WJEC
CBAC

GCE MARKING SCHEME

MATHEMATICS

AS/Advanced

SUMMER 2011

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MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

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INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2011 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

C1

1. (*a*) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -2$ (or equivalent) A1

(*b*) Use of gradient $L_1 \times$ gradient $AB = -1$ M1

A correct method for finding the equation of L_1 using candidate's gradient for L_1 M1

Equation of L_1: $y - (-1) = 1/2\,(x - 9)$ (or equivalent)

(f.t. candidate's gradient for AB) A1

Equation of L_1: $x - 2y - 11 = 0$ (or equivalent)

(f.t. one error if all three M's are awarded) A1

(*c*) (i) An attempt to solve equations of L_1 and L_2 simultaneously M1

$x = 3$, $y = -4$ (convincing.) A1

(ii) A correct method for finding the length of BC M1

$BC = \sqrt{45}$ (or equivalent) A1

(iii) A correct method for finding the coordinates of the mid-point of BC M1

Mid-point of BC has coordinates $(6, -2{\cdot}5)$ A1

(iv) Equation of AC: $x = 3$ B1

2. (*a*) **Either:**

$\dfrac{9(\sqrt{3}+1)+7(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ M1

Numerator: $9\sqrt{3}+9+7\sqrt{3}-7$ A1

Denominator: $3-1$ A1

$\dfrac{9}{\sqrt{3}-1}+\dfrac{7}{\sqrt{3}+1} = 8\sqrt{3}+1$ (c.a.o.) A1

Or:

$\dfrac{9}{\sqrt{3}-1} = \dfrac{9(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$, $\dfrac{7}{\sqrt{3}+1} = \dfrac{7(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

(at least one) M1

Numerators: $9\sqrt{3}+9$, $7\sqrt{3}-7$ (both correct) A1

Denominators: $3-1$ (both correct) A1

$\dfrac{9}{\sqrt{3}-1}+\dfrac{7}{\sqrt{3}+1} = 8\sqrt{3}+1$ (c.a.o.) A1

(*b*) $\dfrac{90}{\sqrt{3}} = 30\sqrt{3}$ B1

$\sqrt{6}\times\sqrt{8} = 4\sqrt{3}$ B1

$(2\sqrt{3})^3 = 24\sqrt{3}$ B1

$\dfrac{90}{\sqrt{3}} - \sqrt{6}\times\sqrt{8} - (2\sqrt{3})^3 = 2\sqrt{3}$ (c.a.o.) B1

3. y-coordinate at $P = -5$ B1

$\dfrac{dy}{dx} = 6x-9$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\dfrac{dy}{dx}$ m1

Use of candidate's numerical value for $\dfrac{dy}{dx}$ as gradient of tangent at P m1

Equation of tangent at P: $y-(-5) = 3(x-2)$ (or equivalent)

(f.t. only candidate's derived value for y-coordinate at P) A1

4. $a = -3$ B1

$b = 2$ B1

A negative quadratic graph M1

Maximum point (3, 2) (f.t. candidate's values for a, b) A1

5. (*a*) $x^2 + (4k + 3)x + 7 = x + k$ M1

$x^2 + (4k + 2)x + (7 - k) = 0$ A1

An attempt to apply $b^2 - 4ac$ to the candidate's quadratic M1

$b^2 - 4ac = (4k + 2)^2 - 4 \times 1 \times (7 - k)$

(f.t. candidate's quadratic) A1

Candidate's expression for $b^2 - 4ac > (\geq)\ 0$ m1

$4k^2 + 5k - 6 > 0$ (convincing) A1

(*b*) Finding critical values $k = -2$, $k = 0{\cdot}75$ B1

A statement (mathematical or otherwise) to the effect that

$k < -2$ or $0{\cdot}75 < k$ (or equivalent) (f.t. only $k = \pm 2$, $k = \pm 0{\cdot}75$) B2

Deduct 1 mark for each of the following errors

the use of $\leq$ rather than $<$

the use of the word 'and' instead of the word 'or'

6. (*a*) $y + \delta y = 7(x + \delta x)^2 - 5(x + \delta x) + 2$ B1

Subtracting y from above to find δy M1

$\delta y = 14x\delta x + 7(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x - 5$ (c.a.o.) A1

(*b*) Required derivative = $4 \times {}^{2}/_{5} \times x^{-3/5} - 9 \times (-1) \times x^{-2}$

(completely correct answer) B2

(one correct term) B1

7. (*a*) $(3 + 2x)^4 = 3^4 + 4 \times 3^3 \times (2x) + 6 \times 3^2 \times (2x)^2 + 4 \times 3 \times (2x)^3 + (2x)^4$

(all terms correct) B2

(three or four terms correct) B1

$(3 + 2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$

(all terms correct) B2

(three or four terms correct) B1

(–1 for incorrect further 'simplification')

(*b*) Coefficient of $x = {}^{n}C_1 \times \frac{1}{4}(x)$ B1

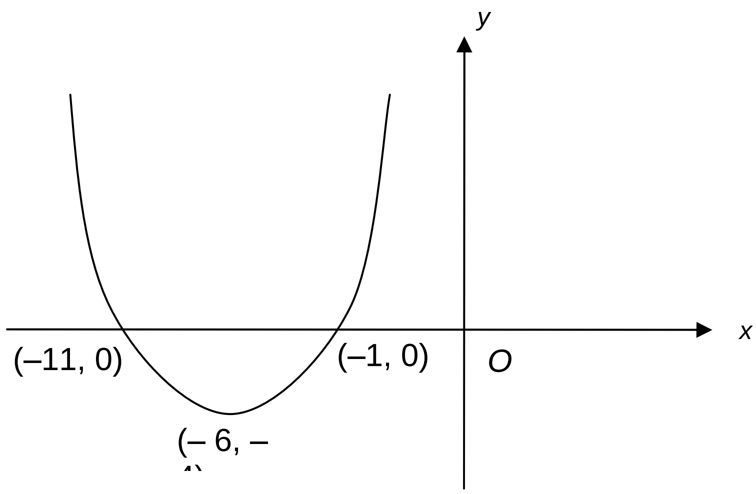
Coefficient of $x^2 = {}^{n}C_2 \times \frac{1}{4^2}(x^2)$ B1

$\frac{n(n-1)}{2} \times \frac{1}{4^m} = k \times n \times \frac{1}{4}$ (o.e.) (m = 1 or 2, k = 5 or ${}^{1}/_{5}$) M1

$n = 41$ (c.a.o.) A1

8.	(a)	Use of $f(-2) = 0$	M1
		$-8p - 4 + 62 + q = 0$	A1
		Use of $f(1) = -36$	M1
		$p - 1 - 31 + q = -36$	A1
		Solving candidate's simultaneous equations for p and q	M1
		$p = 6$, $q = -10$ (convincing)	A1
		Note: Candidates who assume $p = 6$, $q = -10$ and then verify that $x + 2$ is a factor and that dividing the polynomial by $x - 1$ gives a remainder of -36 may be awarded M1 A1 M1 A1 M0 A0	
	(b)	$f(x) = (x + 2)(6x^2 + ax + b)$ with one of a, b correct	M1
		$f(x) = (x + 2)(6x^2 - 13x - 5)$	A1
		$f(x) = (x + 2)(2x - 5)(3x + 1)$	A1
		(f.t. only for $f(x) = (x + 2)(2x + 5)(3x - 1)$ from $6x^2 + 13x - 5$)	

9. (a)



		Concave up curve and y-coordinate of minimum $= -4$	B1
		x-coordinate of minimum $= -6$	B1
		Both points of intersection with x-axis	B1
	(b)	$y = -\frac{1}{2} f(x)$	B2
		If B2 not awarded	
		$y = rf(x)$ with r negative	B1

10. (*a*) $V = x(8 - 2x)(5 - 2x)$ M1

$V = 4x^3 - 26x^2 + 40x$ (convincing) A1

(*b*) $\frac{dV}{dx} = 12x^2 - 52x + 40$ B1

Putting derived $\frac{dV}{dx} = 0$ M1

$x = 1, (10/3)$ (f.t. candidate's $\frac{dV}{dx}$) A1

Stationary value of V at $x = 1$ is 18 (c.a.o) A1

A correct method for finding nature of the stationary point yielding a maximum value (for $0 < x < 2{\cdot}5$) B1

C2

1.

