$ — B1

(b) Under H_0, X (No cured) is B(50,0.7) — B1

and Y (No not cured) is B(50,0.3) (si) — B1

p-value = $P(X \ge 40) \mid H_0)$ — M1

$= P(Y \le 10 \mid H_0) = 0.0789$ — A1

The new drug is no better — B1

(c)(i) X is now B(250,0.7) which is approx N(175,52.5) — M1A1

$z = \frac{189.5 - 175}{\sqrt{52.5}}$ — M1

$= 2.00$ cao — A1

Sig level = 0.02275 (FT from their z) — A1

(ii) X is now B(250,0.8) which is approx N(200,40) — M1A1

$z = \frac{189.5 - 200}{\sqrt{40}}$ — M1

$= -1.66$ cao — A1

Prob = 0.0485 — A1

1. (a)(i) $P(X=5)=\binom{5}{5}\div\binom{8}{5}$ or $\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}=\frac{1}{56}$ (0.018) M1A1

$P(X=4)=\binom{5}{4}\times\binom{3}{1}\div\binom{8}{5}=\frac{15}{56}$ (0.27) A1

$P(X=3)=\binom{5}{3}\times\binom{3}{2}\div\binom{8}{5}=\frac{30}{56}$ (0.54) A1

$P(X=2)=\binom{5}{2}\times\binom{3}{3}\div\binom{8}{5}=\frac{10}{56}$ (0.18) A1

(ii) $E(X)=5\times\frac{1}{56}+4\times\frac{15}{56}+3\times\frac{30}{56}+2\times\frac{10}{56}=\frac{25}{8}$ M1A1

[FT on probabilities]

(b) X is binomial B1

with parameters (5,5/8) B1

Mean value = $5\times\frac{5}{8}=\frac{25}{8}$ B1

[Accept any correct method]

2 (a)(i) $\hat{p}=\frac{78}{120}=0.65$ B1

(ii) ESE = $\sqrt{\frac{0.65\times0.35}{120}}=0.04354..$ M1A1

[M mark needs square root term in denominator]

(iii) 95% confidence limits are

$0.65\pm1.96\times0.0435..$ M1A1

giving [0.56,0.74] A1

[FT values from (i) and (ii) but M mark needs square root in denominator]

(b) Sian is likely to be of above average intelligence. B1

3	(a)	$H_0 : \mu = 1.5; H_1 : \mu \neq 1.5$	B1
	(b)	$\bar{x} = \dfrac{121.2}{80} (= 1.515)$ si	B1
		$s^2 = \dfrac{184.42}{79} - \dfrac{121.2^2}{79 \times 80}$ (= 0.01015…) si	B1
		[Accept division by 80 giving 0.010025]	
		Test stat = $\dfrac{1.515 - 1.5}{\sqrt{0.01015/80}}$	M1A1
		[Award M1 only if $\sqrt{n}$ term present]	
		= 1.33 (1.34)	A1
		Prob from tables = 0.0918 (0.0901)	A1
		p-value = 2 × 0.0918 = 0.1836/5 (0.1802) (FT from line above)	B1
		Accept H_0 (oe) (FT from line above)	B1
	(c)	The sample mean is (approximately) normal.	B1
		The variance estimate is used in place of the actual variance.	B1
4	(a)	$\Sigma x = 61.1; \Sigma x^2 = 373.3412$	B1
		UE of μ = 6.11 cao	B1
		UE of $\sigma^2 = \dfrac{373.3412}{9} - \dfrac{61.1^2}{9 \times 10}$	M1
		= 0.002244… cao	A1
	(b)	DF = 9 si	B1
		At the 95% confidence level, critical value = 2.262	B1
		The 95% confidence limits are	
		$6.11 \pm 2.262\sqrt{\dfrac{0.002244..}{10}}$	M1A1
		[Award the M mark only if *t*-value used – FT on *t* percentile]	
		giving [6.08,6.14]	A1
		[FT from line above as long as $\sqrt{10}$ present]	
5	(a)	$\bar{x} = 65.5; \bar{y} = 67.0$ si	B1B1
		$s_x^2 = \dfrac{258000}{59} - \dfrac{3930^2}{59 \times 60} (= 9.91525..)$ si	M1A1
		$s_y^2 = \dfrac{269900}{59} - \dfrac{4020^2}{59 \times 60} (= 9.49152...)$ si	A1
		[Accept division by 60 which gives 9.75 and 9.3333..]	
		SE = $\sqrt{\dfrac{9.91525..}{60} + \dfrac{9.49152}{60}}$ (= 0.5687 or 0.5640)	M1A1
		The 90% confidence limits for the difference of means are	
		$67.0 - 65.5 \pm 1.645 \times 0.5687$	M1A1
		[M mark requires $\sqrt{60}$, FT from earlier values]	
		giving [0.6,2.4]	A1
	(b)	Yes because the interval does not contain 0.	

6 (a) $E(U) = \lambda\mu + (1-\lambda)\mu = \mu$ M1A1

(b) $\text{Var}(U) = \lambda^2 \text{ Var}(\overline{X}) + (1-\lambda)^2 \text{Var}(\overline{Y})$ M1

$$= \lambda^2 . \frac{\sigma_x^2}{m} + (1-\lambda)^2 . \frac{\sigma_y^2}{n}$$ m1A1

(c)(i) $\dfrac{\text{dVar}(U)}{\text{d}\lambda} = 2\lambda . \dfrac{\sigma_x^2}{m} - 2(1-\lambda) . \dfrac{\sigma_y^2}{n}$ M1A1

This equals zero when m1

$$\frac{\lambda}{1-\lambda} = \frac{\sigma_y^2}{n} . \frac{m}{\sigma_x^2}$$ A1

whence $\lambda = \dfrac{m\sigma_y^2}{m\sigma_y^2 + n\sigma_x^2}$ A1

$\dfrac{\text{d}^2\text{Var}(U)}{\text{d}\lambda^2} = \dfrac{2\sigma_x^2}{m} + \dfrac{2\sigma_y^2}{n} > 0$ therefore minimum B1

[Accept other correct solutions, eg U is a quadratic in λ with positive λ^2 term]

(ii) Min Var = $\dfrac{\sigma_x^2}{m} . \dfrac{m^2\sigma_y^4}{(m\sigma_y^2 + n\sigma_x^2)^2} + \dfrac{\sigma_y^2}{n} . \dfrac{n^2\sigma_x^4}{(m\sigma_y^2 + n\sigma_x^2)^2}$ M1A1

$$= \frac{\sigma_x^2\sigma_y^2(m\sigma_y^2 + n\sigma_x^2)}{(m\sigma_y^2 + n\sigma_x^2)^2}$$ A1

whence the given result.

7 (a) $S_{xy} = 5590.5 - 105 \times 262.6/6 = 995$ B1

$S_{xx} = 2275 - 105^2/6 = 437.5$ B1

$b = \dfrac{995}{437.5} = 2.27$ M1A1

$a = \dfrac{262.6 - 105 \times 2.27..}{6}$ M1

$= 3.97$ A1

[No marks for answer only]

(b) SE of $b = \dfrac{0.5}{\sqrt{437.5}} (= 0.0239)$ M1A1

$z = \dfrac{2.27.. - 2.34}{0.0239}$ M1

[Award M mark only if $\sqrt{S_{xx}}$ present]

$= -2.75$ A1

p-value = 0.00298 A1

Very strong evidence that $\beta < 2.34$ B1

(c) It is clear from the data that β is approximately 2 because y increases by approx 10 when x increases by 5. B1

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