WJEC
CBAC

GCE MARKING SCHEME

MATHEMATICS
AS/Advanced

SUMMER 2010

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2010 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

C1

1. (*a*) (i) Gradient of $AC = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AC = 4/3$ (or equivalent) A1

(ii) A correct method for finding the equation of AC using candidate's gradient for AC M1

Equation of AC: $y - 4 = 4/3[x - (-6)]$ (or equivalent)
(f.t. candidate's gradient for AC) A1

Equation of AC: $4x - 3y + 36 = 0$ (convincing) A1

(iii) [Gradient of $BD = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1]

(to be awarded only if corresponding M1 is not awarded in part (i))

Gradient of $BD = -3/4$ (or equivalent) A1

An attempt to use the fact that the product of perpendicular lines = -1 (or equivalent) M1

Gradient $AC \times$ Gradient $BD = -1 \Rightarrow AC, BD$ perpendicular A1

(iv) [A correct method for finding the equation of BD using the candidate's gradient for BD M1]

(to be awarded only if corresponding M1 is not awarded in part (ii))

Equation of BD: $y - 11 = -{}^{3}/_{4}[x - (-7)]$ (or equivalent)
(f.t. candidate's gradient for BD) A1

Note: Total mark for part (*a*) is 9 marks

(*b*) (i) An attempt to solve equations of AC and BD simultaneously M1

$x = -3$, $y = 8$ (convincing) A1

(ii) A correct method for finding the length of BE M1

$BE = 15$ A1

2. (*a*) $\frac{5\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}} = \frac{(5\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})}{(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})}$ M1

Numerator: $5 \times 7 + 5 \times \sqrt{7} \times \sqrt{3} - \sqrt{7} \times \sqrt{3} - 3$ A1

Denominator: $7 - 3$ A1

$\frac{5\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}} = 8 + \sqrt{21}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{7} - \sqrt{3}$

(*b*) $\sqrt{15} \times \sqrt{20} = 10\sqrt{3}$ B1

$\sqrt{75} = 5\sqrt{3}$ B1

$\frac{\sqrt{60}}{\sqrt{5}} = 2\sqrt{3}$ B1

$(\sqrt{15} \times \sqrt{20}) - \sqrt{75} - \frac{\sqrt{60}}{\sqrt{5}} = 3\sqrt{3}$ (c.a.o.) B1

3. (*a*) $\frac{dy}{dx} = 2x - 8$

An attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = -2$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - (-5) = {}^1/_2(x - 3)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and both m1's awarded) A1

(*b*) Putting candidate's expression for $\frac{dy}{dx} = 4$ M1

x-coordinate of $Q = 6$ A1

y-coordinate of $Q = -2$ A1

$c = -26$ A1

(f.t. candidate's expression for $\frac{dy}{dx}$ and at most one error in the enumeration of the coordinates of Q for all three A marks provided both M1's are awarded)

4. (*a*) $(1 + x)^6 = 1 + 6x + 15x^2 + 20x^3 + \ldots$

All terms correct B2

Three terms correct B1

(*b*) An attempt to substitute $x = -0{\cdot}01$(or $x = -0{\cdot}1$) in the expansion of part (*a*) (f.t. candidate's coefficients from part (*a*)) M1

$(0{\cdot}99)^6 \approx 1 - 6 \times 0{\cdot}01 + 15 \times 0{\cdot}0001 - 20 \times 0{\cdot}000001$

(At least three terms correct, f.t. candidate's coefficients from part (*a*)) A1

$(0{\cdot}99)^6 = 0{\cdot}94148 = 0{\cdot}9415$ (correct to four decimal places) (c.a.o.) A1

5. (*a*) $a = 2$ B1

$b = 3$ B1

$c = -25$ B1

(*b*) $6x^2 + 36x - 17 = 3\,[a(x + b)^2 + c] + k$ ($k \neq 0$, candidate's a, b, c) M1

Least value $= 3c + 4$ (candidate's c) A1

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1

$b^2 - 4ac = k^2 - 4 \times 2 \times 18$ A1

Candidate's expression for $b^2 - 4ac < 0$ m1

$-12 < k < 12$ (c.a.o.) A1

(*b*) Finding critical values $x = -0{\cdot}5$, $x = 0{\cdot}6$ B1

A statement (mathematical or otherwise) to the effect that

$x \leq -0{\cdot}5$ or $0{\cdot}6 \leq x$ (or equivalent) (f.t. only $x = \pm 0{\cdot}5$, $x = \pm 0{\cdot}6$) B2

Deduct 1 mark for each of the following errors

the use of $<$ rather than $\leq$

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = -(x + \delta x)^2 + 5(x + \delta x) - 9$ B1

Subtracting y from above to find δy M1

$\delta y = -2x\delta x - (\delta x)^2 + 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\dfrac{dy}{dx} = \lim_{\delta x \to 0} \dfrac{\delta y}{\delta x} = -2x + 5$ (c.a.o.) A1

(*b*) $\dfrac{dy}{dx} = \dfrac{3}{4} \times \dfrac{1}{3} \times x^{-2/3} + (-2) \times 12 \times x^{-3}$ B1, B1

Either $8^{-2/3} = \dfrac{1}{4}$ **or** second term $= (-)\dfrac{24}{512}$ (or equivalent fraction) B1

$\dfrac{dy}{dx} = \dfrac{1}{64}$ (or equivalent) (c.a.o.) B1

8. (*a*) Use of $f(-2) = 0$ M1

$-96 + 4k + 26 - 6 = 0 \Rightarrow k = 19$ A1

Special case

Candidates who assume $k = 19$ and show $f(-2) = 0$ are awarded B1

(*b*) $f(x) = (x + 2)(12x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x + 2)(12x^2 - 5x - 3)$ A1

$f(x) = (x + 2)(4x - 3)(3x + 1)$(f.t. only $12x^2 + 5x - 3$ in above line) A1

Special case

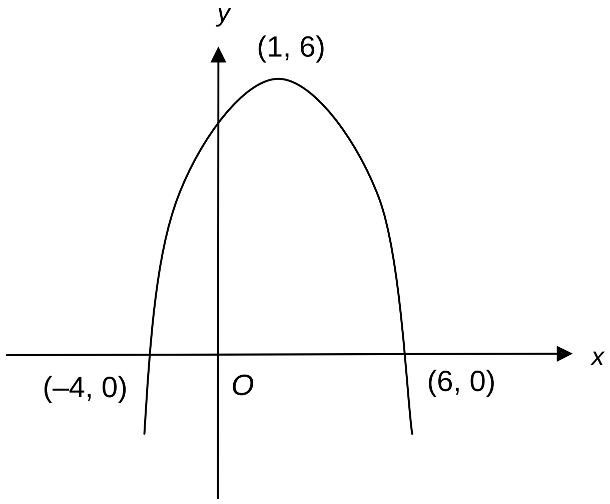
Candidates who find one of the remaining factors,
$(4x - 3)$ or $(3x + 1)$, using e.g. factor theorem, are awarded B1

(*c*) Attempting to find $f(1/2)$ M1

Remainder = $-\frac{25}{4}$ A1

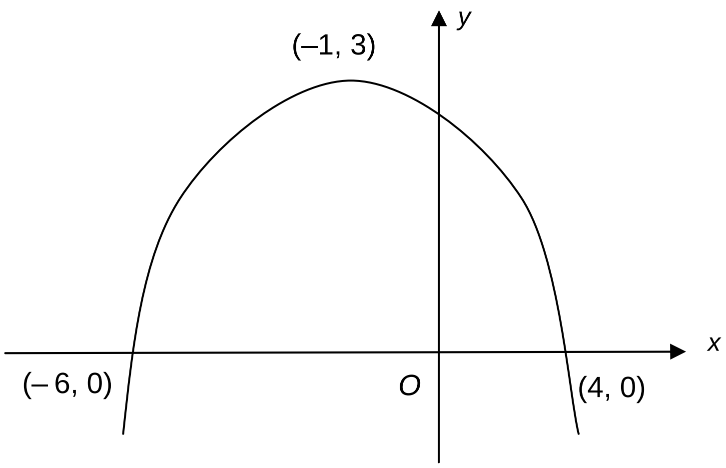
If a candidate tries to solve (*c*) by using the answer to part (*b*), f.t. when candidate's expression is of the form $(x + 2) \times$ two linear factors

9. (*a*)



Concave down curve with maximum at (1, *a*), $a \neq 3$	B1
Maximum at (1, 6)	B1
Both points of intersection with *x*-axis	B1

(*b*)



Concave down curve with maximum at (*b*, 3), $b \neq 1$	B1
Maximum at (–1, 3),	B1
Both points of intersection with *x*-axis	B1

10. (*a*) $\frac{dy}{dx} = \frac{3}{2}x^2 - 6$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = -2, 2$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(-2, 11)$ and $(2, -5)$ (both correct)
(c.a.o.) A1

A correct method for finding nature of stationary points yielding
either $(-2, 11)$ is a maximum point
or $(2, -5)$ is a minimum point (f.t. candidate's derived values) M1
Correct conclusion for other point
(f.t. candidate's derived values) A1

1.

1	1·414213562		
1·25	1·341640787		
1·5	1·290994449		
1·75	1·253566341	(5 values correct)	B2
2	1·224744871	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ — M1

$I \approx \frac{0{\cdot}25}{2} \times \{1{\cdot}414213562 + 1{\cdot}224744871 + 2(1{\cdot}341640787 + 1{\cdot}290994449 + 1{\cdot}253566341)\}$

$I \approx 10{\cdot}41136159 \div 8$

$I \approx 1{\cdot}301420198$

$I \approx 1{\cdot}301$ (f.t. one slip) — A1

Special case for candidates who put $h = 0{\cdot}2$

1	1·414213562		
1·2	1·354006401		
1·4	1·309307341		
1·6	1·274754878		
1·8	1·247219129		
2	1·224744871	(all values correct)	B1

Correct formula with $h = 0{\cdot}2$ — M1

$I \approx \frac{0{\cdot}2}{2} \times \{1{\cdot}414213562 + 1{\cdot}224744871 + 2(1{\cdot}354006401 + 1{\cdot}309307341 + 1{\cdot}274754878 + 1{\cdot}247219129)\}$

$I \approx 13{\cdot}00953393 \div 10$

$I \approx 1{\cdot}300953393$

$I \approx 1{\cdot}301$ (f.t. one slip) — A1

Note: Answer only with no working earns 0 marks

2. (*a*) $12(1 - \sin^2\theta) - 5\sin\theta = 10$ (correct use of $\cos^2\theta = 1 - \sin^2\theta$) — M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = coefficient of $\sin^2\theta$ and $b \times d$ = constant — m1

$12\sin^2\theta + 5\sin\theta - 2 = 0 \Rightarrow (4\sin\theta - 1)(3\sin\theta + 2) = 0$

$\Rightarrow \sin\theta = \frac{1}{4}, \sin\theta = -\frac{2}{3}$ (c.a.o.) — A1

$\theta = 14{\cdot}48°, 165{\cdot}52°$ — B1

$\theta = 221{\cdot}81°, 318{\cdot}19°$ — B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks

$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $2x = -58°, 122°, 302°$ (at least one value) B1

$x = 61°, 151°$, (both values) B1

Note: Subtract a maximum of 1 mark for additional roots in range, ignore roots outside range.

(*c*) $\sin\phi + 2\sin\phi\cos\phi = 0$ **or** $\tan\phi + 2\tan\phi\cos\phi = 0$

or $\sin\phi\left(\frac{1}{\cos\phi} + 2\right) = 0$ M1

$\sin\phi = 0$ (or $\tan\phi = 0$) , $\cos\phi = -\frac{1}{2}$ (both values) A1

$\phi = 0°, 180°$ (both values) A1

$\phi = 120°$ A1

Note: Subtract a maximum of 1 mark for each additional root in range for each branch, ignore roots outside range.