1·6	0·203915171		
1·7	0·244678248		
1·8	0·315656565		
1·9	0·467071461	(5 values correct)	B2
2	1	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}1$ M1

$I \approx \frac{0{\cdot}1}{2} \times \{0{\cdot}203915171 + 1 + 2(0{\cdot}244678248 + 0{\cdot}315656565 + 0{\cdot}467071461)\}$

$I \approx 3{\cdot}258727719 \div 20$

$I \approx 0{\cdot}162936386$

$I \approx 0{\cdot}163$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}08$

1·6	0·203915171		
1·68	0·234831747		
1·76	0·281831135		
1·84	0·360946198		
1·92	0·520261046		
2	1	(all values correct)	B1

Correct formula with $h = 0{\cdot}08$ M1

$I \approx \frac{0{\cdot}08}{2} \times \{0{\cdot}203915171 + 1 + 2(0{\cdot}234831747 + 0{\cdot}281831135 + 0{\cdot}360946198 + 0{\cdot}520261046)\}$

$I \approx 3{\cdot}999655423 \div 25$

$I \approx 0{\cdot}159986216$

$I \approx 0{\cdot}160$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) $\sin\theta + 12(1 - \sin^2\theta) = 6$ (correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = coefficient of $\sin^2\theta$ and $b \times d$ = constant m1

$12\sin^2\theta - \sin\theta - 6 = 0 \Rightarrow (4\sin\theta - 3)(3\sin\theta + 2) = 0$

$\Rightarrow \sin\theta = \frac{3}{4}$, $\sin\theta = -\frac{2}{3}$ (c.a.o.) A1

$\theta = 48{\cdot}59°, 131{\cdot}41°$ B1

$\theta = 221{\cdot}81°, 318{\cdot}19°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $2x - 35° = -27°, 27°, 333°$ (one value) B1

$x = 4°, 31°$ B1, B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) Correct use of $\tan\phi = \frac{\sin\phi}{\cos\phi}$ (o.e.) M1

$\phi = 135°$ A1

$\phi = 315°$ A1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $\frac{y}{{}^3/_5} = \frac{x}{{}^5/_{13}}$ (o.e.) (correct use of sine rule) M1

$y = 1{\cdot}56x$ (convincing) A1

(*b*) ${}^1/_2 \times x \times y \times {}^{56}/_{65} = 4{\cdot}2$ (correct use of area formula) M1

Substituting $1{\cdot}56x$ for y in candidate's equation of form $axy = b$ M1

$1{\cdot}56x^2 = 9{\cdot}75$ (o.e.) A1

$x = 2{\cdot}5$ (f.t. candidate's quadratic equation provided both M's awarded) A1

$y = 3{\cdot}9$ (f.t. provided both M's awarded) A1

4. (*a*) $\frac{15}{2} \times [2a + 14d] = 780$ B1

Either $[a + d] + [a + 3d] + [a + 9d] = 100$

or $[a + 2d] + [a + 4d] + [a + 10d] = 100$ M1

$3a + 13d = 100$ (seen or implied by later work) A1

An attempt to solve candidate's derived linear equations simultaneously by eliminating one unknown M1

$a = 3$, $d = 7$ (both values) (c.a.o.) A1

(*b*) $d = 9$ B1

A correct method for finding $(p + 7)$ th term M1

$(p + 7)$ th term = 1086 (c.a.o.) A1

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) (i) $\frac{a}{1 - r} = ka$, (k = 4 or ¼) M1

$r = 0{\cdot}75$ (c.a.o.) A1

(ii) $a + 0{\cdot}75a = 35$ (f.t. candidate's derived value for r, provided $r \neq 1$) M1

$a = 20$ (f.t. candidate's derived value for r, provided $r \neq 1$) A1

$S_9 = \frac{20(1 - 0{\cdot}75^9)}{1 - 0{\cdot}75}$

(f.t. candidate's derived values for r and a, provided $r \neq 1$) M1

$S_9 = 73{\cdot}99 = 74$ (c.a.o.) A1

6. (*a*) $\frac{x^{4/3}}{4/3} - 2 \times \frac{x^{1/4}}{1/4} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $x^2 - 4x + 6 = -x + 10$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b$ = candidate's constant m1

$(x - 4)(x + 1) = 0 \Rightarrow x = 4, -1$ (both values, c.a.o.) A1

$y = 6, y = 11$ (both values, f.t. candidate's x-values) A1

Note: Answer only with no working earns 0 marks

(ii) **Either:**

Total area $= \int_{-1}^{4}(-x + 10)\,dx - \int_{-1}^{4}(x^2 - 4x + 6)\,dx$

(use of integration) M1

$\int x^2\,dx = \frac{x^3}{3}$ B1

Either: $\int x\,dx = \frac{x^2}{2}$ **and** $\int 4x\,dx = \frac{4x^2}{2}$ or: $\int 3x\,dx = \frac{3x^2}{2}$ B1

Either: $\int 10\,dx = 10x$ **and** $\int 6\,dx = 6x$ or: $\int 4\,dx = 4x$ B1

Total area $= [-(1/2)x^2 + 10x]_{-1}^{4} - [(1/3)x^3 - (4/2)x^2 + 6x]_{-1}^{4}$ (o.e.)

$= \{(-16/2 + 40) - (-1/2 - 10)\}$

$- \{(64/3 - 32 + 24) - (-1/3 - 2 - 6)\}$

(substitution of candidate's limits in at least one integral) m1

Subtraction of integrals with correct use of candidate's x_A, x_B as limits m1

Total area = 125/6 (c.a.o.) A1

Or:

Area of trapezium = 85/2 (f.t. candidate's x_A, x_B) B1

Area under curve $= \int_{-1}^{4}(x^2 - 4x + 6)\,dx$

(use of integration) M1

$= [(1/3)x^3 - (4/2)x^2 + 6x]_{-1}^{4}$

(correct integration) B2

$= (64/3 - 32 + 24) - (-1/3 - 2 - 6)$

(substitution of candidate's limits) m1

$= 65/3$

Use of candidate's, x_A, x_B as limits and trying to find total area by subtracting area under curve from area of trapezium m1

Total area = 85/2 – 65/3 = 125/6 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indices) B1

$\log_a x/y = p - q$ (the relationship between log and power)

$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) $\frac{1}{2}\log_a x^8 = \log_a x^4$, $3\log_a 2/x = \log_a 2^3/x^3$

(one use of power law) B1

$\frac{1}{2}\log_a x^8 - \log_a 4x + 3\log_a 2/x = \log_a \frac{x^4 \times 2^3}{4x \times x^3}$ (addition law) B1

(subtraction law) B1

$\frac{1}{2}\log_a x^8 - \log_a 4x + 3\log_a 2/x = \log_a 2$ (c.a.o.) B1

8. (*a*) $A(2, -1)$ B1

A correct method for finding the radius M1

Radius = 5 A1

(*b*) (i) A correct method for finding the length of AB M1

$AB = 10$ (f.t. candidate's coordinates for A) A1

Difference in radii = distance between centres,

$\therefore$ circles touch A1

(ii) Gradient $BP(AP)(AB) = \frac{\text{inc in } y}{\text{inc in } x}$ M1

Gradient $BP = \frac{11 - 3}{-7 - (-1)} = \frac{8}{-6}$ (o.e)

(f.t. candidate's coordinates for A) A1

Use of $m_{\tan} \times m_{\text{rad}} = -1$ M1

Equation of common tangent is:

$y - 3 = \frac{3}{4}[x - (-1)]$ (o.e)

(f.t. one slip provided both M's are awarded) A1

9. $r\theta = 7{\cdot}6$ B1

$\frac{r^2\theta}{2} = 36{\cdot}1$ B1

An attempt to eliminate θ M1

$\frac{r}{2} = \frac{36{\cdot}1}{7{\cdot}6} \Rightarrow r = 9{\cdot}5$ A1

$\theta = \frac{7{\cdot}6}{9{\cdot}5} \Rightarrow \theta = 0{\cdot}8$ (f.t. candidate's value for r) A1

C3

1. (*a*)

1	1·386294361		
1·25	1·517870719		
1·5	1·658228077		
1·75	1·802122256	(5 values correct)	B2
2	1·945910149	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{3} \times \{1{\cdot}386294361 + 1{\cdot}945910149 + 4(1{\cdot}517870719 + 1{\cdot}802122256) + 2(1{\cdot}658228077)\}$

$I \approx 19{\cdot}92863256 \div 12$

$I \approx 1{\cdot}66071938$

$I \approx 1{\cdot}6607$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\displaystyle\int_1^2 \ln\left(\frac{1}{3 + x^2}\right) \mathrm{d}x \approx -1{\cdot}6607$ (f.t. candidate's answer to (*a*)) B1

2. $2\,\mathrm{cosec}^2\theta + 3\,(\mathrm{cosec}^2\theta - 1) + 4\,\mathrm{cosec}\,\theta = 9$

(correct use of $\cot^2\theta = \mathrm{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\,\mathrm{cosec}\,\theta + b)(c\,\mathrm{cosec}\,\theta + d)$, with $a \times c$ = coefficient of $\mathrm{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$5\,\mathrm{cosec}^2\theta + 4\,\mathrm{cosec}\,\theta - 12 = 0 \Rightarrow (5\,\mathrm{cosec}\,\theta - 6)(\mathrm{cosec}\,\theta + 2) = 0$

$\Rightarrow \mathrm{cosec}\,\theta = \frac{6}{5}$, $\mathrm{cosec}\,\theta = -2$