Special Case:

No factorisation but division throughout by $\sin\phi$ (or $\tan\phi$) to yield

$1 + 2\cos\phi = 0$ (or equivalent) M1

$\phi = 120°$ A1

3. (*a*) $\frac{1}{2} \times AC \times 11 \times \sin 110° = 31$

(substituting the correct values in the correct places in the area formula) M1

$AC = 5{\cdot}998$ ($AC = 6$) A1

$BC^2 = 6^2 + 11^2 - 2 \times 6 \times 11 \times \cos 110°$

(substituting the correct values in the correct places in the cos rule, (f.t. candidate's value for AC) M1

$BC = 14{\cdot}22$ (f.t. candidate's value for AC) A1

(*b*) $\frac{\sin XZY}{2} = \frac{\sin 60°}{2\sqrt{3} - 1}$ (substituting the correct values in the correct places in the sine formula) M1

$\sin XZY = \frac{2 \times \sin 60°}{2\sqrt{3} - 1}$ m1

$\sin XZY = \frac{6 + \sqrt{3}}{11}$ A1

4. $3 \times \frac{x^{3/2}}{3/2} - 6 \times \frac{x^{-3}}{-3} - x + c$ B1, B1, B1

(–1 if no constant term present)

5. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$

Or

$2S_n = [a + a + (n-1)d] +$ (n times) M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $\frac{n}{2}[2 \times 4 + (n-1) \times 2] = 460$ M1

Either: Rewriting above equation in a form ready to be solved

$2n^2 + 6n - 920 = 0$ or $n^2 + 3n - 460 = 0$ or $n(n + 3) = 460$

or: $n = 20, n = -23$ A1

$n = 20$ (c.a.o.) A1

(*c*) $a + 4d = 9$ B1

$(a + 5d) + (a + 9d) = 42$ B1

An attempt to solve the candidate's two linear equations

Simultaneously by eliminating one unknown M1

$d = 4$ (c.a.o.) A1

$a = -7$ (f.t. candidate's value for d) A1

6. (*a*) $r = -0{\cdot}6$ B1

$S_\infty = \frac{40}{1 - (-0{\cdot}6)}$ M1

$S_\infty = 25$ (c.a.o.) A1

(*b*) (i) $ar^3 = 8$ B1

$ar^2 + ar^3 + ar^4 = 28$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$\frac{r^3}{r^2 + r^3 + r^4} = \frac{8}{28} \Rightarrow 2r^2 - 5r + 2 = 0$ (convincing) A1

(ii) $r = 0{\cdot}5$ ($r = 2$ discarded, c.a.o.) B1

$a = 64$ (f.t. candidate's value for r, provided $|r| < 1$) B1

7.

Area $= \int^{3} \left(3x + \frac{x^3}{5}\right) dx$ (use of integration) M1

$\frac{3x^2}{2} + \frac{x^4}{4 \times 5}$ (correct integration) B1, B1

Area $= (27/2 + 81/20) - (3/2 + 1/20)$ (an attempt to substitute limits) m1

Area $= 16$ (c.a.o.) A1

8. (*a*) Let $p = \log_a x$

Then $x = a^p$ (relationship between log and power) B1

$x^n = a^{pn}$ (the laws of indices) B1

$\therefore \log_a x^n = pn$ (relationship between log and power)

$\therefore \log_a x^n = pn = n \log_a x$ (convincing) B1

(*b*) **Either:**

$(2y - 1) \log_{10} 6 = \log_{10} 4$

(taking logs on both sides and using the power law) M1

$$y = \frac{\log_{10} 4 + \log_{10} 6}{2 \log_{10} 6}$$ A1

$y = 0{\cdot}887$ (f.t. one slip, see below) A1

Or:

$2y - 1 = \log_6 4$ (rewriting as a log equation) M1

$$y = \frac{\log_6 4 + 1}{2}$$ A1

$y = 0{\cdot}887$ (f.t. one slip, see below) A1

Note: an answer of $y = -0{\cdot}113$ from $y = \dfrac{\log_{10} 4 - \log_{10} 6}{2 \log_{10} 6}$

earns M1 A0 A1

an answer of $y = 1{\cdot}774$ from $y = \dfrac{\log_{10} 4 + \log_{10} 6}{\log_{10} 6}$

earns M1 A0 A1

(*c*) $\log_a 4 = \dfrac{1}{2} \Rightarrow 4 = a^{1/2}$ (rewriting log equation as power equation)

M1

$a = 16$ A1

9. (*a*) $A(4, -1)$ B1

A correct method for finding radius M1

Radius = $\sqrt{10}$ A1

(*b*) (i) **Either:**

An attempt to substitute the coordinates of P in the equation of C M1

Verification that $x = 7$, $y = -2$ satisfy equation of C and hence P lies on C A1

Or:

An attempt to find AP^2 M1

$AP^2 = 10 \Rightarrow P$ lies on C A1

(ii) A correct method for finding Q M1

$Q(1, 0)$ (f.t. candidate's coordinates for A) A1

(*c*) An attempt to substitute $(2x - 4)$ for y in the equation of the circle M1

$x^2 - 4x + 3 = 0$ (or $5x^2 - 20x + 15 = 0$) A1

$x = 1, x = 3$ (correctly solving candidate's quadratic, both values) A1

Points of intersection are $(1, -2)$, $(3, 2)$ (c.a.o.) A1

10. (*a*) (i) $L = R\theta - r\theta$ B1

(ii) $K = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$ B1

(*b*) An attempt to eliminate θ M1

Use of $R^2 - r^2 = (R + r)(R - r)$ m1

$r = \frac{2K}{L} - R$ A1

C3

1.

0	0·5		
0·2	0·401312339		
0·4	0·310025518		
0·6	0·231475216	(3 values correct)	B1
0·8	0·167981614	(5 values correct)	B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{3} \times \{0{\cdot}5 + 0{\cdot}167981614 + 4(0{\cdot}401312339 + 0{\cdot}231475216) + 2(0{\cdot}310025518)\}$

$I \approx 0{\cdot}2 \times 3{\cdot}819182871 \div 3$

$I \approx 0{\cdot}254612191$

$I \approx 0{\cdot}2546$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) e.g. $\theta = \frac{\pi}{2}$

$\cos\theta + \cos 4\theta = 1$ (choice of θ and one correct evaluation) B1

$\cos 2\theta + \cos 3\theta = -1$ (both evaluations correct but different) B1

(*b*) $2(\sec^2\theta - 1) = \sec\theta + 8$ (correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in sec θ, either by using the quadratic formula or by getting the expression into the form $(a \sec\theta + b)(c \sec\theta + d)$, with $a \times c$ = coefficient of $\sec^2\theta$ and $b \times d$ = constant m1

$2\sec^2\theta - \sec\theta - 10 = 0 \Rightarrow (2\sec\theta - 5)(\sec\theta + 2) = 0$

$\Rightarrow \sec\theta = \frac{5}{2}, \sec\theta = -2$

$\Rightarrow \cos\theta = \frac{2}{5}, \cos\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 66{\cdot}42°, 293{\cdot}58°$ B1

$\theta = 120{\cdot}0°, 240{\cdot}0°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{d}{dx}(4x^2y) = 4x^2\frac{dy}{dx} + 8xy$ B1

$\frac{d}{dx}(3x^3 - 5x) = 9x^2 - 5$ B1

$\frac{dy}{dx} = \frac{9x^2 - 5 - 8xy}{4y^3 + 4x^2}$ (c.a.o.) B1 (*b*) $\frac{dx}{dt} = 4 - 2\sin 2t$, B1

$\frac{dy}{dt} = 3\cos 3t$ B1

Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Substituting $\frac{\pi}{12}$ for t in expression for $\frac{dy}{dx}$ m1

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ A1

4. $f(x) = 4x^3 - 2x - 5$

An attempt to check values or signs of $f(x)$ at $x = 1$, $x = 2$ M1

$f(1) = -3 < 0$, $f(2) = 23 > 0$

Change of sign $\Rightarrow f(x) = 0$ has root in (1, 2) A1

$x_0 = 1{\cdot}2$

$x_1 = 1{\cdot}227601026$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 1{\cdot}230645994$

$x_3 = 1{\cdot}230980996$

$x_4 = 1{\cdot}231017841 = 1{\cdot}23102$ (x_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 1{\cdot}231015$, $x = 1{\cdot}231025$ M1

$f(1{\cdot}231015) = -1{\cdot}197 \times 10^{-4} < 0$, $f(1{\cdot}231025) = 4{\cdot}218 \times 10^{-5} > 0$ A1

Change of sign $\Rightarrow \alpha = 1{\cdot}23102$ correct to five decimal places A1

Note: 'Change of sign' must appear at least once.

5. (*a*) (i) $\frac{dy}{dx} = 13 \times (7 + 2x)^{12} \times f(x)$, $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = 26 \times (7 + 2x)^{12}$ A1

(ii) $\frac{dy}{dx} = \frac{5}{\sqrt{1 - (5x)^2}}$ **or** $\frac{1}{\sqrt{1 - (5x)^2}}$ **or** $\frac{5}{\sqrt{1 - 5x^2}}$ M1

$\frac{dy}{dx} = \frac{5}{\sqrt{1 - 25x^2}}$ A1

(iii) $\frac{dy}{dx} = x^3 \times f(x) + e^{4x} \times g(x)$ M1

$\frac{dy}{dx} = x^3 \times f(x) + e^{4x} \times g(x)$ (either $f(x) = 4e^{4x}$ or $g(x) = 3x^2$) A1

$\frac{dy}{dx} = x^3 \times 4e^{4x} + e^{4x} \times 3x^2$ (all correct) A1 (*b*) $\frac{d}{dx}(\tan x) = \frac{\cos x \times m\cos x - \sin x \times k\sin x}{\cos^2 x}$ ($m = \pm 1$, $k = \pm 1$) M1

$\frac{d}{dx}(\tan x) = \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$ A1

$\frac{d}{dx}(\tan x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x} = \sec^2 x$ (convincing) A1

6. (*a*) (i) $\int (7x-9)^{1/2}\,dx = k \times \frac{(7x-9)^{3/2}}{3/2} + c$ ($k = 1, 7, {}^1/_7$) M1

$\int (7x-9)^{1/2}\,dx = {}^1/_7 \times \frac{(7x-9)^{3/2}}{3/2} + c$ A1

(ii) $\int e^{x/6}\,dx = k \times e^{x/6} + c$ ($k = 1, 6, {}^1/_6$) M1

$\int e^{x/6}dx = 6 \times e^{x/6} + c$ A1

(iii) $\int \frac{4}{5x-1}\,dx = 4 \times k \times \ln|5x-1| + c$ ($k = 1, 5, {}^1/_5$) M1

$\int \frac{4}{5x-1}\,dx = 4 \times {}^1/_5 \times \ln|5x-1| + c$ A1

(*b*) $\int (3x-4)^{-3}\,dx = k \times \frac{(3x-4)^{-2}}{-2}$ ($k = 1, 3, {}^1/_3$) M1

$\int_2^4 8 \times (3x-4)^{-3}\,dx = \left[8 \times {}^1/_3 \times \frac{(3x-4)^{-2}}{-2}\right]_2^4$ A1

Correct method for substitution of limits M1

$\int_2^4 8 \times (3x-4)^{-3}\,dx = \frac{5}{16} = 0{\cdot}3125$ (f.t. for $k = 1, 3$ only) A1

7. (*a*) Trying to solve either $3x + 1 \leq 5$ or $3x + 1 \geq -5$ M1

$3x + 1 \leq 5 \Rightarrow x \leq {}^{4}/_{3}$

$3x + 1 \geq -5 \Rightarrow x \geq -2$ (both inequalities) A1

Required range: $-2 \leq x \leq {}^{4}/_{3}$ (f.t. one slip) A1

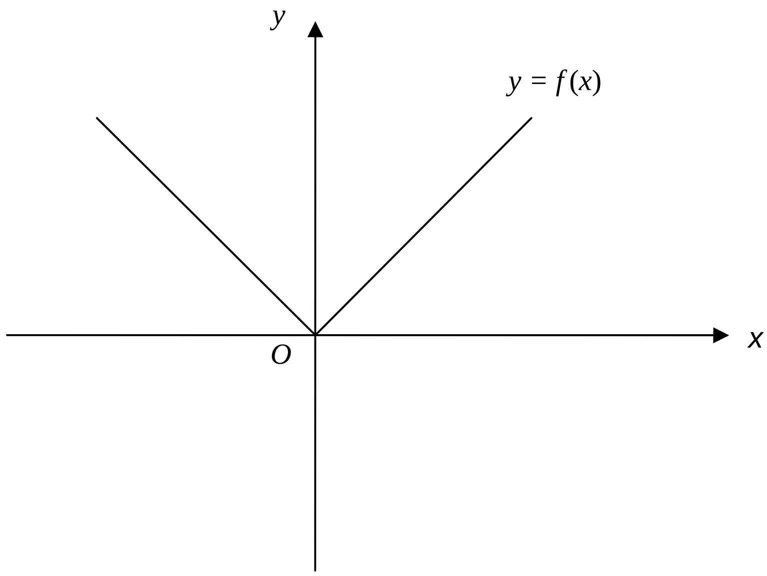
Alternative mark scheme

$(3x + 1)^2 \leq 25$ (forming and trying to solve quadratic) M1

Critical points $x = -2$ and $x = {}^{4}/_{3}$ A1

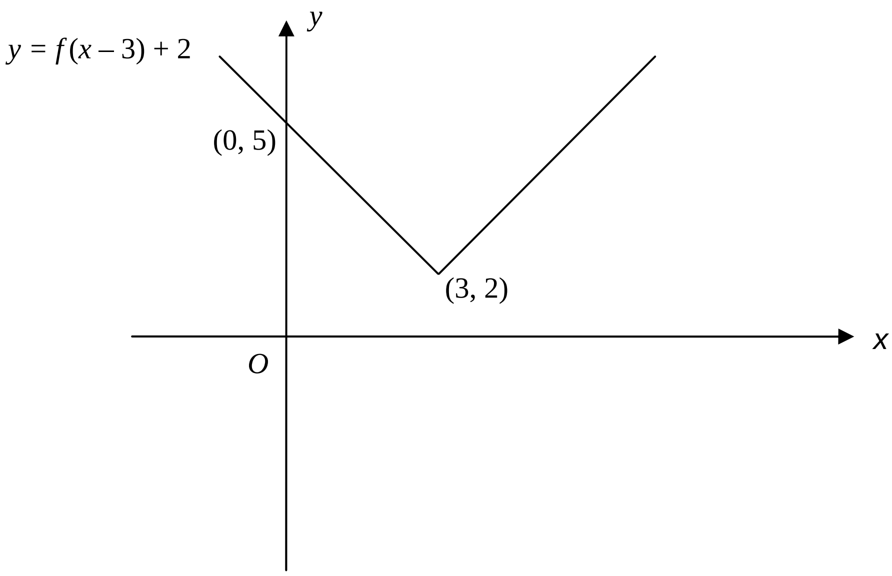
Required range: $-2 \leq x \leq {}^{4}/_{3}$ (f.t. one slip in critical points) A1