$\Rightarrow \sin\theta = \frac{5}{6}$, $\sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 56{\cdot}44°, 123{\cdot}56°$ B1

$\theta = 210°, 330°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks

$\sin\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(2x^3) = 6x^2$, $\frac{d}{dx}(2x) = 2$, $\frac{d}{dx}(25) = 0$ B1

$\frac{d}{dx}(x^2 \cos y) = x^2(-\sin y)\frac{dy}{dx} + 2x(\cos y)$ B1

$\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{6x^2 + 2x\cos y + 2}{x^2 \sin y - 4y^3}$ (c.a.o.) B1

(*b*) (i) candidate's x-derivative = $3t^2$

candidate's y-derivative = $4t + 20t^3$

(one term correct B1, all three terms correct B2)

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{4 + 20t^2}{3t}$ (c.a.o.) A1

(ii) $\frac{dy}{dx} = 5 \Rightarrow 20t^2 - 15t + 4 = 0$

(f.t. candidate's expression for $\frac{dy}{dx}$ from (i)) B1

Considering $b^2 - 4ac$ for candidate's quadratic M1

$b^2 - 4ac = 225 - 320 < 0$ and hence no such real value of t exists

(f.t. candidate's quadratic) A1

4. (*a*) $f'(x) = (11) \times g(x) - 6x$

where $g(x) =$ **either** $\frac{2}{1 + (2x)^2}$ **or** $\frac{1}{1 + (2x)^2}$ **or** $\frac{2}{1 + 2x^2}$ M1

$f'(x) = 11 \times \frac{2}{1 + 4x^2} - 6x$ A1

$f'(x) = 0 \Rightarrow 12x^3 + 3x - 11 = 0$ (convincing) A1

(*b*) $x_0 = 0{\cdot}9$

$x_1 = 0{\cdot}884366498$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 0{\cdot}886029122$

$x_3 = 0{\cdot}885852598$

$x_4 = 0{\cdot}885871344 = 0{\cdot}88587$ (x_4 correct to 5 decimal places) B1

Let $h(x) = 12x^3 + 3x - 11$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}885865$,

$x = 0{\cdot}885875$ M1

$h(0{\cdot}885865) = -1{\cdot}42 \times 10^{-4} < 0$, $h(0{\cdot}885875) = 1{\cdot}70 \times 10^{-4} > 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}88587$ correct to five decimal places A1

5. (*a*) $\frac{dy}{dx} = \frac{1}{3} \times (9 - 2x)^{-2/3} \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = -\frac{2}{3} \times (9 - 2x)^{-2/3}$ A1

(*b*) $\frac{dy}{dx} = \frac{f(x)}{\cos x}$ (including $f(x) = 1$) M1

$\frac{dy}{dx} = \frac{\pm \sin x}{\cos x}$ A1

$\frac{dy}{dx} = -\tan x$ (c.a.o.) A1

(*c*) $\frac{dy}{dx} = x^3 \times f(x) + \tan 4x \times g(x)$ M1

$\frac{dy}{dx} = x^3 \times f(x) + \tan 4x \times g(x)$ (either $f(x) = 4\sec^2 4x$ or $g(x) = 3x^2$) A1

$\frac{dy}{dx} = x^3 \times 4\sec^2 4x + \tan 4x \times 3x^2$ (all correct) A1

(*d*) $\frac{dy}{dx} = \frac{(3x+2)^4 \times k \times e^{6x} - e^{6x} \times 4 \times (3x+2)^3 \times m}{[(3x+2)^4]^2}$

with either $k = 6$, $m = 3$ or $k = 6$, $m = 1$ or $k = 1$, $m = 3$ M1

$\frac{dy}{dx} = \frac{(3x+2)^4 \times 6 \times e^{6x} - e^{6x} \times 4 \times (3x+2)^3 \times 3}{[(3x+2)^4]^2}$ A1

$\frac{dy}{dx} = \frac{18x \times e^{6x}}{(3x+2)^5}$ (c.a.o.) A1

6. (*a*) (i) $\displaystyle\int \frac{9}{4x+3}\,dx = k \times 9 \times \ln|4x+3| + c$ $(k = 1, 4, {}^{1}/_{4})$ M1

$\displaystyle\int \frac{9}{4x+3}\,dx = {}^{9}/_{4} \times \ln|4x+3| + c$ A1

(ii) $\displaystyle\int 3e^{5-2x}\,dx = k \times 3 \times e^{5-2x} + c$ $(k = 1, -2, -{}^{1}/_{2})$ M1

$\displaystyle\int 3e^{5-2x}\,dx = -{}^{3}/_{2} \times e^{5-2x} + c$ A1

(iii) $\displaystyle\int \frac{5}{(7x-1)^3}\,dx = \frac{k \times 5 \times (7x-1)^{-2}}{-2} + c$ $(k = 1, 7, {}^{1}/_{7})$ M1

$\displaystyle\int \frac{5}{(7x-1)^3}\,dx = \frac{5 \times (7x-1)^{-2}}{-2 \times 7} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\displaystyle\int \cos\left(3x - \frac{\pi}{6}\right)dx = \left[k \times \sin\left(3x - \frac{\pi}{6}\right)\right]$ $(k = 1, 3, \pm\,{}^{1}/_{3})$ M1

$\displaystyle\int \cos\left(3x - \frac{\pi}{6}\right)dx = \left[{}^{1}/_{3} \times \sin\left(3x - \frac{\pi}{6}\right)\right]$ A1

$\displaystyle\int_0^{\pi/3} \cos\left(3x - \frac{\pi}{6}\right)dx = k \times \left[\sin\left(\frac{5\pi}{6}\right) - \sin\left(-\frac{\pi}{6}\right)\right]$

(A correct method for substitution of limits
f.t. only candidate's value for k, $k = 1, 3, \pm\,{}^{1}/_{3}$) m1

$\displaystyle\int_0^{\pi/3} \cos\left(3x - \frac{\pi}{6}\right)dx = \frac{1}{3}$ (c.a.o.) A1

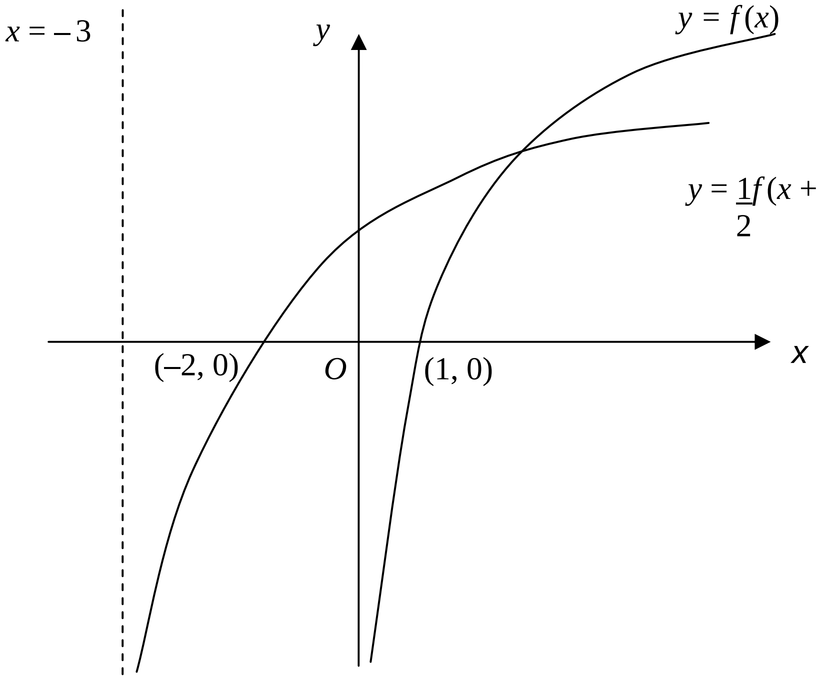
Note: Answer only with no working earns 0 marks

7. (*a*) Choice of a, b, with one positive and one negative and one side correctly evaluated M1
Both sides of identity evaluated correctly A1

(*b*) Trying to solve $2x + 1 = 3x - 4$ M1
Trying to solve $2x + 1 = -(3x - 4)$ M1
$x = 5$, $x = 0{\cdot}6$ (both values) A1

Alternative mark scheme
$(2x + 1)^2 = (3x - 4)^2$ (squaring both sides) M1
$5x^2 - 28x + 15 = 0$ (c.a.o.) A1
$x = 5$, $x = 0{\cdot}6$ (both values, f.t. one slip in quadratic) A1

8.



Correct shape, including the fact that the y-axis is an asymptote for
$y = f(x)$ at $-\infty$ B1

$y = f(x)$ cuts x-axis at (1, 0) B1

Correct shape, including the fact that $x = -3$ is an asymptote for
$y = \frac{1}{2} f(x + 3)$ at $-\infty$ B1

$y = \frac{1}{2} f(x + 3)$ cuts x-axis at (–2, 0) (f.t. candidate's x-intercept for $f(x)$) B1

The diagram shows that the graph of $y = f(x)$ is steeper than the graph of
$y = \frac{1}{2} f(x + 3)$ in the first quadrant B1

9. (*a*) $y + 3 = e^{2x+1}$ B1

An attempt to express equation as a logarithmic equation and to isolate x M1

$x = \frac{1}{2}[\ln(y + 3) - 1]$ (c.a.o.) A1

$f^{-1}(x) = \frac{1}{2}[\ln(x + 3) - 1]$ (f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = (a, b)$ with

$a = -3$ B1

$b = -2$ B1

10. (*a*) $R(f) = (-19, \infty)$ B1

$R(g) = (-\infty, -2)$ B1

(*b*) $D(fg) = (6, \infty)$ B1

$R(fg) = (-15, \infty)$ B1

(*c*) (i) $fg(x) = \left(1 - \frac{1}{2}x\right)^2 - 19$ B1

(ii) Putting expression for $fg(x)$ equal to $2x - 26$ and setting up a quadratic in x of the form $ax^2 + bx + c = 0$ M1

$\frac{1}{4}x^2 - 3x + 8 = 0 \Rightarrow x = 4, 8$ (c.a.o.) A1

Rejecting $x = 4$ and thus $x = 8$ (c.a.o.) A1

1. (*a*) $f(x) \equiv \dfrac{A}{(x+2)^2} + \dfrac{B}{(x+2)} + \dfrac{C}{(x-3)}$ (correct form) M1

$x^2 + x + 13 \equiv A(x-3) + B(x+2)(x-3) + C(x+2)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3$, $C = 1$, $B = 0$ (all three coefficients correct) A2

(at least one coefficient correct) A1

(*b*) $\int f(x)\,dx = \dfrac{3}{(x+2)} + \ln(x-3)$ B1 B1

(f.t. candidates values for A, B, C)

$\int_6^7 f(x)\,dx = \left[\dfrac{3}{9} - \dfrac{3}{8}\right] - [\ln 4 - \ln 3] = 0{\cdot}246(015405)$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. $4x^3 - 2x^2\dfrac{dy}{dx} - 4xy + 2y\dfrac{dy}{dx} = 0$

$\left(-2x^2\dfrac{dy}{dx} - 4xy\right)$ B1

$\left(4x^3 + 2y\dfrac{dy}{dx}\right)$ B1

Either $\dfrac{dy}{dx} = \dfrac{4xy - 4x^3}{2y - 2x^2}$ **or** $\dfrac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 3$ in candidate's expression **and** the use of $grad_{normal} \times grad_{tangent} = -1$ M1

Equation of normal: $y - 3 = -\dfrac{1}{2}(x-1)$

$\left(\text{f.t. candidate's value for } \dfrac{dy}{dx}\right)$ A1

3. (*a*) $\dfrac{2\tan x}{1-\tan^2 x} = 4\tan x$ (correct use of formula for $\tan 2x$) M1

$\tan x = 0$ A1

$2\tan^2 x - 1 = 0$ A1

$x = 0°, 180°$ (both values) A1

$x = 35{\cdot}26°, 144{\cdot}74°$ (both values) A1

(*b*) $R = 25$ B1

Expanding $\cos(\theta - \alpha)$ and using either $25\cos\alpha = 7$ **or** $25\sin\alpha = 24$ **or** $\tan\alpha = \dfrac{24}{7}$ to find α (f.t. candidate's value for R) M1

$\alpha = 73{\cdot}74°$ (c.a.o.) A1

$\cos(\theta - \alpha) = \dfrac{16}{25} = 0{\cdot}64$ (f.t. candidate's value for R) B1

$\theta - \alpha = 50{\cdot}21°, -50{\cdot}21°$ (at least one value, f.t. candidate's value for R) B1

$\theta = 23{\cdot}53°, 123{\cdot}95°$ (c.a.o.) B1

4. (*a*) candidate's x-derivative $= -3\sin t$

candidate's y-derivative $= 4\cos t$ (at least one term correct) B1

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = -\dfrac{4\cos t}{3\sin t}$ (o.e.) (c.a.o.) A1

At P, $y - 4\sin p = -\dfrac{4\cos p}{3\sin p}(x - 3\cos p)$ (o.e.)

(f.t. candidate's expression for $\dfrac{dy}{dx}$) M1

$(3\sin p)y - 12\sin^2 p = (-4\cos p)x + 12\cos^2 p$

$(3\sin p)y = (-4\cos p)x + 12\cos^2 p + 12\sin^2 p$

$(3\sin p)y + (4\cos p)x - 12 = 0$ (convincing) A1

(*b*) (i) $A = (2\sqrt{3}, 0)$ B1

$B = (0, 8)$ B1

(ii) Correct use of Pythagoras Theorem to find AB M1

$AB = 2\sqrt{19}$ (convincing) A1

5. (*a*) $A(-3, 0)$, $B(3, 0)$, $C(0, 3)$ B1

(*b*) (i)

$$\text{Volume} = \pi\int_{-3}^{3}(9 - x^2)\,dx$$

(f.t candidate's x-coordinates for A, B) M1

$$\int(9 - x^2)\,dx = 9x - \frac{x^3}{3}$$ B1

Volume = 36π (c.a.o.) A1

Note: Answer only with no working earns 0 marks

(ii) This is the volume of a sphere of radius 3 E1

6.

$$(1 + 2x)^{1/2} = 1 + (1/2) \times (2x) + \frac{(1/2) \times (1/2 - 1) \times (2x)^2}{1 \times 2} + \ldots$$

(– 1 each incorrect term) B2

$$\frac{1}{(1 + 3x)^2} = 1 + (-2) \times (3x) + \frac{(-2) \times (-3) \times (3x)^2}{1 \times 2} + \ldots$$

(– 1 each incorrect term) B2

$$4(1 + 2x)^{1/2} - \frac{1}{(1 + 3x)^2} = 3 + 10x - 29x^2 + \ldots$$

(– 1 each incorrect term) B2

Expansion valid for $|x| < 1/3$ B1

7. (*a*) $\int x \sin 2x \, dx = x \times k \times \cos 2x - \int k \times \cos 2x \times g(x)\, dx$

($k = \pm ^1/_2, \pm 2$ or ± 1) M1

$k = -\frac{1}{2}$, $g(x) = 1$ A1, A1

$\int x \sin 2x \, dx = -\frac{1}{2} \times x \times \cos 2x + \frac{1}{4} \times \sin 2x + c$ (c.a.o.) A1

(*b*) $\int \frac{x}{(5 - x^2)^3} \, dx = \int \frac{k}{u^3} \, du$ ($k = \pm ^1/_2$ or ± 2) M1

$\int \frac{a}{u^3} \, du = -\frac{a}{2} u^{-2}$ B1

$\int_0^2 \frac{x}{(5 - x^2)^3} \, dx = -\frac{k}{2}\left[u^{-2} \right]_5^1$ or $-\frac{k}{2}\left[\frac{1}{(5 - x^2)^2} \right]_0^2$

(f.t. candidate's value for k, $k = \pm ^1/_2$ or ± 2) A1

$\int_0^2 \frac{x}{(5 - x^2)^3} \, dx = \frac{6}{25}$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $\frac{dN}{dt} = kN$ B1

(*b*) $\int \frac{dN}{N} = \int k \, dt$ M1

$\ln N = kt + c$ A1

$N = e^{kt + c} = Ae^{kt}$ (convincing) A1

(*c*) (i) $100 = Ae^{2k}$

$160 = Ae^{12k}$ (both values) B1

Dividing to eliminate A M1

$1{\cdot}6 = e^{10k}$ A1

$k = \frac{1}{10} \ln 1{\cdot}6 = 0{\cdot}047$ (convincing) A1

(ii) $A = 91({\cdot}0283)$ (o.e.) B1

When $t = 20$, $N = 91({\cdot}0283) \times e^{0{\cdot}94}$

(f.t. candidate's derived value for A) M1

$N = 233$ (c.a.o.) A1

9. (*a*) Use of $(5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}).(4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}) = 0$ M1

$5 \times 4 + (-8) \times 6 + 4 \times a = 0$ m1

$a = 7$ A1

(*b*) (i) $\mathbf{r} = 8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.) B1

(ii) $8 + 2\lambda = 4 - 2\mu$

$3 + \lambda = 7 + \mu$

$-7 + 2\lambda = 5 + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

$\lambda = 1$, $\mu = -3$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ do not satisfy third equation B1