(*b*) (i)



Correct graph B1

(ii)



Translation of graph of $f(x) = |x|$ with vertex at $(\pm 3, \pm 2)$ M1

Coordinates of vertex = (3, 2) A1

Crosses y-axis at (0, 5) A1

8. (*a*) $g'(x) = \dfrac{3 \times f(x)}{4x^2 + 9} + 2 \quad f(x) \neq 1$ M1

$g'(x) = \dfrac{3 \times 8x}{4x^2 + 9} + 2$ A1

$g'(x) = \dfrac{24x + 8x^2 + 18}{4x^2 + 9} = \dfrac{2(2x + 3)^2}{4x^2 + 9}$ (convincing) A1

(*b*) (i) At stationary point, $\dfrac{2(2x + 3)^2}{4x^2 + 9} = 0$

or $\dfrac{3 \times 8x}{4x^2 + 9} + 2 = 0$ M1

$\dfrac{2(2x + 3)^2}{4x^2 + 9} = 0$ only when $x = -\,^3/_2$ A1

(ii) $g'(x) > 0$ either side of $x = -\,^3/_2$ (or at all other points) M1

Stationary point is a point of inflection A1

9. (*a*) $y - 5 = \ln(3x - 2)$ B1

An attempt to express candidate's equation as an exponential equation M1

$x = \dfrac{(e^{y-5} + 2)}{3}$ (f.t. one slip) A1

$f^{-1}(x) = \dfrac{(e^{x-5} + 2)}{3}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [5, \infty)$ B1

10. (*a*) $R(f) = [1, \infty)$ B1

$R(g) = [-3, \infty)$ B1

(*b*) $gf(x) = 2\sqrt{(x + 4)}^2 - 3.$ M1

$gf(x) = 2x + 5$ A1

(*c*) $fg(x) = \sqrt{(2x^2 - 3 + 4)}$ (correct composition) B1

$[fg(x)]^2 = 17^2$ (candidate's $fg(x)$) M1

$x^2 = 144$ (f.t. one numerical slip) A1

$x = \pm 12$ (c.a.o.) A1

C4

1. (*a*) $f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$8 - x - x^2 \equiv A(x-2)^2 + Bx(x-2) + Cx$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 2$, $C = 1$, $B = -3$ (2 coefficients, c.a.o.) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = \frac{-2}{x^2} + \frac{3}{(x-2)^2} - \frac{2}{(x-2)^3}$ (at least one of first two terms) B1

(third term) B1

(f.t. candidates values for *A*, *B*, *C*)

$f'(1) = 3$ (c.a.o.) B1

2. $10x + 4x\frac{dy}{dx} + 4y - 3y^2\frac{dy}{dx} = 0$ $\left(4x\frac{dy}{dx} + 4y\right)$ B1

$\left(-3y^2\frac{dy}{dx}\right)$ B1

$\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - (-2) = -4(x - 1)$

$\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

3. (*a*) $2(2\cos^2\theta - 1) = 9\cos\theta + 7$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant m1

$4\cos^2\theta - 9\cos\theta - 9 = 0 \Rightarrow (4\cos\theta + 3)(\cos\theta - 3) = 0$

$\Rightarrow \cos\theta = \frac{-3}{4}$, ($\cos\theta = 3$) (c.a.o.) A1

$\theta = 138{\cdot}59°, 221{\cdot}41°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $4\cos\theta + 3 = 0$, ignore roots outside range.

$\cos\theta = -$, f.t. for 2 marks, $\cos\theta = +$, f.t. for 1 mark

(*b*)	(i)	$R = 13$	B1
		Correctly expanding $\sin(x - \alpha)$ and using either $13\cos\alpha = 5$ **or** $13\sin\alpha = 12$ **or** $\tan\alpha = \frac{12}{5}$ to find α (f.t. candidate's value for R)	M1
		$\alpha = 67{\cdot}38°$ (c.a.o)	A1
	(ii)	Least value of $\frac{1}{5\sin x - 12\cos x + 20} = \frac{1}{13 \times (\pm 1) + 20}$. (f.t. candidate's value for R)	M1
		Least value = $\frac{1}{33}$ (f.t. candidate's value for R)	A1
		Corresponding value for $x = 157{\cdot}38°$ (o.e.) (f.t. candidate's value for α)	A1

4.

Volume = $\pi \int_{\pi/6}^{\pi/3} \sin^2 x \, dx$	B1
Use of $\sin^2 x = \frac{(\pm 1 \pm \cos 2x)}{2}$	M1
Correct integration of candidate's $\frac{(\pm 1 \pm \cos 2x)}{2}$	A1
Correct substitution of correct limits in candidate's integrated expression	M1
Volume = $\frac{\pi^2}{12} = 0{\cdot}822(467...)$ (c.a.o.)	A1

5.

$\left(1 - \frac{x}{4}\right)^{1/2} = 1 - \frac{x}{8} - \frac{x^2}{128}$	$\left(1 - \frac{x}{8}\right)$	B1
	$\left(- \frac{x^2}{128}\right)$	B1
$\lvert x \rvert < 4$ or $-4 < x < 4$		B1
$\frac{\sqrt{3}}{2} \approx 1 - \frac{1}{8} - \frac{1}{128}$	(f.t. candidate's coefficients)	B1
$\sqrt{3} \approx \frac{111}{64}$.	(convincing)	B1

6. (*a*) Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and at least one of $\frac{dx}{dt} = -\frac{2}{t^2}$, $\frac{dy}{dt} = 4$ correct M1

$\frac{dy}{dx} = -2t^2$ (o.e.) A1

Equation of tangent at P: $y - 4p = -2p^2\left(x - \frac{2}{p}\right)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y = -2p^2x + 8p$ (convincing) A1

(*b*) Substituting $x = 2$, $y = 3$ in equation of tangent M1

$4p^2 - 8p + 3 = 0$ A1

$p = \frac{1}{2}, \frac{3}{2}$ (both values, c.a.o.) A1

Points are (4, 2), ($^4/_3$, 6) (f.t. candidate's values for p) A1

7. (*a*) $\int x^3 \ln x \, dx = f(x) \ln x - \int f(x)\, g(x)\, dx$ M1

$f(x) = \frac{x^4}{4}$, $g(x) = \frac{1}{x}$ A1, A1

$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + c$ (c.a.o.) A1

(*b*) $\int x(2x-3)^4 \, dx = \int f(u) \times u^4 \times k \, du$

($f(u) = pu + q$, $p \neq 0$, $q \neq 0$ and $k = {}^1/_2$ or 2) M1

$\int x(2x-3)^4 \, dx = \int \frac{(u+3)}{2} \times u^4 \times \frac{du}{2}$ A1

$\int (au^5 + bu^4)\, du = \frac{au^6}{6} + \frac{bu^5}{5}$ ($a \neq 0$, $b \neq 0$) B1

Either: Correctly inserting limits of –1, 1 in candidate's $\frac{au^6}{6} + \frac{bu^5}{5}$

or: Correctly inserting limits of 1, 2 in candidate's

$\frac{a(2x-3)^6}{6} + \frac{b(2x-3)^5}{5}$ m1

$\int_1^2 x(2x-3)^4 \, dx = \frac{3}{10}$ (c.a.o.) A1

8. (*a*) $\frac{dV}{dt} = -kV^2$ B1

(*b*) $\int \frac{dV}{V^2} = -\int k\,dt$ (o.e.) M1

$-\frac{1}{V} = -kt + c$ A1

$c = -\frac{1}{12000}$ (c.a.o.) A1

$V = \frac{12000}{12000kt + 1} = \frac{12000}{at + 1}$ (convincing) A1

(*c*) Substituting $t = 2$ and $V = 9000$ in expression for V M1

$a = \frac{1}{6}$ A1

Substituting $t = 4$ in expression for V with candidate's value for a M1

$V = 7200$ (c.a.o) A1

9. (*a*) $(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = 18$ B1

$|2\mathbf{i} - 2\mathbf{j} + \mathbf{k}| = \sqrt{9}$, $|\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}| = \sqrt{81}$ (one correct) B1

Correctly substituting in the formula $\mathbf{a.b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos\theta$ M1

$\theta = 48{\cdot}2°$ (c.a.o.) A1

(*b*) (i) $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $2 - \lambda = -1 + \mu$

$-2 - 2\lambda = -4 + \mu$

$1 + 7\lambda = -2 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = -1$, $\mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $3\mathbf{i} - 6\mathbf{k}$ (f.t. one slip) A1

10. Assume that positive real numbers a, b exist such that $a + b < 2\sqrt{ab}$.

Squaring both sides we have: $(a + b)^2 < 4ab \Rightarrow a^2 + b^2 + 2ab < 4ab$ B1

$a^2 + b^2 - 2ab < 0 \Rightarrow (a - b)^2 < 0$ B1

This contradicts the fact that a, b are real and thus $a + b \geq 2\sqrt{ab}$ B1

FP1

1.

$$f(x+h)-f(x)=\frac{1}{1+(x+h)^2}-\frac{1}{1+x^2}$$ M1A1

$$=\frac{1+x^2-1-x^2-2xh-h^2}{[1+(x+h)^2](1+x^2)}$$ A1

$$=\frac{-2xh-h^2}{[1+(x+h)^2](1+x^2)}$$ A1

$$f'(x)=\lim_{h\to 0}\frac{-2xh-h^2}{h[1+(x+h)^2](1+x^2)}$$ M1

$$=\frac{-2x}{(1+x^2)^2}$$ A1

2.

$$z-\frac{5\bar{z}}{z}=2-\mathrm{i}-\frac{5(2+\mathrm{i})}{2-\mathrm{i}}$$ B1

$$=2-\mathrm{i}-\frac{5(2+\mathrm{i})^2}{(2+\mathrm{i})(2-\mathrm{i})}$$ M1

$$=2-\mathrm{i}-\frac{5(4+4\mathrm{i}-1)}{5}$$ A1

[Award A1 for either a correct numerator or denominator]

$= -1-5\mathrm{i}$ A1

Modulus = $\sqrt{26}$ (5.10) B1

$\tan^{-1}(y/x)$ = 1.37 (78.7°) B1

Argument = 4.51 (258.7°) B1

3. (a) Determinant = $2(10-5\lambda)+\lambda(4\lambda-5)+3(5-8)$ M1

$= 4\lambda^2 - 15\lambda + 11$ A1

Singular when

$4\lambda^2 - 15\lambda + 11 = 0$ M1

λ= 1, 11/4 A1

(b) When λ = 3,

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 5 \end{bmatrix}$$

(i) Determinant = 2 B1

Cofactor matrix = $\begin{bmatrix} -5 & 7 & -3 \\ 0 & -2 & 2 \\ 3 & -3 & 1 \end{bmatrix}$ M1

Adjugate = $\begin{bmatrix} -5 & 0 & 3 \\ 7 & -2 & -3 \\ -3 & 2 & 1 \end{bmatrix}$ A1

$\mathbf{A}^{-1} = \frac{1}{2}\begin{bmatrix} -5 & 0 & 3 \\ 7 & -2 & -3 \\ -3 & 2 & 1 \end{bmatrix}$ A1

(ii) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -5 & 0 & 3 \\ 7 & -2 & -3 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ M1

$= \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ A1

4. $\alpha+\beta=-2, \alpha\beta=3$ B1

Consider

$$\alpha-\frac{1}{\beta^2}+\beta-\frac{1}{\alpha^2}=\alpha+\beta-\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)$$ M1

$$=\alpha+\beta-\frac{[(\alpha+\beta)^2-2\alpha\beta]}{\alpha^2\beta^2}$$ A1

$$=-2-\frac{[4-6]}{9}=-\frac{16}{9}$$ A1

$$\left(\alpha-\frac{1}{\beta^2}\right)\left(\beta-\frac{1}{\alpha^2}\right)=\alpha\beta+\frac{1}{\alpha^2\beta^2}-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$ M1

$$=\alpha\beta+\frac{1}{\alpha^2\beta^2}-\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$ A1

$$=3+\frac{1}{9}+\frac{2}{3}=\frac{34}{9}$$ A1

The required equation is $x^2+\frac{16}{9}x+\frac{34}{9}=0$ B1

5. The statement is true for n = 1 since $4^2-1=15$ B1

Let the statement be true for $n = k$, ie

$4^{2k}-1$ is divisible by 15 or $4^{2k}-1=15N$ M1

Consider

$$4^{2(k+1)}-1=16\times 4^{2k}-1$$ M1

$$=16(1+15N)-1$$ A1

$$=16\times 15N+15$$ A1

This is divisible by 15 so true for $n = k \Rightarrow$ true for $n = k + 1$, hence proved by induction. A1

6. (a) Let $\frac{1}{r(r+2)} = \frac{A}{r} + \frac{B}{r+2} = \frac{A(r+2)+Br}{r(r+2)}$ M1

$A = 1/2, B = -1/2$ A1A1

(b)

$$S_n = \frac{1}{2}\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right]$$

$$-\frac{1}{2}\left[\frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2}\right]$$ M1A1

$= \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$ A1

$= \frac{3}{4} - \frac{n+2+n+1}{2(n+1)(n+2)}$ A1

$= \frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)}$

7. (a) $\ln f(x) = -2x\ln x$ M1A1

$\frac{f'(x)}{f(x)} = -2\ln x - 2x \times \frac{1}{x}$ M1A1

$f'(x) = -2(\ln x + 1) \times x^{-2x}$ A1

(b) At a stationary point, $f'(x) = 0$ so

$\ln x = -1$ M1
$x = 1/\mathrm{e}$ (= 0.368) A1
$y = (\mathrm{e}^2)^{1/\mathrm{e}} = 2.09$ A1
It is a maximum because $f(0.3) = 2.06$ and $f(0.4) = 2.08$ M1A1
[Accept $f'(0.3) = 0.84$ and $f'(0.4) = -0.11$]

8. (a) Rotation matrix

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ B1

Translation matrix $= \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$\mathbf{T} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ M1

$= \begin{bmatrix} 0 & -1 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & -1 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

$-y - 3 = x$

$x + 1 = y$ A1

$(x,y) = (-2,-1)$ M1A1

(c) Let $(x, y) \to (x', y')$ under T.

$x' = -y - 3$

$y' = x + 1$ M1A1

We are given that $x' + 2y' = 3$ so that M1

$-y - 3 + 2(x + 1) = 3$

$y = 2x - 4$ A1

[Special case: $x + 2y = 3$ giving $y = 2\underline{x} + 10$ – award 3/4]

9. (a) $u+\mathrm{i}v=\dfrac{1}{1-x-\mathrm{i}y}$ M1

$=\dfrac{1-x+\mathrm{i}y}{(1-x)^2+y^2}$ A1

$u=\dfrac{1-x}{(1-x)^2+y^2}$ A1

$v=\dfrac{y}{(1-x)^2+y^2}$ A1

(b) Dividing,

$\dfrac{v}{u}=\dfrac{y}{1-x}$ M1

$=1$ A1

The equation of the locus is $v=u$.

(c) Fixed points satisfy

$z=\dfrac{1}{1-z}$

or $z^2-z+1=0$ M1A1

Solutions are

$z=\dfrac{1\pm\sqrt{-3}}{2}=\dfrac{1\pm\mathrm{i}\sqrt{3}}{2}$ M1

corresponding to points $\left(\dfrac{1}{2},\pm\dfrac{\sqrt{3}}{2}\right)$ A1

Alternative solution:

Fixed points satisfy

$x=\dfrac{1-x}{(1-x)^2+y^2}\,;\,y=\dfrac{y}{(1-x)^2+y^2}$ M1A1

$x=1-x$ giving $x=½$ A1

$\dfrac{1}{4}+y^2=1$ giving $y=\pm\dfrac{\sqrt{3}}{2}$ A1

1. $u = x\sqrt{x} \Rightarrow du = \frac{3}{2}\sqrt{x}dx$ B1

and $[0,2] \rightarrow [0, 2\sqrt{2}]$ B1

$I = \frac{2}{3}\int_0^{2\sqrt{2}} \frac{du}{\sqrt{9-u^2}}$ M1

$= \frac{2}{3}\left[\sin^{-1}(\frac{u}{3})\right]_0^{2\sqrt{2}}$ A1

= 0.821 A1

2. (a) $r = 5$ B1

$\theta = \tan^{-1}(4/3) = 0.927$ (53.1°) B1

(b) First root = $(5^{1/3}, 0.309)$ M1

= 1.63 + 0.520i A1A1

Second root = $(5^{1/3}, 0.309 + 2\pi/3)$ M1

= – 1.26 + 1.15i A1

Third root = $(5^{1/3}, 0.309 + 4\pi/3)$ M1

= – 0.364 – 1.67i A1

3. Substituting for sin and cos,

$\frac{5 \times 2t}{1+t^2} - \frac{5(1-t^2)}{1+t^2} = 1$ M1A1

$10t - 5 + 5t^2 = 1 + t^2$ A1

$2t^2 + 5t - 3 = 0$

Solving, t = ½, – 3. M1A1

$\tan\left(\frac{x}{2}\right) = \frac{1}{2} \Rightarrow \frac{x}{2} = 0.464 + n\pi$ (26.6° + 180n°) M1A1

x = 0.927 + 2$n\pi$ (53.1° + 360n°) A1

$\tan\left(\frac{x}{2}\right) = -3 \Rightarrow \frac{x}{2} = -1.25 + n\pi$ (– 71.6° + 180n°) M1

x = – 2.50 + 2$n\pi$ (– 143° + 360n°) A1

4. (a) Let $\frac{3x^2}{(x+2)(x^2+2)} \equiv \frac{A}{x+2} + \frac{Bx+C}{x^2+2}$

$= \frac{A(x^2+2)+(Bx+C)(x+2)}{(x+2)(x^2+2)}$ M1

A = 2, B = 1, C = – 2 A1A1A1

(b) $I = 2\int_1^2 \frac{dx}{x+2} + \int_1^2 \frac{xdx}{x^2+2} - 2\int_1^2 \frac{dx}{x^2+2}$ M1

$= 2[\ln(x+2)]_1^2 + \frac{1}{2}[\ln(x^2+2)]_1^2 - \frac{2}{\sqrt{2}}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_1^2$ A1A1A1

$= 2\ln 4 - 2\ln 3 + \frac{1}{2}(\ln 6 - \ln 3) - \frac{2}{\sqrt{2}}\left(\tan^{-1}\left(\frac{2}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$ A1

= 0.441 cao A1

5. $\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ B1

$= 5\cos^4\theta.i\sin\theta + 10\cos^2\theta.i^3\sin^3\theta + i^5\sin^5\theta$ + real terms M1A1

Considering imaginary terms,

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ A1

$\frac{\sin 5\theta}{\sin\theta} = 5\cos^4\theta - 10\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$ A1

$= 5\cos^4\theta - 10\cos^2\theta + 10\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$

$= 16\cos^4\theta - 12\cos^2\theta + 1$ A1

As $\theta \to 0, \cos\theta \to 1$ so the required limiting value is 5. M1A1

[FT their previous expression]

6. (a) $f'(x) = \dfrac{(x-1)^2 - 2x(x-1)}{(x-1)^4}$ M1A1

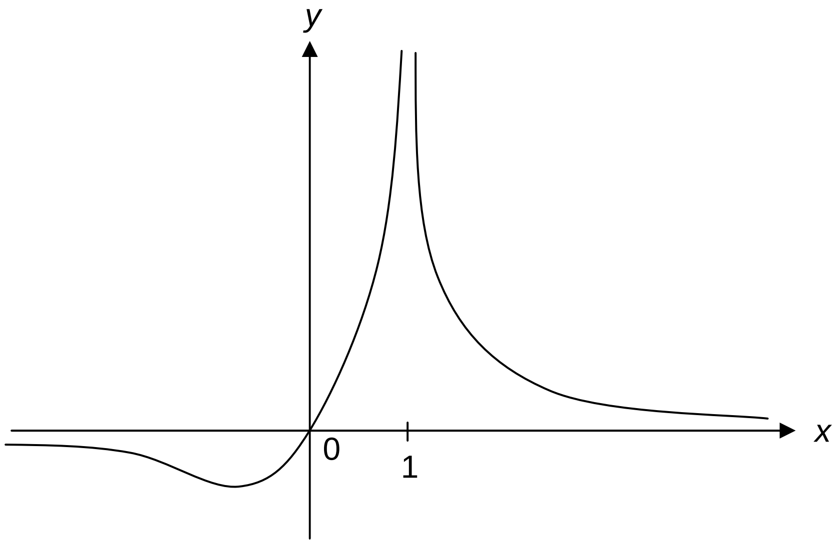
Stationary points occur when

$(x-1)^2 = 2x(x-1)$ m1

$x = -1, y = -1/4$ A1

(b) The asymptotes are $x = 1$ and $y = 0$. B1B1

(c) G1G1



(d) Consider

$\dfrac{x}{(x-1)^2} = 2$ M1

$2x^2 - 5x + 2 = 0$ A1

$x = ½, 2$ A1

$f^{-1}(A) = [0,1/2] \cup [2,\infty)$ A1A1

7. (a) $g(-x) = f(-x) + f(x) = g(x)$, therefore even B1

$h(-x) = f(-x) - f(x) = -h(x)$ therefore odd B1

The result follows from the fact that

$$f(x) = \frac{1}{2}g(x) + \frac{1}{2}h(x)$$ B1

(b) (i) $g(x) = \ln(1+\sin x) + \ln(1-\sin x)$ M1

$= \ln\cos^2 x$ A1

$= 2\ln\cos x$ A1

(ii) $h(x) = \ln(1+\sin x) - \ln(1-\sin x)$ M1

$= \ln\left(\frac{1+\sin x}{1-\sin x}\right)$ A1

$= \ln\left(\frac{(1+\sin x)^2}{\cos^2 x}\right)$ A1

$= 2\ln\left(\frac{1+\sin x}{\cos x}\right)$ A1

$= 2\ln(\sec x + \tan x)$

8. (a) Writing the equation in the form

$x^2 = -8y$

we note that, in the usual notation, a = 2 and x,y are interchanged with the negative sign indicating that the graph is below the x-axis. M1

The focus is (0, – 2) and the directrix y = 2. A1A1

(b) (i) The result follows since

$(4p)^2 + 8(-2p^2) = 0$ B1

(ii) $\frac{dy}{dx} = \frac{-4p}{4} = -p$ B1

The equation of the tangent is

$y + 2p^2 = -p(x - 4p)$ M1

$y + px = 2p^2$ A1

(iii) This passes through $(\lambda, 2)$ if

$2 + p\lambda = 2p^2$ M1

or $2p^2 - p\lambda - 2 = 0$

The 2 roots p_1, p_2 satisfy $p_1 p_2 = -1$ M1

And since the gradient of the tangent, m, satisfies $m = -p$, it follows that $m_1 m_2 = -1$ which is the condition for perpendicularity. A1

FP3

1. (a) $f'(x) = 2\cosh 2x - 14\cosh x + 8$ M1

$= 2(2\cosh^2 x - 1) - 14\cosh x + 8$ A1

$= 2(2\cosh^2 x - 7\cosh x + 3)$

(b) We solve

$2\cosh^2 x - 7\cosh x + 3 = 0$ M1

$(2\cosh x - 1)(\cosh x - 3) = 0$ M1

cosh$x$ = ½, 3 A1

cosh$x$ = ½ has no solution (so only one stationary point given by cosh$x$ = 3) A1

$x = \cosh^{-1} 3 = 1.76$ A1

(c) $f''(x) = 2(4\cosh x \sinh x - 7\sinh x)$ B1

$f''(1.76) = 28$ (exact value 20√2) B1

Positive therefore minimum. B1

2. dx = cosh$u$du ; $[0,3] \to [0,\theta]$ where $\theta = \sinh^{-1} 3$ B1B1

$$\mathrm{I} = \int_0^{\theta} \frac{\sinh^2 u \cosh u \mathrm{d}u}{\sqrt{\sinh^2 u + 1}}$$ M1

$$= \int_0^{\theta} \sinh^2 u \mathrm{d}u$$ A1

$$= \frac{1}{2}\int_0^{\theta} (\cosh 2u - 1)\, \mathrm{d}u$$ A1

$$= \frac{1}{2}\left[\frac{1}{2}\sinh 2u - u\right]_0^{\theta}$$ A1

= 3.83 A1

3. (a) Putting $f(x) = x^x$ and taking logs,

$\ln f(x) = x \ln x$ M1

$\dfrac{f'(x)}{f(x)} = 1 + \ln x$ A1

$f'(x) = x^x(1 + \ln x)$

(b) (i) The Newton-Raphson formula is

$x_{n+1} = x_n - \dfrac{(x_n^{\,x_n} - 2)}{x_n^{\,x_n}(1 + \ln x_n)}$ M1A1

Starting with 1.5, one application of the formula gives 1.56 A1

(ii) $f(1.555) - 2 = -0.01326...$; $f(1.565) - 2 = .01564...$

The change of sign indicates that α = 1.56 correct to 2 dps.