10. Assume that there is a real and positive value of x such that $4x + \frac{9}{x} < 12$

$4x^2 - 12x + 9 < 0$ B1

$(2x - 3)^2 < 0$ B1

This contradicts the fact that x is real and thus $4x + \frac{9}{x} \geq 12$ B1

FP1

1. $f(x+h) - f(x) = \dfrac{1}{(x+h)^3} - \dfrac{1}{x^3}$ M1

$= \dfrac{x^3 - (x+h)^3}{x^3(x+h)^3}$ A1

$= \dfrac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{x^3(x+h)^3}$ A1

$= \dfrac{-3x^2h - 3xh^2 - h^3}{x^3(x+h)^3}$ A1

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{-3x^2 - 3xh - h^2}{x^3(x+h)^3}$ M1

$f'(x) = \lim_{h \to 0}\left(\dfrac{-3x^2 - 3xh - h^2}{x^3(x+h)^3}\right)$ A1

$= -\dfrac{3}{x^4}$

2. $S_n = 2\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r$ M1

$= \dfrac{2n(n+1)(2n+1)}{6} - \dfrac{n(n+1)}{2}$ A1A1

$= \dfrac{n(n+1)}{6}(4n+2-3)$ m1

$= \dfrac{n(n+1)(4n-1)}{6}$ A1

3. $(1+2\text{i})(2-3\text{i}) = 2 - 6\text{i}^2 + 4\text{i} - 3\text{i} = 8 + \text{i}$ M1A1

$2(x - \text{i}y) + \text{i}(x + \text{i}y) = 8 + \text{i}$ M1

$2x - y = 8$ m1

$x - 2y = 1$ A1

$x = 5, y = 2$ cao A1A1

4. (a) Det = 1(7 + 3) + 2(12 – 14) + 1(– 2 – 4) M1A1

= 0 as required

(b)(i) Using row operations, M1

$x + 2y + z = 1$

$-3y + z = 0$ A1

$-9y + 3z = \lambda - 4$ A1

The third line is three times the second line so

$\lambda = 4$ A1

(ii) Put $z = \alpha$. M1

Then $y = \alpha/3$ A1

$x = 1 - 5\alpha/3$ A1

5. 2 – i is a root. B1

$x^2 - 4x + 5$ is a factor. M1A1

$x^4 - 2x^3 - 2x^2 + 6x + 5 = (x^2 + 2x + 1)(x^2 - 4x + 5)$ M1A1

The other root is – 1. M1A1

[Award M0M0M0 if no working]

6. The statement is true for $n = 1$ since 6 + 4 is divisible by 10. B1

Let the statement be true for $n = k$, ie $6^k + 4$ is divisible by 10 so that $6^k + 4 = 10N$. M1

Consider $6^{k+1} + 4 = 6 \times 6^k + 4$ M1A1

$= 6(10N - 4) + 4$ A1

$= 60N - 20$ A1

This is divisible by 10 so true for $n = k \Rightarrow$ true for $n = k + 1$ (and since true for $n = 1$) therefore the statement is proved by induction. A1

7. (a) Anticlock rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Translation matrix = $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Reflection matrix in $y + x = 0$ = $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$ A1

$$= \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) The general point on the line is $(\lambda, 2\lambda - 1)$. M1

The image of this point is given by

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \lambda \\ 2\lambda - 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\lambda - 1 \\ 2\lambda + 1 \\ 1 \end{bmatrix}$$ m1

$x = -\lambda - 1, y = 2\lambda + 1$ A1

Eliminating λ, M1

The equation of the image is $y = -2x - 1$. A1

8. (a) $\mathbf{A}^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$ M1A1

$5\mathbf{A} + 2\mathbf{I} = 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} = \mathbf{A}^2$ M1A1

(b) $\mathbf{A}^3 = 5\mathbf{A}^2 + 2\mathbf{A}$ B1

$= 5(5\mathbf{A} + 2\mathbf{I}) + 2\mathbf{A}$ B1

$= 27\mathbf{A} + 10\mathbf{I}$ B1

9. Let the roots be $\alpha, \alpha\beta, \alpha\beta^2$. Then M1

$$\alpha + \alpha\beta + \alpha\beta^2 = -f$$

$$\alpha^2\beta + \alpha^2\beta^2 + \alpha^2\beta^3 = g$$ M1

$$\alpha^3\beta^3 = -h$$ A1

[Award M1A0 if roots not given in geometric progression]

Divide the second equation by the first:

$$\alpha\beta = -\frac{g}{f}$$ M1A1

Cubing and comparing with the third equation, M1

$$\left(-\frac{g}{f}\right)^3 = -h$$ A1

$$g^3 = f^3h$$

10. (a) $u + \mathrm{i}v = \dfrac{1}{(x + \mathrm{i}y)^2}$ M1

$= \dfrac{(x - \mathrm{i}y)^2}{(x^2 + y^2)^2}$ m1

$= \dfrac{x^2 - y^2 - 2\mathrm{i}xy}{(x^2 + y^2)^2}$

$u = \dfrac{x^2 - y^2}{(x^2 + y^2)^2}, v = -\dfrac{2xy}{(x^2 + y^2)^2}$ A1

(b)(i) Putting $y = mx$, M1

$u = \dfrac{x^2(1 - m^2)}{(x^2 + m^2x^2)^2}, v = \dfrac{-2mx^2}{(x^2 + m^2x^2)^2}$ A1

Dividing,

$\dfrac{v}{u} = -\dfrac{2m}{1 - m^2}$ A1

So $v = m'u$ where $m' = -\dfrac{2m}{1 - m^2}$ m1

(ii) The gradients are equal if

$$m = -\frac{2m}{1 - m^2}$$ M1

Solving, $m = 0, \pm\sqrt{3}$. M1A1

FP2

1. $u = \sqrt{x} \Rightarrow \mathrm{d}u = \frac{1}{2}\frac{1}{\sqrt{x}}\mathrm{d}x$ B1

and $[1,4] \to [1, 2]$ B1

$I = 2\int_1^2 \frac{\mathrm{d}u}{9+u^2}$ M1

$= \frac{2}{3}\left[\tan^{-1}(\frac{u}{3})\right]_1^2$ A1

$= 0.1775$ A1

2. Combining the first and third terms,

$2\cos 2\theta \cos 3\theta + \cos 3\theta = 0$ M1A1

$\cos 3\theta(2\cos 2\theta + 1) = 0$ A1

EITHER $\cos 3\theta = 0 \Rightarrow 3\theta = (2n+1).\frac{\pi}{2}$ M1

$\theta = (2n+1).\frac{\pi}{6}$ (n an integer) A1

OR $\cos 2\theta = -\frac{1}{2} \Rightarrow 2\theta = (2n+1)\pi \pm \frac{\pi}{3}$ M1

$\theta = (2n+1).\frac{\pi}{2} \pm \frac{\pi}{6}$ A1

Alternatively, combining the second and third terms,

$2\cos\theta \cos 4\theta + \cos\theta = 0$ M1A1

$\cos\theta(2\cos 4\theta + 1) = 0$ A1

EITHER $\cos\theta = 0 \Rightarrow \theta = (2n+1).\frac{\pi}{2}$ (n an integer) M1A1

OR $\cos 4\theta = -\frac{1}{2} \Rightarrow 4\theta = (2n+1)\pi \pm \frac{\pi}{3}$ M1

$\theta = (2n+1).\frac{\pi}{4} \pm \frac{\pi}{12}$ A1

[Accept equivalent forms and answers in degrees]

3. (a) As $x \to 2$ from above $f(x) \to 4$ and $f(2) = 1$ M1

There is therefore a jump at $x = 2$ so not continuous. A1

(accept informal notation)

(b) For $x < 2$, $f'(x) = 6 - 2x > 0$ throughout. B1

For $x > 2$, $f'(x) = 2x - 2 > 0$ throughout. B1

Furthermore, $f(x)$ increases from 1 to 4 going through $x = 2$. B1

So f is a strictly increasing function. B1

(c) $f(A) = [-2,1] \cup (4,7]$ B1B1B1

4. (a) $|z| = \sqrt{2}$ B1

$\theta = \tan^{-1}(-1) + \pi = 3\pi/4$ M1A1

(b) First root = $(2^{1/6}, \pi/4)$ M1

= 0.794 + 0.794i A1A1

Second root = $(2^{1/6}, \pi/4 + 2\pi/3)$ M1

= – 1.084 + 0.291i A1

Third root = $(2^{1/6}, \pi/4 + 4\pi/3)$ M1

= 0.291 – 1.084i A1

(c) We require $3\pi/4 \times n$ to be a multiple of 2π. Using any valid method including trial and error, M1

$n = 8$ [Award M1A0 for $n = 4$] A1

5. (a) $z^n = (\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$ B1

$z^{-n} = \dfrac{1}{\cos n\theta + \mathrm{i}\sin n\theta}$

$= \dfrac{\cos n\theta - \mathrm{i}\sin n\theta}{\cos^2 n\theta + \sin^2 n\theta}$ M1

$= \cos n\theta - \mathrm{i}\sin n\theta$ A1

$z^n - z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta - (\cos n\theta - \mathrm{i}\sin n\theta) = 2\mathrm{i}\sin n\theta$ A1