M1A1

(c) (i) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\frac{\ln 2}{x}}\right) = -\dfrac{\ln 2}{x^2}\mathrm{e}^{\frac{\ln 2}{x}}$ B1

Putting x = 1.5 gives – 0.489 (1.56 gives – 0.444) B1

The iteration converges because this is less than 1 in modulus.

B1

(iii) Successive values are

1.56

1.559437401 M1

1.559687398

1.559576282

1.559625664

The required value is 1.5596. A1

[Award A1 only if calculations done to reasonable accuracy]

4. $2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2}{2y} \quad (\frac{3}{2}\sqrt{x})$ B1

Arc length = $\int_0^1 \sqrt{1 + \dfrac{9x}{4}}\,\mathrm{d}x$ M1A1

$= \left[\dfrac{2}{3} \times \dfrac{4}{9}\left(1 + \dfrac{9x}{4}\right)^{3/2}\right]_0^1$ M1A1

$= \dfrac{8}{27}\left(\left(\dfrac{13}{4}\right)^{3/2} - 1\right)$ A1

= 1.44 A1

5. (a) (i) $f(0) = 0$ B1

$$f'(x) = \frac{\cosh x}{1+\sinh x};\ f'(0) = 1$$ B1B1

$$f''(x) = \frac{\sinh x(1+\sinh x) - \cosh^2 x}{(1+\sinh x)^2};f''(0) = -1$$ B1B1

$$f'''(x) = \frac{\cosh x(1+\sinh x)^2 - 2(1+\sinh x)\cosh x(\sinh x - 1)}{(1+\sinh x)^4};\ f'''(0) = 3$$ B1B1

The Maclaurin series is

$$0 + x - \frac{x^2}{2} + \frac{3x^3}{6} + ... = x - \frac{x^2}{2} + \frac{x^3}{2} + ...$$ M1A1

(ii) Both even and odd powers of x are present or equivalent argument based on series. B1

(b) $$x - \frac{x^2}{2} + \frac{x^3}{2} = 10x^2$$ M1

$x^2 - 21x + 2 = 0$ A1

$x = 0.096$ A1

[FT from (a)]

6. (a) Consider

$x = r\cos\theta = \cos^2\theta + 2\sin\theta\cos\theta$ M1

$$\frac{dx}{d\theta} = -2\cos\theta\sin\theta + 2\cos^2\theta - 2\sin^2\theta$$ A1

At the required point, $\frac{dx}{d\theta} = 0$, giving M1

$\sin 2\theta = 2\cos 2\theta$ A1

$\tan 2\theta = 2$ A1

$\theta = \frac{1}{2}\tan^{-1} 2 = 0.55$ A1

$r = 1.9$ A1

[Treat consideration of $y = r\sin\theta$ as a special case and award B2B2 for (2.18,1.34)]

(b) Area = $\frac{1}{2}\int_0^{\pi/2}(\cos\theta + 2\sin\theta)^2 \mathrm{d}\theta$ M1

$= \frac{1}{2}\int_0^{\pi/2}(\cos^2\theta + 4\sin\theta\cos\theta + 4\sin^2\theta)\mathrm{d}\theta$ A1

$= \frac{1}{2}\int_0^{\pi/2}(\frac{1}{2} + \frac{1}{2}\cos 2\theta + 2\sin 2\theta + 2 - 2\cos 2\theta)\mathrm{d}\theta$ M1A1

$= \frac{1}{2}\left[\frac{5\theta}{2} - \cos 2\theta - \frac{3}{4}\sin 2\theta\right]_0^{\pi/2}$ A1

$= 2.96 \ \left(1 + \frac{5\pi}{8}\right)$ A1

7. (a) $I_n = \int_0^{\pi/2}\cos^{n-1}x\mathrm{d}(\sin x)$ M1

$= \left[\cos^{n-1}x\sin x\right]_0^{\pi/2} + \int_0^{\pi/2}\sin x.(n-1)\cos^{n-2}x\sin x\mathrm{d}x$ A1

$= (n-1)\int_0^{\pi/2}(1-\cos^2 x)\cos^{n-2}x\mathrm{d}x$ M1A1

$= (n-1)I_{n-2} - (n-1)I_n$ A1

$I_n = \left(\frac{n-1}{n}\right)I_{n-2}$

(b) (i) $I_4 = \frac{3}{4}I_2 = \frac{3}{4}\times\frac{1}{2}I_0$ M1

$= \frac{3}{8}\int_0^{\pi/2} \mathrm{d}x$ A1

$= \frac{3\pi}{16}$ (0.589) A1

(ii) Integral = $\int_0^{\pi/2}\cos^5 x(1-\cos^2 x)\mathrm{d}x = I_5 - I_7$ M1A1

$= \left(\frac{4}{5}\times\frac{2}{3} - \frac{6}{7}\times\frac{4}{5}\times\frac{2}{3}\right)\int_0^{\pi/2}\cos x\mathrm{d}x$ M1

$= \left(\frac{4}{5}\times\frac{2}{3} - \frac{6}{7}\times\frac{4}{5}\times\frac{2}{3}\right)\times 1$ A1

$= \frac{8}{105}$ A1

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2010 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics M1

Notes: cao = correct answer only, oe = or equivalent, si = seen or implied,
ft = follow through
(c) = candidate's value acceptable

1. (a) Use of $v^2 = u^2 + 2as$ with $u = (\pm)2.1$, $a = (\pm)9.8$, $s = (\pm)15.4$ M1

$v^2 = 2.1^2 + 2 \times 9.8 \times 15.4$ A1

$v = \underline{17.5\ (\text{ms}^{-1})}$ cao A1

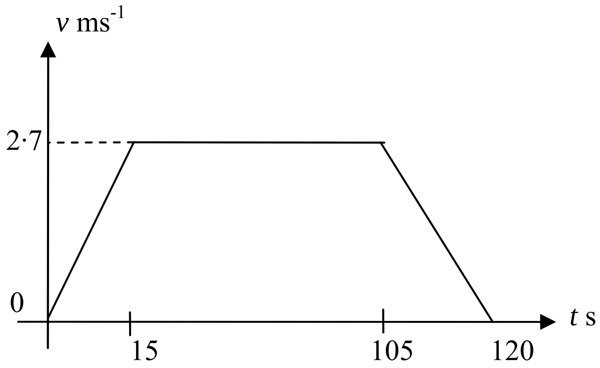
(b) Use of $v = u + at$ with $v = 17.5$(c), $a = (\pm)9.8$, $u = (\pm)2.1$ oe M1

$17.5 = 2.1 + 9.8t$ A1

$t = \dfrac{11}{7}$ cao A1

2.

(a)



attempt at v-t graph with one correct section and axes M1

second correct section A1

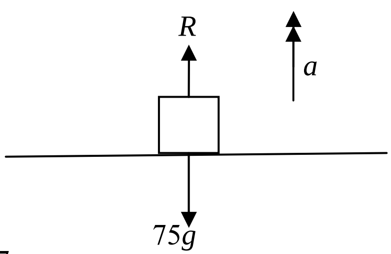
completely correct graph with labels A1

(b) Distance = 0.5(90 + 120) × 2.7 attempt to calculate total area M1

any correct value for an area B1

= <u>283.5 (m)</u> cao A1

(c)



$a = \dfrac{2.7}{15} = (0 \cdot 18)$ B1

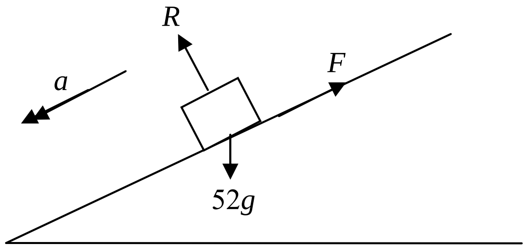
Apply N2L to woman $R - 75g = 75a$ all forces, dim correct M1

correct equation A1

$R = 75(9.8 + 0.18)$

$= \underline{748.5\ (\text{N})}$ ft a A1

3.



$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

Resolve perpendicular to plane — M1

$R = 52g\cos\alpha$

Use of $F = \mu R$ — m1

$= 0.2 \times 52 \times 9.8 \times \frac{12}{13}$ — si A1

$= \underline{94.08\ (N)}$

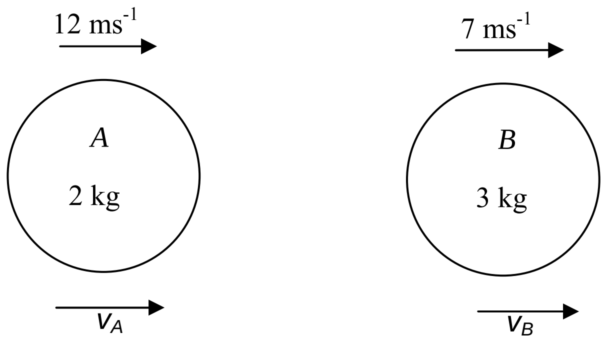
Apply N2L to object down slope — Dim correct, all forces M1

$52g\sin\alpha - F = 52a$ — A1

$52 \times 9.8 \times \frac{5}{13} - 94.08 = 52a$

$a = \underline{1.96\ (ms^{-2})}$ — cao A1

4.



(a) attempt at conservation of momentum equation — M1

$2 \times 12 + 3 \times 7 = 2v_A + 3v_B$ — A1

$2v_A + 3v_B = 45$

attempt at restitution equation — M1

$v_B - v_A = -0.6(7 - 12)$ — A1

$-3v_A + 3v_B = 9$

attempt to solve simultaneously dep. — Both M's m1

$5\,v_A = 36$

$v_A = \underline{7.2\ (ms^{-1})}$ — cao A1

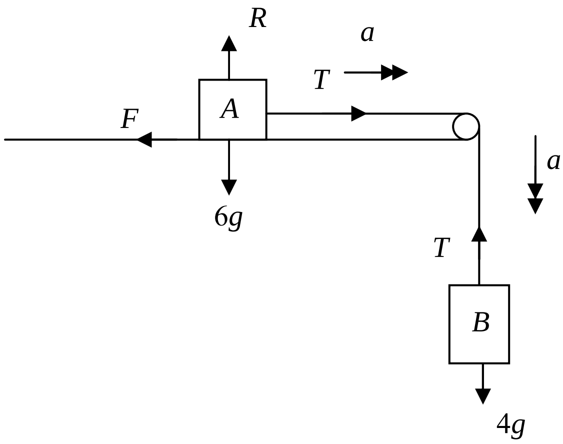
$v_B = \underline{10.2\ (ms^{-1})}$ — cao A1

(b) Use of Impulse = change in momentum — M1

$I = 3(10.2 - 7)$

$= \underline{9.6\ (Ns)}$ — ft sensible results only A1

5.



(a) Apply N2L to *B/A* — M1

$4g - T = 4a$ — A1

Apply N2l to other particle — M1

$T - F = 6a$ — A1

Resolve vertically, particle *A*

$R = 6g$ — si B1

$F = \mu R = 0.4 \times 6\text{g} = 2.4g$ — B1

attempt to solve equations simultaneously — m1

$4g - 2.4g = 10a$

$a = \underline{0.16g} = \underline{1.568\ (\text{ms}^{-2})}$ — cao A1

$T = \underline{32.928\ (\text{N})}$ — cao A1

(b) Light strings enable the assumption that tension is constant throughout the string to be used. — B1

6. Attempt to resolve in direction of 12 N force — M1

$Y = 12 - 5\sqrt{3}\sin 60° - 3\sqrt{2}\sin 45°$ — A1

$Y = 1.5$

Attempt to resolve in perpendicular direction — M1

$X = 5\sqrt{3}\cos 60° - 3\sqrt{2}\cos 45°$ — A1

$X = 1.33$

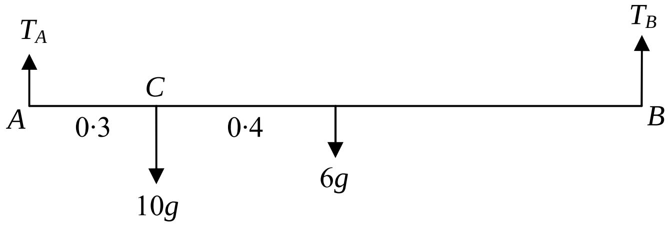
Resultant $R = \sqrt{(1.5)^2 + 1.33^2}$ — M1

$= \underline{2.(0048)\ (\text{N})}$ — ft A1

$\theta = \tan^{-1}\left(\frac{1.33}{1.50}\right) = 41.6°$ — M1

Dir of *R* is 41.6° to the right with the 12 N force — ft A1

7.



Moments about A	dim. correct equation, all forces M1
	any correct moment B1
$1.4\, T_B = 0.7 \times 6g + 0.3 \times 10g$	A1
$T_B = \underline{50.4\text{ (N)}}$	cao A1
Resolve vertically dim correct, all forces	oe M1
$T_A + T_B = 16g$	A1
$T_A = \underline{106.4\text{ (N)}}$	ft T_B A1

8.

Use of $s = ut + 0.5at^2$ with $s = 95$, $t = 5$	M1
$95 = 5u + 0.5 \times a \times 25$	A1
Use of $v = u + at$ with $t = 7$. $v = 29.8$	M1
$29.8 = u + 7a$	A1
attempt to solve simultaneously	m1
$10.8 = 4.5a$	
$a = \underline{2.4}$	cao A1
$u = \underline{13}$	cao A1

9. (a)

Lamina	Area	from AD	from AB
$ABCD$	80	4	5
XYZ	9	3	3
Decoration	89	x	y

one correct pair of distances B1
all four correct B1
correct areas B1

Moments about AD	M1
$89x = 80 \times 4 + 9 \times 3$	ft A1
$x = \underline{3.90\text{ (cm)}}$	cao A1
Moments about AB	M1
$89y = 80 \times 5 + 9 \times 3$	ft A1
$y = \underline{4.80\text{ (cm)}}$	cao A1

(b)

$\theta = \tan^{-1}\left(\dfrac{x}{10 - y}\right)$	correct triangle M1
$= \tan^{-1}\left(\dfrac{3.9}{10 - 4.8}\right)$	ft A1
$= \underline{36.9°}$	

Mathematics M2

1. (a) Attempt to integrate a $v = \int 3 - 4t \, dt$ M1

$v = 3t - 2t^2$ (+C) A1

When $t = 0$, $v = -1$ use of initial conditions m1

Therefore C = -1 A1

$v = \underline{-2t^2 + 3t - 1}$

(b) When P is at rest, $v = 0$ M1

$-2t^2 + 3t - 1 = 0$

$2t^2 - 3t + 1 = 0$

$(2t - 1)(t - 1) = 0$

$t = \underline{0.5, 1}$ ft C ≠ 0 A1

(c) Attempt to integrate v M1

Distance required $= \int_{\frac{1}{2}}^{1} -2t^2 + 3t - 1 \, dt$ limits oe m1

$= \left[-\frac{2}{3}t^3 + \frac{3}{2}t^2 - t \right]_{\frac{1}{2}}^{1}$ ft v A1

$= \frac{1}{24}$ cao A1

2. (a) $\mathbf{v} = \frac{dr}{dt}$ used M1

$\mathbf{v} = 6t\,\mathbf{i} + (13 - 4t)\,\mathbf{j}$ A1

Speed = $\sqrt{(6t)^2 + (13 - 4t)^2}$ M1

When $t = 2$, speed $= \sqrt{144 + 25} = \underline{13}$ ft A1

(b) velocity is perpendicular to (2**i** – **j**) when $\boldsymbol{v}.(2\mathbf{i} - \mathbf{j}) = 0$ M1

$12t - 13 + 4t = 0$ method for dot product M1

$t = \frac{13}{16}$ cao A1

(c) Acceleration of P = $\mathbf{a} = \frac{dv}{dt}$ used M1

$\mathbf{a} = 6\mathbf{i} - 4\mathbf{j}$ independent of t ft v A1

Magnitude $= \sqrt{36 + 16} = \sqrt{52}$ ft A1

(d) Let θ be the required angle.

Use of $\mathbf{a}.\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta$ with $\mathbf{b} = \boldsymbol{v}$ when $t = 2$ M1

$(6i - 4j).(12i + 5j) = \sqrt{52} \times 13\cos\theta$ ft A1

$72 - 20 = \sqrt{52} \times 13\cos\theta$

$\theta = \underline{56.3°}$ cao A1

3. (a) Use of Hooke's Law $T = \frac{\lambda x}{l}$ M1

$3 \times 9.8 = \frac{294x}{2}$ A1

$x = \underline{0.2\ (\text{m})}$ cao A1

(b) Use of loss in potential energy $= mgh$ M1

Loss in PE $= 3 \times 9.8 \times (0.8 + 0.2)$ si ft x A1

$= 29.4$ (J)

Gain in KE $= 0.5 \times 3v^2 = 1.5\,v^2$ (J) B1

Use of gain in elastic energy $= \frac{1}{2} \times \frac{\lambda \times x^2}{l}$ 3 energies M1

Gain in EE $= \frac{1}{2} \times \frac{294 \times 0.2^2}{2}$ ft x A1

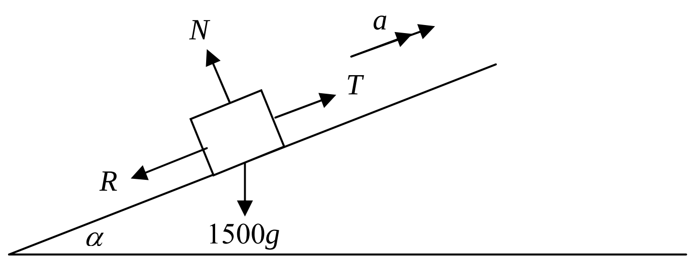
$= 2.94$ (J)

Use of conservation of energy M1

$29.4 = 1.5\,v^2 + 2.94$ ft A1

$v = \underline{4.2\ (\text{ms}^{-1})}$ cao A1

4. (a)



Use of $T = \frac{P}{v}$ M1

$T = \frac{30 \times 1000}{8}$ (= 3750 N) si A1

N2L up slope all forces, dim correct equation M1

$T - R - 1500g\sin\alpha = 1500a$ -1 each error A2

$3750 - 600 - 1500 \times 9.8 \times \frac{6}{49} = 1500a$

$a = \underline{0.9\ (\text{ms}^{-2})}$ cao A1

(b) At maximum attainable speed, $a = 0$ used M1

Apply N2L to particle up the slope

M1

$T = R + mg\sin\alpha$ A1

$\frac{30000}{v} = 600 + 1500 \times 9.8 \times \frac{6}{49}$

$v = \underline{12.5\ (\text{ms}^{-1})}$ cao A1

5. (a) Initial vertical speed $= V\sin30° = (0.5V)$ B1

Use of $v^2 = u^2 + 2as$ with $u = V\sin30°$, $v = 0$, $a = (\pm)9.8$, $s = (\pm)4.9$ M1

$0 = 0.25V^2 + 2(-9.8)(4.9)$ A1

$V = \underline{19.6}$ A1

(b) Use of $s = ut + 0.5at^2$ with $s = (\pm)39.2$, $a = (\pm)9.8$, $u = (\pm)0.5\times19.6$(c)

M1

$-39.2 = 9.8t - 4.9t^2$ A1

$t^2 - 2t - 8 = 0$ attempt to solve quadratic m1

$t = \underline{4\text{s}}$ A1

(c) initial horizontal velocity $= 19.6\cos30° = (16.97)$ B1

Use of $v = u + at$ with $u = 9.8$(c), $a = (\pm)9.8$, $t = 3$ M1

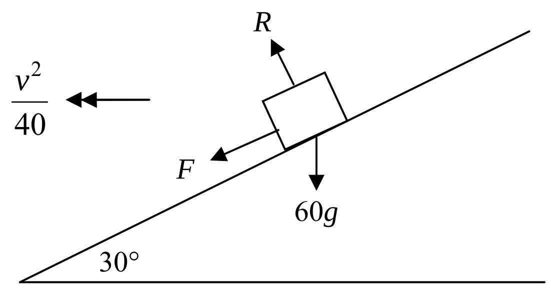
$v = 9.8 - 9.8 \times 3$ A1

$v = -19.6$

Speed $= \sqrt{16\cdot97^2 + 19\cdot6^2}$ M1

$= \underline{25.9\ (\text{ms}^{-1})}$ A1

6.



Let the maximum speed be v ms^{-1}.

On the point of slipping, friction is limiting and $F = \mu R = 0.25R$ M1

Resolving vertically dim correct equation M1

$R\cos30° = mg + F\sin30°$ A1

$\frac{R\sqrt{3}}{2} - \frac{R}{8} = 60g$

$R(4\sqrt{3} - 1) = 480g$

$R = 793.495$ (N)

Apply N2L horizontally all forces, dim correct M1

$R\sin 30° + F\cos30° = ma$ A1

$a = \frac{v^2}{40}$ M1

$\frac{R}{2} + \frac{1}{4}R\frac{\sqrt{3}}{2} = \frac{3}{2}v^2$

$1.5v^2 = 568.54$

$v = \underline{19.5\ (\text{ms}^{-1})}$ cao A1

7. (a) Use of conservation of energy — M1

$0.5m \times 13^2 = 0.5mv^2 + mg(2.5)(1 - \cos\theta)$ — KE A1, PE A1

$v^2 = 169 - 2 \times 9.8 \times 2.5(1 - \cos\theta)$

$v^2 = 120 + 49\cos\theta$

$v = \sqrt{(120 + 49\cos\theta)}$ — cao A1

When $\cos\theta = 0.5$ $\quad v^2 = 120 + 24.5$

$= 144.5$

$v = 12(.02)$ — ft A1

(b) Apply N2L towards centre — all forces, dim correct M1

$T - mg\cos\theta = \frac{mv^2}{r}$ — A1

$T = 3 \times 9.8\cos\theta + \frac{3 \times (120 + 49\cos\theta)}{2.5}$ — substitution m1

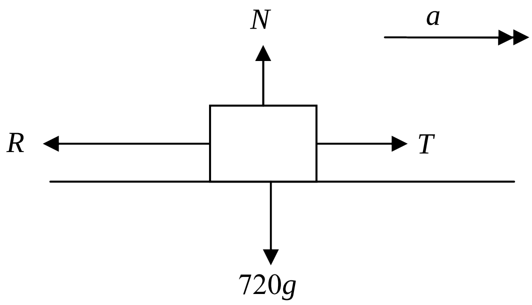
$T = 144 + 88.2\cos\theta$ — cao A1

(c) Minimum value of $\cos\theta$ is -1. Therefore $T > 0$ for all values of θ. — M1

Therefore P describes complete circles. — A1

Mathematics M3

1.



(a) Use of $T = \dfrac{P}{v}$ M1

$T = \dfrac{81 \times 1000}{v}$ si A1

Apply N2L to car dim correct equation M1

$T - R = ma$

$\dfrac{81000}{v} - 90v = 720\dfrac{\mathrm{d}v}{\mathrm{d}t}$ A1

Divide by 90 and multiply by v throughout

$900 - v^2 = 8v\dfrac{\mathrm{d}v}{\mathrm{d}t}$ A1

(b) Attempt to separate variables M1

$\int \dfrac{8v}{900 - v^2}\,\mathrm{d}v = \int \mathrm{d}t$ A1

Integrating

$-4 \ln|900 - v^2| = t + C$ correct ln term A1

all correct A1

$t = -4 \ln |900 - v^2| - C$

Required time $= \left[-4\ln|900 - v^2|\right]_5^{20}$ subtraction of t values M1

correct limits oe A1

$= 4\left[\ln\left(\dfrac{900 - 25}{900 - 400}\right)\right]$

$= 4\left[\ln\left(\dfrac{875}{500}\right)\right]$

$= 4\ln(1.75)$

$= \underline{2.24\text{ (s)}}$ cao A1

2. (a) At equilibrium $12g = \dfrac{\lambda \times 0 \cdot 05}{0 \cdot 75}$ use of Hook's Law M1

$\lambda = \underline{1764\ (\text{N})}$ A1

(b) Consider a displacement x from the equilibrium position.