$z^n + z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta = 2\cos n\theta$ A1

(b) $\left(z - \dfrac{1}{z}\right)^4 = z^4 - 4z^3.\dfrac{1}{z} + 6z^2.\dfrac{1}{z^2} - 4.\dfrac{1}{z^3} + \dfrac{1}{z^4}$ M1A1

$= z^4 + \dfrac{1}{z^4} - 4\left(z^2 + \dfrac{1}{z^2}\right) + 6$ A1

$(2\mathrm{i}\sin\theta)^4 = 2\cos 4\theta - 8\cos 2\theta + 6$ M1

$\sin^4\theta = \dfrac{1}{8}\cos 4\theta - \dfrac{1}{2}\cos 2\theta + \dfrac{3}{8}$ A1

6. (a) The equation can be rewritten

$2(x-1)^2 + 3(y+2)^2 = 6$ M1A1

The centre is (1, – 2) A1

(b) The equation can be rewritten

$\dfrac{(x-1)^2}{3} + \dfrac{(y+2)^2}{2} = 1$ M1

$a = \sqrt{3}, b = \sqrt{2}$ A1

$1 - e^2 = \dfrac{2}{3},\quad e = \dfrac{1}{\sqrt{3}}$ M1A1

(c) The foci are (0, – 2) ; (2, – 2) B1B1

(d) The directrices are $x = -2$, $x = 4$. B1B1

7. (a) Derivative = $\sin(e^x)$ B1

(b) Derivative = $\frac{d}{du}\left[\int_0^u \sin(e^t)dt\right]\times\frac{du}{dx}$ M1

$= 2x\sin(e^{x^2})$ A1

8. (a) EITHER

$f(x)=\frac{x^2+2x+1}{x^2-3x+2}=\frac{x^2-3x+2+5x-1}{(x-1)(x-2)}$ M1

$= 1+\frac{5x-1}{(x-1)(x-2)}$ A1

Let $\frac{5x-1}{(x-1)(x-2)}=\frac{B}{x-1}+\frac{C}{x-2}=\frac{B(x-2)+C(x-1)}{(x-1)(x-2)}$ M1

Putting x =1,2, B = – 4, C = 9 A1

OR

$1-\frac{4}{x-1}+\frac{9}{x-2}=\frac{(x-1)(x-2)-4(x-2)+9(x-1)}{(x-1)(x-2)}$ M1A1

$=\frac{x^2-3x+2-4x+8+9x-9}{(x-1)(x-2)}$ M1

$=\frac{x^2+2x+1}{(x-1)(x-2)}$ A1

$= f(x)$

$f'(x)=\frac{4}{(x-1)^2}-\frac{9}{(x-2)^2}$ B1B1

$f''(x)=-\frac{8}{(x-1)^3}+\frac{18}{(x-2)^3}$ B1

(b) Stationary points occur when

$\frac{4}{(x-1)^2}=\frac{9}{(x-2)^2}$ M1

$\frac{x-1}{x-2}=\pm\frac{2}{3}$ A1

x = – 1, y = 0 cao A1

$f''(-1)>0$ therefore minimum (FT their coords) A1

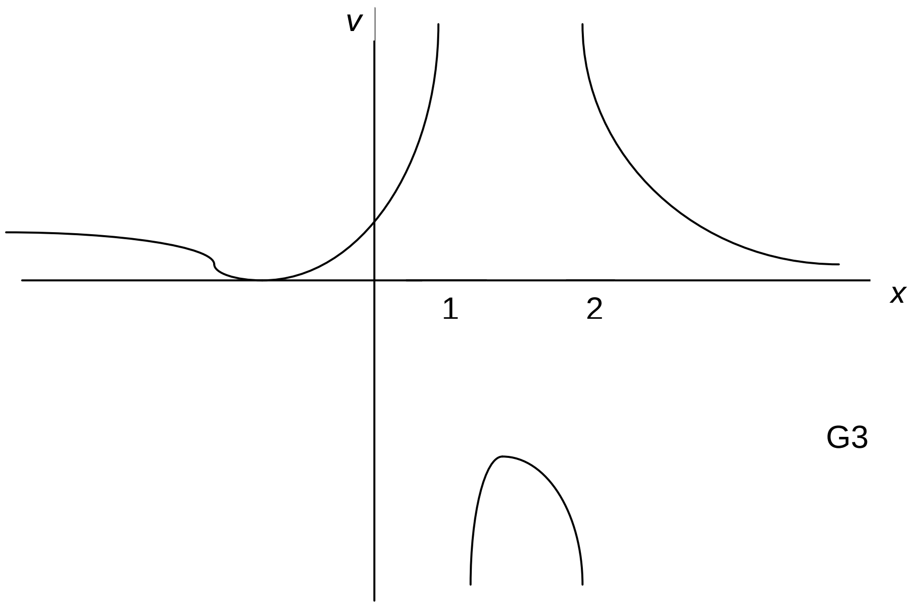
x = 7/5, y = – 24 cao A1

$f''(7/5)<0$ therefore maximum (FT their cords) A1

(c) The asymptotes are x = 1 and x = 2 B1

and y = 1 B1

(d)



G3

FP3

1. EITHER

Using $\text{sech}^2\theta + \tanh^2\theta = 1$ to give M1

$3\text{sech}^2\theta + 5\text{sech}\theta - 2 = 0$ A1

Use of formula or factorisation to give M1

$\text{sech}\theta = -2, 1/3$ A1

$\text{sech}\theta$ cannot equal -2 B1

$\text{sech}\theta = 1/3 \Rightarrow \cosh\theta = 3$ B1

OR

Division by $\cosh^2\theta$ to give M1

$3\sinh^2\theta = 5\cosh\theta + \cosh^2\theta$ A1

leading to

$2\cosh^2\theta - 5\cosh\theta - 3 = 0$ A1

Use of formula or factorisation to give M1

$\cosh\theta = -½, 3$ A1

$\cosh\theta$ cannot equal $-1/2$ B1

THEN

$\theta = \cosh^{-1}3 = \ln(3 + \sqrt{8})$ M1A1

2. Putting $t = \tan(x/2)$ gives $\text{d}x = \dfrac{2\text{d}t}{1+t^2}$ B1

$(0, \pi/2) \to (0,1)$ B1

$I = \displaystyle\int_0^1 \frac{2\text{d}t/(1+t^2)}{2 + 2t/(1+t^2)}$ M1

$= \displaystyle\int_0^1 \frac{\text{d}t}{t^2 + t + 1}$ A1

$= \displaystyle\int_0^1 \frac{\text{d}t}{(t+1/2)^2 + 3/4}$ m1

$= \dfrac{2}{\sqrt{3}}\left[\tan^{-1}\left(\dfrac{2(t+1/2)}{\sqrt{3}}\right)\right]_0^1$ A1

$= \dfrac{2}{\sqrt{3}}(\tan^{-1}\sqrt{3} - \tan^{-1}(1/\sqrt{3}))$ A1

$= \dfrac{2}{\sqrt{3}}\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right)$ A1

$= \dfrac{\pi}{3\sqrt{3}}$

3. $y^2 = 4a(x-a) \Rightarrow 2y\frac{dy}{dx} = 4a$ M1A1

$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{4a^2}{y^2}$ M1

$= 1+ \frac{a}{x-a} = \frac{x}{x-a}$ A1

Arc length = $\int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ B1

$= \int_{2a}^{4a} \sqrt{\frac{x}{x-a}}\,dx$

$x = a\cosh^2 u \Rightarrow dx = 2a\cosh u \sinh u\, du$ B1

$(2a, 4a) \to (\cosh^{-1}\sqrt{2}, \cosh^{-1} 2)$ B1

AL = $\int_{\cosh^{-1}\sqrt{2}}^{\cosh^{-1}2} \sqrt{\frac{a\cosh^2 u}{a\cosh^2 u - a}}\, 2a\cosh u \sinh u\,du$ M1A1

$= 2a \int_{\cosh^{-1}\sqrt{2}}^{\cosh^{-1}2} \frac{\cosh u}{\sinh u}.\cosh u \sinh u\,du$ A1

$= 2a \int_{\cosh^{-1}\sqrt{2}}^{\cosh^{-1}2} \cosh^2 u\,du$

$= a \int_{\cosh^{-1}\sqrt{2}}^{\cosh^{-1}2} (1+\cosh 2u)\,du$ M1A1

$= a\left[u + \frac{1}{2}\sinh 2u\right]_{\cosh^{-1}\sqrt{2}}^{\cosh^{-1}2}$ A1

$= 2.49a$ cao A1

4. (a) $f'(x) = e^x\cos x - e^x \sin x$ B1

$f''(x) = e^x\cos x - e^x\sin x - e^x\sin x - e^x\cos x = -2e^x\sin x$ B1

(b) $f'''(x) = -2e^x\sin x - 2e^x\cos x$ B1

$f''''(x) = -2e^x\sin x - 2e^x\cos x - 2e^x\cos x + 2e^x\sin x (= -4e^x\cos x)$ B1