Apply N2L $12g - T = 12\,x$ M1

$12g - \dfrac{\lambda(0 \cdot 05 + x)}{0 \cdot 75} = 12\ x$ ft λ A1

$x = -(14)^2\,x$

Therefore is SHM (with $\omega = 14$). A1

Amplitude = $\underline{0.05\ (\text{m})}$ B1

Period = $\dfrac{2\pi}{\omega} = \dfrac{\pi}{7}$ s B1

(c) Maximum speed = $a\omega$ used M1

$= 0.05 \times 14$

$= \underline{0.7\ (\text{ms}^{-1})}$ ft a A1

(d) Use of $v^2 = \omega^2(a^2 - x^2)$ with $\omega = 14$, $a = 0.05$(c), $x = 0.03$ M1

$v^2 = 14^2(0.05^2 - 0.03^2)$ ft a A1

$= 14^2 \times 0.04^2$

$v = \underline{0.56\ (\text{ms}^{-1})}$ cao A1

(e) Displacement from Origin = x

$x = (-)0.05\cos(14t)$ M1

When $t = 1.6$

$x = (-)\ 0.05\ \cos(14 \times 1.6)$ ft a A1

$x = (-)\underline{0.046\ (\text{m})}$ cao A1

3. Auxiliary equation $4m^2 - 12m + 9 = 0$ B1

$(2m - 3)^2 = 0$

$m = 1.5$ (twice) B1

Complementary function $x = (A + Bt)e^{1.5t}$ ft B1

For PI, try $x = at + b$, $\frac{dx}{dt} = a$ M1

$-12a + 9(at + b) = 18t - 87$ A1

$9a = 18$ comparing coefficients m1

$a = 2$

$-24 + 9b = -87$

$b = -7$ both A1

General solution $x = (A + Bt)e^{1.5t} + 2t - 7$ ft B1

Use of initial conditions $t = 0$, $x = 5$, $\frac{dx}{dt} = 10$ in general solution M1

$A - 7 = 5$

$A = 12$ cao A1

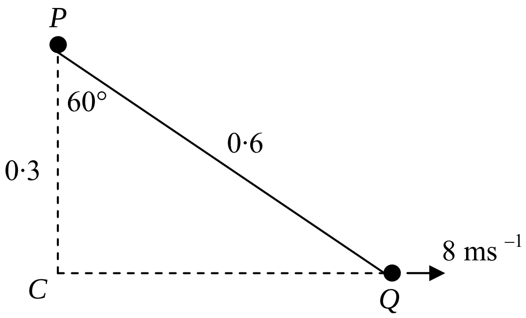
$\frac{dx}{dt} = (A + Bt)(1.5)e^{1.5t} + Be^{1.5t} + 2$ correct diff. ft B1

$1.5A + B + 2 = 10$

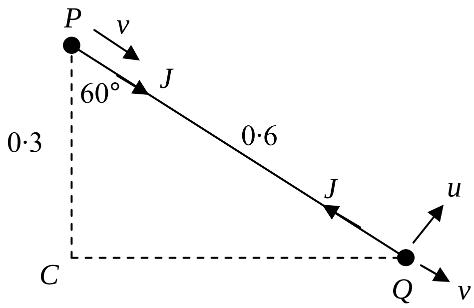
$B = -10$ cao A1

$\underline{x = (12 - 10t)e^{1.5t} + 2t - 7}$

4. When the string jerks tight, each particle begins to move in direction PQ with equal speeds v.



Just before jerk



Just after jerk

$$\cos \angle CPQ = \frac{1}{2}$$

$$\sin \angle CPQ = \frac{\sqrt{3}}{2}$$ si B1

Use of impulse = change in momentum M1

Applied to P $J = 3v$ B1

Applied to Q $J = 5 \times 8\sin 60° - 5v$ A1

Attempt to solve simultaneously m1

$$3v = 40 \times \frac{\sqrt{3}}{2} - 5v$$

$$v = \frac{5\sqrt{3}}{2} = \underline{4.33 \text{ (ms}^{-1})}$$ cao A1

Speed of particle P is 4.33 ms^{-1}.

Magnitude of impulsive tension $= J = 3v$

$$= \frac{15\sqrt{3}}{2} = \underline{12.99 \text{ (Ns)}}$$ cao A1

units B1

Perpendicular to PQ, there is no impulse

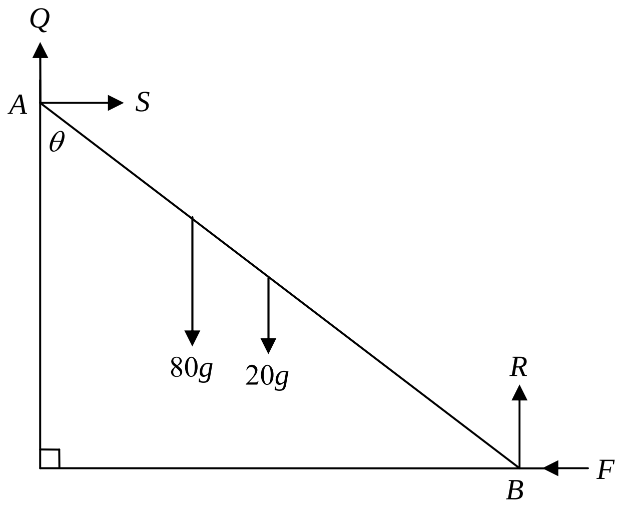
Speed of particle Q perpendicular to PQ = $8\cos 60° = 4$ ms^{-1} B1

Speed of particle Q = $\sqrt{4^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$ M1

$= \underline{5.89 \text{ ms}^{-1}}$ cao A1

5.	(a)	Use of N2L	M1
		$150g - 10v^2 = 150a$	A1
		$15g - v^2 = 15v\dfrac{dv}{ds}$	A1
	(b)	Attempt to separate variables	M1
		$\displaystyle\int \frac{15v\,dv}{v^2 - 15g} = -\int ds$	A1
		$\dfrac{15}{2}\ln\lvert v^2 - 15g\rvert = -s\,(+C)$	correct ln A1
			all correct A1
		Use of boundary conditions $s = 0$, $v = 30$	m1
		$\dfrac{15}{2}\ln\lvert 900 - 15g\rvert = C$	
		$s = \dfrac{15}{2}\ln\left\lvert\dfrac{753}{v^2 - 15g}\right\rvert$	cao A1
	(c)	$v = 14$	used M1
		$s = \dfrac{15}{2}\ln\left(\dfrac{753}{14^2 - 15\times 9\cdot 8}\right)$	
		$s = \underline{20.49}$	cao A1
	(d)	Removing ln	M1
		$\exp\left(\dfrac{2}{15}s\right) = \dfrac{753}{v^2 - 15g}$	ft A1
		$v^2 - 15g = 753\exp\left(-\dfrac{2}{15}s\right)$	
		$v^2 = 15g + 753\exp\left(-\dfrac{2}{15}s\right)$	cao A1
		$v^2 = 147 + 753\exp\left(-\dfrac{2}{15}s\right)$	

6.



(a) Use of Friction = μ × Normal reaction — si M1

$Q = 0.3\,S$ — A1

Attempt at taking mom. about B 4 terms, dim correct equation M1

$20g \times 2.5 \sin\theta + 80g \times 3\sin\theta = 4S + 3Q$ — -1 each error A2

$294 + 1411.2 = 4S + 0.9S$

$4.9S = 1705.2$

$S = \underline{348 \text{ (N)}}$ — cao A1

(b) Resolve vertically — 4 terms, dim correct M1

$Q + R = 80g + 20g$ — A1

$R = 100g - 0.3 \times 348$

$R = 875.6$ N

Resolve horizontally

$F = S \ (= 348)$ — B1

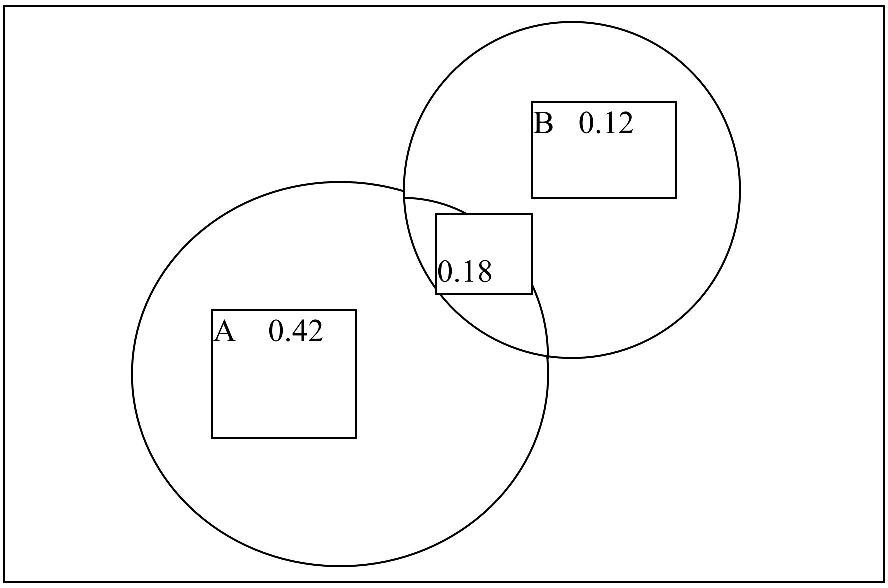
Use of $F \leq \mu R$ — M1

$$\mu \geq \frac{348}{875 \cdot 6} = 0 \cdot 39744$$

$\mu \geq \underline{0.397}$ — cao A1

Mathematics S1

1.	(a)	$P(A \cap B) = 0.6 \times 0.3$	B1
		$P(A \cup B) = 0.6 + 0.3 - 0.6 \times 0.3$	M1
		$= 0.72$	A1
	(b)	EITHER	
		$P(B') = 1 - P(B) = 0.7$	B1
		$P(A \cup B') = P(A) + P(B') - P(A)P(B')$	M1
		$= 0.6 + 0.7 - 0.6 \times 0.7 = 0.88$	A1
	OR		



		Correct Venn diagram	B1
		Required prob = 1 – 0.12 = 0.88	M1A1
2.	(a)	$E(Y) = 3 \times 4 - 1 = 11$	M1A1
		$Var(Y) = 9 \times 2 = 18$	M1A1
	(b)	$E(Y^2) = Var(Y) + \{E(Y)\}^2$	M1
		$= 18 + 121 = 139$	A1
		[FT 1 arithmetic slip in (a)]	
3.	(a)	Mean = 6	si B1
		(i) Prob $= e^{-6} \times \frac{6^3}{3!} = 0.0892$	M1A1
		(ii) Prob = 1 – 0.7149 = 0.2851 [FT on mean]	M1A1
	(b)	Prob of no arrivals $= e^{-0.1t}$	B1
		Attempting to solve $e^{-0.1t} = 0.25$	M1
		$-0.1t \log e = \log 0.25$	A1
		$t = -\frac{\log 0.25}{0.1 \log e} = 13.86$	A1
		[Award 2 marks for 14 using tables]	

4.	(a)	Prob wins on 1st throw = 0.8 × 0.3 = 0.24	M1A1
	(b)	Prob wins on 2nd throw = 0.8 × 0.7 × 0.8 × 0.3 = 0.1344 [FT from (a) if M1 awarded in (a)]	M1A1
	(c)	Prob wins = $0.24 + 0.24 \times 0.56 + 0.24 \times 0.56^2 + \ldots$	M1A1
		$= \dfrac{0.24}{1-0.56} = 6/11$ (0.55)	M1A1
		[For candidates who solve for Bill first, award M0A0 for (a), M1A1 for 0.168 in (b), M1A1 for 0.3 + 0.3 × 0.56 + ... and M1A1 for 0.3/(1 – 0.56) = 15/22 (0.68) in (c)]	
5.	(a)	P(correct ans) = $0.6 \times 1 + 0.4 \times 0.25$	M1A1
		$= 0.7$	A1
		[Award M1 if 1 and 0.25 reversed]	
	(b)	Reqd prob = $\dfrac{0.6}{0.7}$ [FT denominator from (a) if answer < 1]	B1B1
		$= \dfrac{6}{7}$ cao	B1
6.	(a)	$\sum p_x = 16k = 1$ so $k = 1/16$	M1A1
	(b)	(i) $E(X) = \dfrac{1}{16}(1\times1 + 3\times3 + 5\times5 + 7\times7)$	M1
		$= 5.25$ [Accept $84k$]	A1
		(ii) $\mathrm{E}\left(\dfrac{1}{X}\right) = \dfrac{1}{16}\left(1\times\dfrac{1}{1} + 3\times\dfrac{1}{3} + 5\times\dfrac{1}{5} + 7\times\dfrac{1}{7}\right)$	M1A1
		$= 0.25$ [Accept $4k$]	A1
	(c)	(i) Possibilities are 1,5 and 3,3	si B1
		Prob = $\dfrac{1}{256}(1\times5 + 5\times1 + 3\times3)$	M1A1
		[Award M1 for 2 or 3 terms]	
		$= \dfrac{19}{256}$ (0.074) [Accept $19/k^2$]	A1
		(ii) Possibilities are 1,1 ; 3,3 ; 5,5 ; 7,7	si B1
		Prob = $\dfrac{1}{256}(1^2 + 3^2 + 5^2 + 7^2)$	M1
		[Award M1 for 3 or 4 terms]	
		= 0.328 (21/64) [Accept $84/k^2$]	A1