$f(0) = 1, f'(0) = 1, f''(0) = 0, f'''(0) = -2, f''''(0) = -4$ B1

$f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + \frac{x^4}{24}f''''(0) + \ldots$ M1

$= 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \ldots$ cao A1

(c) Differentiating both sides,

$e^x\cos x - e^x\sin x = 1 - x^2 - \frac{2x^3}{3} + \ldots$ M1A1

$e^x\sin x = 1 + x - \frac{x^3}{3} - 1 + x^2 + \frac{2x^3}{3} + \ldots$ (FT series from (a)) M1

$= x + x^2 + \frac{x^3}{3} + \ldots$ A1

5. (a) When $x = 0.6, x\sin x - 0.5 = -0.161...$, when $x = 0.8, x\sin x = 0.073...$ M1

Because of the sign change, there is a root between 0.6 and 0.8. A1

(b)(i) The Newton-Raphson iteration is

$$x_{n+1} = x_n - \frac{(x_n \sin x_n - 0.5)}{(\sin x_n + x_n \cos x_n)} \quad \text{M1A1}$$

$$= \frac{x_n \sin x_n + x_n^2 \cos x_n - x_n \sin x_n + 0.5}{\sin x_n + x_n \cos x_n} \quad \text{A1}$$

$$= \frac{x_n^2 \cos x_n + 0.5}{\sin x_n + x_n \cos x_n}$$

(ii) Successive values are

0.7
0.7415796192 B1
0.7408411726
0.7408409551 B1

The required value is 0.74084 B1

(c)(i) $$f'(x) = \frac{1}{\sqrt{1-(0.5/x)^2}} \times \frac{-0.5}{x^2} \quad \text{M1A1}$$

[Only award M1 if chain rule used]

$f'(0.7) = -1.45...$ cao (Accept any argument to which 0.74084 rounds) A1

This is greater than 1 in modulus so the sequence is divergent and cannot be used to find α. (FT on their $f'(x)$) B1

6. (a) Area = $\frac{1}{2}\int r^2 \mathrm{d}\theta$ M1

$$= \frac{1}{2}\int_0^{\pi/2} \sin^2 2\theta \mathrm{d}\theta \quad \text{A1}$$

$$= \frac{1}{4}\int_0^{\pi/2} (1-\cos 4\theta)\mathrm{d}\theta \quad \text{A1}$$

$$= \frac{1}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\pi/2} \quad \text{A1}$$

$$= \frac{\pi}{8} \quad \text{A1}$$

(b) Consider

$$y = r\sin\theta$$

$$= \sin 2\theta \sin\theta \quad \text{M1}$$

$$\frac{dy}{d\theta} = \sin 2\theta\cos\theta + 2\cos 2\theta\sin\theta \quad \text{A1}$$

At P, $\sin 2\theta\cos\theta + 2\cos 2\theta\sin\theta = 0$ M1

EITHER

$$2\tan\theta = -\tan 2\theta = -\frac{2\tan\theta}{1-\tan^2\theta} \quad \text{A1}$$

$$\tan^2\theta = 2 \quad \text{A1}$$

OR

$$2\sin\theta\cos^2\theta + 2\sin\theta(2\cos^2\theta - 1) = 0 \quad \text{A1}$$

$$\cos^2\theta = 1/3 \text{ or } \sin^2\theta = 2/3 \quad \text{A1}$$

THEN

$$\theta = 0.955 \ (54.7^\circ) \quad \text{cao} \quad \text{A1}$$

$$r = 0.943 \ (2\sqrt{2}/3) \quad \text{cao} \quad \text{A1}$$

7. (a)

$$I_n = \int_0^a \tanh^{n-2}x\tanh^2 x\,dx \quad \text{M1}$$

$$= \int_0^a \tanh^{n-2}x(1-\text{sech}^2 x)\,dx \quad \text{m1A1}$$

$$= I_{n-2} - \frac{1}{n-1}\left[\tanh^{n-1}x\right]_0^a \quad \text{A1A1}$$

$$= I_{n-2} - \frac{0.5^{n-1}}{n-1}$$

(b)

$$I_0 = \int_0^\alpha dx \quad \text{M1}$$

$$= [x]_0^{\tanh^{-1}0.5} = \tanh^{-1}0.5 = 0.549 \quad \text{A1}$$

$$I_4 = I_2 - \frac{0.5^3}{3} \quad \text{M1}$$

$$= I_0 - 0.5 - \frac{0.5^3}{3} \quad \text{m1}$$

$$= 0.00764 \quad \text{A1}$$

GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3 AS/Advanced

SUMMER 2011

INTRODUCTION

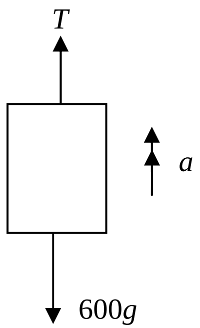
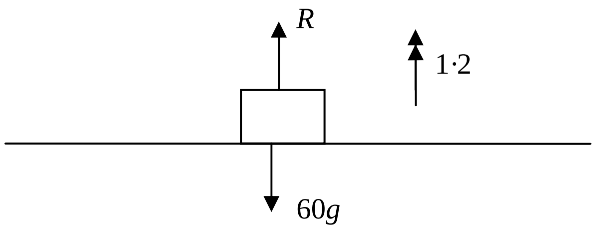
The marking schemes which follow were those used by WJEC for the Summer 2011 examination in GCE MATHEMATICS - M1-M3 & S1-S3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

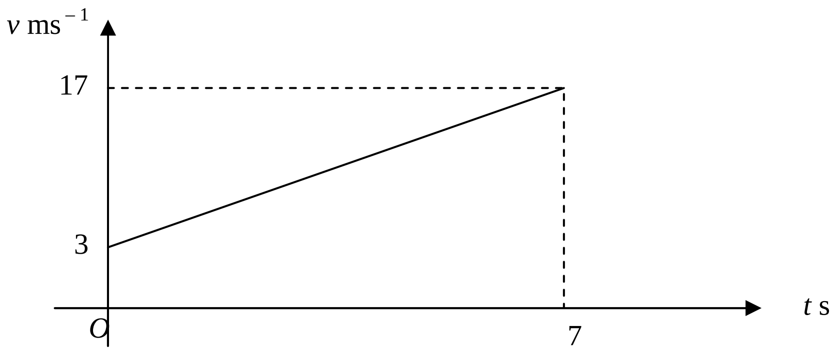
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

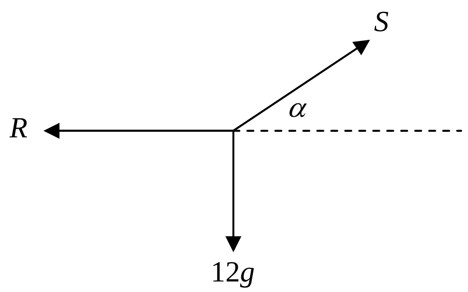
WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

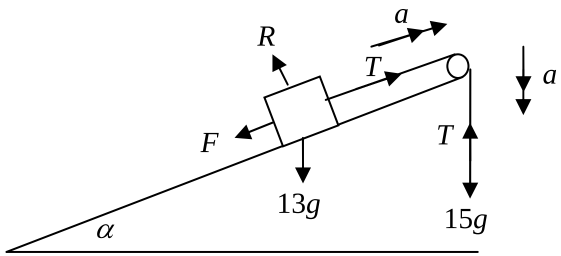
M1

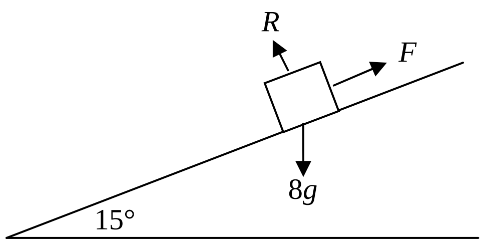
Question	Solution	Mark	Notes
1(a)	$v = u + at$, $u = 1$, $a = 9.8$, $t = 2.5$ $v = 1 + 9.8 \times 2.5$ $= \underline{25.5\ (\text{ms}^{-1})}$	M1 A1 A1	Accept ± values for *u* and *a*. Correct equation, accept ± accept ±
1(b)	$s = ut + 0.5at^2$, $u = 1$, $a = 9.8$, $t = 2.5$ $= 1 \times 2.5 + 0.5 \times 9.8 \times 2.5^2$ $= \underline{33.125(\text{m})}$	M1 A1 A1	Accept ± values for *u* and *a*. equivalent method Correct equation, accept ±. ft (a) if applicable. accept ±. ft (a) if applicable.