7.	(a)	Number of 6s obtained, X, is B(50,0.2)	B1
		(i) Prob = $\binom{50}{12} \times 0.2^{12} \times 0.8^{38} = 0.1033$	
		or 0.8139 – 0.7107 or 0.2893 – 0.1861 = 0.1032	M1A1
		(ii) P(at least 10) = 0.5563 or 1 – 0.4437 = 0.5563	M1A1
	(b)	Prob of 2 6s = $0.2^2 = 0.04$	B1
		X is now B(200, 0.04) which is approx P(8)	B1
		[FT p from previous line]	
		$P(5 \le X \le 10) = 0.8159 - 0.0996$ or $0.9004 - 0.1841$	B1B1
		= 0.7163 cao	B1
8.	(a)	$\int_0^1 kx(1-x^2)\mathrm{d}x = 1$	M1
		Integral = $k\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$	B1
		[Limits must appear somewhere for M1]	
		$= \frac{k}{4}$	A1
		so $k = 4$	
	(b)	$E(X) = \int_0^1 x.4x(1-x^2)\mathrm{d}x$	M1A1
		$= \left[\frac{4x^3}{3} - \frac{4x^5}{5}\right]_0^1$	A1
		[Limits not required until 2nd line]	
		$= \frac{8}{15}$	A1

(c) (i) $F(x) = \int_0^x 4t(1-t^2)\mathrm{d}t$ M1

[Limits not required for M1]

$= \left[2t^2 - t^4\right]_0^x$ A1

[Limits must appear here]

$= 2x^2 - x^4$ A1

(ii) Prob = $F(0.75) - F(0.25)$

$= 2\times 0.75^2 - 0.75^4 - (2\times 0.25^2 - 0.25^4)$ M1

= 0.6875 A1

[FT if M1 awarded in (c)(i) and answer sensible]

(iii) The median m satisfies

$2m^2 - m^4 = 0.5$ M1

[FT from (c)(i) if M1 awarded there]

$2m^4 - 4m^2 + 1 = 0$ A1

$m^2 = \dfrac{4 \pm \sqrt{8}}{4}$ m1

$m = 0.541$ cao A1

Mathematics S2

1.	(a)	$z = \frac{120-106}{8} = 1.75$ (Accept ±)	M1A1
		Prob = 0.0401	A1
	(b)	Distribution of total weight T is N(1060, 640)	M1A1A1
		$z = \frac{1000-1060}{\sqrt{640}} = -2.37$	M1A1
		$P(T < 1000) = 0.0089$	A1
		[No FT on incorrect variance]	
2.	(a)	Under H_0, mean = 15	si B1
		$P(X \le 9) = 0.0699$ (Accept 0.0778 from Normal app)	B1
		$P(X \ge 22) = 0.0531$ (Accept 0.0465 from Normal app)	B1
		Sig level = 0.0699 + 0.0531 = 0.123	M1A1
		[FT one slip but treat Normal apps as incorrect here]	
	(b)	X is now Po(150) which is approx N(150,150)	B1
		[FT their mean]	
		$z = \frac{169.5-150}{\sqrt{150}}$	M1A1
		[Award M1A0 for incorrect continuity correction]	
		= 1.59	A1
		Prob from tables = 0.0559	A1
		p-value = 0.1118	B1
		Insufficient evidence to reject H_0 (Accept 'Accept H_0').	B1
		[No c/c gives z = 1.63, prob = 0.0516 and p-value = 0.1032	
		Incorrect c/c gives z = 1.67. prob = .0475 and pv = 0.095]	

3. (a) $\bar{x} = \dfrac{11.5 + 11.7 + 11.6}{3}$ (= 11.6) B1

SE of $\bar{X} = \dfrac{0.2}{\sqrt{3}}$ (= 0.115…) B1

95% conf limits are

$11.6 \pm 1.96 \times 0.2/\sqrt{3}$ M1A1

[M1 correct form, A1 1.96]

giving [11.4,11.8] cao A1

(b) $H_0 : \mu = 12; H_1 : \mu > 12$ B1

$\bar{y} = \dfrac{12.1 + 12.2 + 12.4 + 12.1}{4}$ (=12.2) B1

Test stat = $\dfrac{12.2 - 12}{\sqrt{0.2^2 / 4}}$ A1

[Award M1 only if there is division by 4 in the denominator]

[FT on slip in calculating $\bar{y}$]

= 2.0 1

p-value = 0.0228 A1

Strong evidence for thinking that μ exceeds 12. B1

(c) SE of $\bar{y} - \bar{x} = \sqrt{\dfrac{0.2^2}{3} + \dfrac{0.2^2}{4}}$ (= 0.152..) M1A1

90% confidence limits are

12.2 – 11.6 ± 1.645 × 0.152… m1A1

giving [0.35,0.85] cao A1

4. (a) E(X) = 3 ⇒ np = 3 B1

Using Var(X) = $E(X^2) - [E(X)]^2$ M1

$np(1 - p) = 11.1 - 9 = 2.1$ A1

Solving,

$1 - p = 0.7$ so $p = 0.3$ m1A1

$n = 10$ A1

(b) E(Y) = 6 B1

E(XY) = 3 × 6 = 18 B1

$E(Y^2) = 3.6 + 6^2 = 39.6$ M1A1

$E(X^2Y^2) = 11.1 \times 39.6 = 439.56$ M1A1

Var(XY) = 439.56 – 18^2 = 115.56 m1A1

5. (a) $A = 0.5 \times PQ\cos\theta . PQ\sin\theta = 8\sin\theta\cos\theta = 4\sin 2\theta$ B1

(b) $P(A \le 2) = P(4\sin 2\theta \le 2)$ M1

$= P(2\theta \le \sin^{-1}[1/2])$ A1

$= P(\theta \le \pi/12)$ [Accept 15°] A1

$= \dfrac{\pi/12}{\pi/4}$ M1

$= 1/3$ A1

(c) $f(\theta) = \dfrac{4}{\pi}$ [Only award if quoted or used in (c)] B1

$E(A) = \int_0^{\pi/4} \dfrac{4}{\pi} \times 4\sin 2\theta \, d\theta$ M1

$= \dfrac{8}{\pi}\left[-\cos 2\theta\right]_0^{\pi/4}$ A1

$= \dfrac{8}{\pi}$ A1

6. (a) $H_0 : p = 0.75$ versus $H_1 : p < 0.75$ B1

(b) (i) Under H_0, X (No germ) is B(50,0.75) B1

and Y (No not germ) is B(50,0.25) (si) B1

Sig level = $P(X < 30) \mid H_0)$ M1

$= P(Y > 20 \mid H_0) = 0.0063$ A1

[Award B1B0M1A0 if Normal approx used]

(ii) Required prob = $P(X \ge 30 \mid p = 0.5)$ M1

$= P(Y \le 20) = 0.1013$ A1

[Award M1A0 if Normal approx used]

(c) X is now B(200,0.75) which is approx N(150,37.5) B1B1

$z = \dfrac{140.5 - 150}{\sqrt{37.5}}$ M1

$= -1.55$ A1

[Award M1A0 for incorrect continuity correction]

p-value = 0.0606

A1

[No c/c gives p = 0.0516, incorrect c/c gives p = 0.0436]

Insufficient evidence to doubt the statement on the packet B1

Mathematics S3

1. (a) $\hat{p} = \frac{140}{250} = 0.56$ B1

(b) ESE = $\sqrt{\frac{0.56 \times 0.44}{250}}$ (= 0.031394..) si M1A1

99% confidence limits are

$0.56 \pm 2.576 \times 0.031394..$ M1A1

giving [0.48,0.64] A1

(c) No because 0.5 lies within the interval. B1

2. (a) $H_0 : \mu = 1; H_1 : \mu < 1$ B1

(b) $\bar{x} = \frac{99.6}{100} = 0.996$ B1

$s^2 = \frac{99.24}{99} - \frac{99.6^2}{99 \times 100}$ = 0.000387878… B1

[Accept division by 100 giving 0.000384]

Test stat = $\frac{0.996 - 1}{\sqrt{0.000387878/100}}$ [M0 if square root omitted] M1A1

= –2.03 (–2.04) A1

p-value = 0.021 A1

Strong evidence to reject H_0 (or accept H_1) [FT on p-value] B1

(c) We assume that the sample mean is normally distributed B1

3. (a) (i) $P(20p, 10p, 10p) = \frac{1}{6} \times \frac{3}{5} \times \frac{2}{4} \times 3 = \frac{3}{20}$ M1A1

$P(\text{20p,10p,5p}) = \frac{1}{6} \times \frac{3}{5} \times \frac{2}{4} \times 6 = \frac{6}{20}$ A1

$P(\text{20p,5p,5p}) = \frac{1}{6} \times \frac{2}{5} \times \frac{1}{4} \times 3 = \frac{1}{20}$ A1

$P(\text{10p,10p,10p}) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ A1

$P(\text{10p,10p,5p}) = \frac{3}{6} \times \frac{2}{5} \times \frac{2}{4} \times 3 = \frac{6}{20}$ A1

$P(\text{10p,5p,5p}) = \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times 3 = \frac{3}{20}$ A1

The sampling distribution of *T* is

t	20p	25p	30p	35p	40p
P(T = t)	3/20	6/20	2/20	6/20	3/20

M1A1

4. (a) $\Sigma x = 18\,;\Sigma x^2 = 312$ si B1

UE of μ = 1.5 B1

$$\text{UE of } \sigma^2 = \frac{312}{11} - \frac{18^2}{11\times 12}$$ M1

= 25.909… A1

(b) DF = 11 si B1

At the 95% confidence level, critical value = 2.201 B1

The 95% confidence limits are

$$1.5 \pm 2.201\sqrt{\frac{25.909..}{12}}$$ M1A1

[Only award M1 if *t*-distribution used and square root present]

[FT values from (a)]

giving [– 1.7,4.7] A1

(c) The claim is justified since all the possible means within the interval are less than 5 in modulus. [FT from confidence interval] B1

5. (a) $H_0 : \mu_x = \mu_y\,; H_1 : \mu_x \neq \mu_y$ B1

(b) $\bar{x} = 1.1013..;\bar{y} = 1.1506..$ B1B1

$$s_x^2 = \frac{92.4}{74} - \frac{82.6^2}{74\times 75} = 0.01932..\ \ (0.01906..)$$ B1

$$s_y^2 = \frac{102.2}{74} - \frac{86.3^2}{74\times 75} = 0.03915...\ \ (0.03863)$$ B1

[Accept division by 75]

$$\text{SE} = \sqrt{\frac{0.01932..}{75} + \frac{0.03915}{75}}\ \ (= 0.02792.., 0.02773)$$ M1A1

$$\text{Test stat} = \frac{1.1506 - 1.1013}{0.02792}$$ [M0 if no square root] M1

= 1.77 (accept 1.8 or 1.79) A1

Prob from tables = 0.0384 (0.0375, 0.0367) A1

p-value = 0.0768 (0.075, 0.734) [FT from line above] B1

(c) Insufficient evidence to state there is a difference in mean life-times. B1

6.	(a)	$E(X) = -\theta + 1 - 3\theta = 1 - 4\theta$	B1
		$Var(X) = \theta + 1 - 3\theta - (1-4\theta)^2$	M1
		$= \theta + 1 - 3\theta - 1 + 8\theta - 16\theta^2$	A1
		$= 2\theta(3 - 8\theta)$	
	(b)	$E(U) = \frac{1 - E(\bar{X})}{4}$ [M1A0 if E omitted]	M1
		$= \frac{1-(1-4\theta)}{4}$	A1
		$= \theta$	
		$Var(U) = \frac{Var(\bar{X})}{16}$	M1
		$= \frac{2\theta(3-8\theta)}{16n}$	A1
	(c)	N is $B(n,2\theta)$; $E(N) = 2n\theta$ si	B1
		$E(V) = \frac{2n\theta}{2n} = \theta$ [B0 if E omitted]	B1
		$Var(N) = 2n\theta(1 - 2\theta)$ si	B1
		$Var(V) = \frac{Var(N)}{4n^2}$	M1
		$= \frac{\theta(1-2\theta)}{2n}$	A1
	(d)	$Var(V) - Var(U) = \frac{1}{n}\left(\frac{\theta}{2} - \theta^2 - \frac{3\theta}{8} + \theta^2\right)$	M1
		$= \frac{\theta}{8n}$ (> 0)	A1
		[FT from previous results]	
		U is better because $Var(U) < Var V)$	B1
7.	(a)	$\sum x = 210, \sum x^2 = 9100, \sum y = 14.92, \sum xy = 554.4$	B2
		[B1 for 2 or 3 correct]	
		$S_{xy} = 554.4 - 210 \times 14.92 / 6 = 32.2$	B1
		$S_{xx} = 9100 - 210^2 / 6 = 1750$	B1
		$b = \frac{32.2}{1750} = 0.0184$	M1A1
		$a = \frac{14.92 - 210 \times 0.0184}{6}$	M1
		= 1.84	A1
		[Working must be seen for marks to be awarded]	
	(b)	SE of $a = 0.02\sqrt{\frac{9100}{6 \times 1750}}$ (= 0.0186..)	M1A1
		The 90% confidence interval for α is given by	
		1.84 ± 1.645 × 0.0186	M1A1
		(1.81, 1.87)	cao A1

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