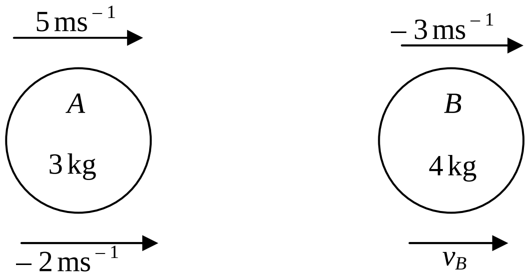
Question	Solution	Mark	Notes
2(a)	 N2L applied to lift $T - 600g = 600a$ $a = \underline{1.2}$	M1 A1 A1	dim correct, opposing T and $600g$ correct equation cao
2(b)	 N2L applied to person $R - 60g = 60 \times 1.2$ $R = \underline{660\ (\text{N})}$	M1 A1 A1	Dim correct, opposing R and $60g$. Correct equation. FT a ft candidate's a, both Ms required.

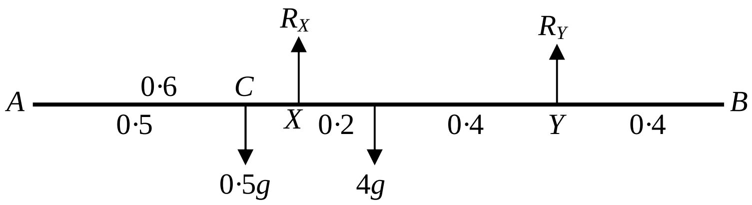
Question	Solution	Mark	Notes
3(a)	Consider motion from *A* to *B* $s = ut + 0.5at^2$, $t = 2$, $s = 10$ $10 = 2u + 0.5a \times 2^2$ $10 = 2u + 2a$	 M1 A1	 Correct substitution of values
	Consider motion from *A* to *C* $v = u + at$, $v = 17$, $t = 7$ $17 = u + 7a$	M1 A1	
	Solve simultaneously $a = 2$ $u = 3$	m1 A1 A1	Depends on both previous Ms cao ft slip if both equations correct
3(b)		M1 A1	 ft *u*
3(c)	Distance $AC = 0.5(3 + 17) \times 7$ $= \underline{70(\text{m})}$	M1 A1	correct method for area under graph oe ft *u* if appropriate

Question	Solution	Mark	Notes
4.			
4(a)	Resolve vertically $S \sin\alpha = 12g$ S = <u>196(N)</u>	M1 A1 A1	attempt at resolution to get equ, accept cos correcr equation cao
4(b)	Resolve horizontally $S \cos \alpha = R$ R = <u>156.8 (N)</u>	M1 A1 A1	attempt at resolution to get equ, accept sin correct equation ft S, depends on both previous Ms

Question	Solution	Mark	Notes
5.			
	N2L applied to *B* $15g - T = 15a$	M1 A1	dim correct, opposing *T* and 15*g*. correct equation
	N2L applied to *A* $T - 13g\sin\alpha = 13a$ $T - 5g = 13a$	M1 A1	dim correct, opposing *T* and 13*g* resolved. Correct equation
	Solve equations simultaneously Adding $15g - 5g = 28a$	m1	depends on both Ms
	a = <u>3.5 (ms^{-2})</u> T = <u>94.5 (N)</u>	A1 A1	cao ft if both equations correct.

Question	Solution	Mark	Notes
6.			
6(a)	Resolve perpendicular to plane $R = 8g\cos 15°$	M1 A1	dim correct, accept sin
	Resolve parallel to plane $F = 8g\sin 15°$	M1 A1	dim correct, accept cos
	Least $\mu = F/R$ Least $\mu = \tan 15° = 0.26795 = \underline{0.28}$(to 2 d. p.)	M1 A1	award if seen in (a) or (b) cao. do not penalise unrounded correct answers.
6(b)	$F = 0.1 \times 8g\cos 15°$ $8g\sin 15° - 0.1 \times 8g\cos 15° = 8a$ $a = \underline{1.59(14)}$	A1 M1 A1 A1	 Attempt at N2L. correct equation. cao

Question	Solution	Mark	Notes
7.			
7(a)	Conservation of momentum $3 \times 5 \ + \ 4 \times (-3) \ = \ 3 \times (-2) \ + \ 4v_B$ $15 - 12 \ = \ -6 + 4v_B$ $v_B \ = \ \underline{2.25\ (ms^{-1})}$	M1 A1 A1	Attempted, no more than 1 sign error correct equation cao
7(b)	Restitution $2.25 - (-2) \ = \ -e(-3 - 5)$ $4.25 \ = \ 8e$ $e \ = \ \underline{0.53125}$	M1 A1 A1	Attempted. Only one sign error in vel. any correct equation ft (a) if >-3
7(c)	Required Impulse $= \ 3(5 + 2)$ $= \ \underline{21\ (Ns)}$	M1 A1	 allow negative answer.

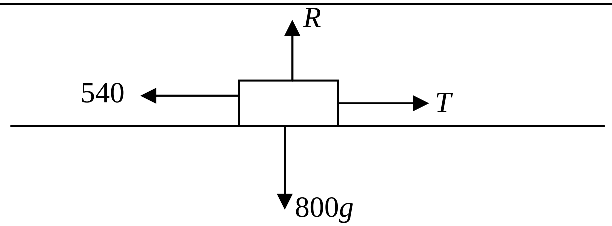
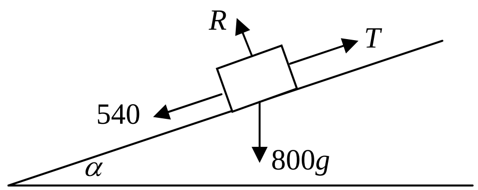
Question	Solution	Mark	Notes
8.			
8(a)	Moments about X $0.5g \times 0.1 = 4g \times 0.2 - R_Y \times 0.6$ $0.6R_Y = 0.8g - 0.05g$ $R_Y = \underline{1.25g} = \underline{12.25 \text{ (N)}}$	M1 B1 A1 A1	Attempt at equation, oe correct equation A1, one correct mom B1 cao
	Resolve vertically $R_X + R_Y = 0.5g + 4g$ $R_X = 4.5\text{g} – 1.25g$ $= \underline{3.25g} = \underline{31.85 \text{ (N)}}$	M1 A1 A1	Attempted. dim correct. any correct equation ft R
8(b)	On point of turning about X, $R_Y = 0$ Moments about X $(0.5 + M)g \times 0.1 = 4g \times 0.2$ $0.5 + M = 8$ $M = \underline{7.5 \text{ (kg)}}$	M1 m1 A1 A1	Any equivalent method to obtain equation correct equation

Question	Solution	Mark	Notes
9.			
	Area — from *Oy* — from *Ox*		
	OAP — 108 — 12 — 3	B1	B1 for 3
	PBQ — 12 — 12 — 7	B1	B1 for 7
	Lamina — 96 — x — y	B1	B1 for 108, 12, 96
	$x = 12$	B1	
	Moments about *Ox*	M1	
	$108 \times 3 = 12 \times 7 + 96y$	A1	ft values from table
	$y =$ <u>2.5</u>	A1	cao
9(b)	$\tan\theta = (6 - 25)/4$	M1A1	ft (a)
	$\theta =$ <u>41.2°</u>	A1	ft (a)

Question	Solution	Mark	Notes
1(a)	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	M1	sin to cos, t retained
	$a = 36\cos 3t + 16\sin 2t$	A1A1	one mark for each correct term
1(b)	$x = \int 12\sin 3t - 8\cos 2t \quad dt$		
	$x = -4\cos 3t - 4\sin 2t + (C)$	M1 A1A1	sin to cos, t retained one mark for each correct term
	$t = 0, x = 0$	m1	use of initial conditions
	$0 = -4 + C$ $C = 4$	A1	ft one error only

Question	Solution	Mark	Notes
2(a)	Speed = v = $r\omega$ $v = 0.6 \times 5$ v = $\underline{3\ (\text{ms}^{-1})}$	M1 A1	use of correct formula, oe
2(b)	tension in string = $m \times$ acceleration towards centre $T = mr\omega^2$ $T = 0.5 \times 0.6 \times 5^2$ T = $\underline{7.5\ (\text{N})}$	M1 A1	use of formula ft v

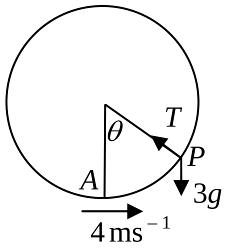
Question	Solution	Mark	Notes
3(a)	Attempt to differentiate v to find the acceleration **a** = 6**j** + 12t^2**k** **F** = 12**j** + 24t^2**k**	M1 A1 A1	powers of t decreased once. vector ft **a**
3(b)	When t = 1, **v** = 2**i** + 6**j** + 4**k** and **F** = 12**j** + 24**k** **F.v** = $(2 \times 0) + (6 \times 12) + (4 \times 24)$ **F.v** = <u>168</u> Units: watts	M1 M1 A1 B1	use ot t=1 in **v,F** or **v.F** correct method for dot product ft **F, v**

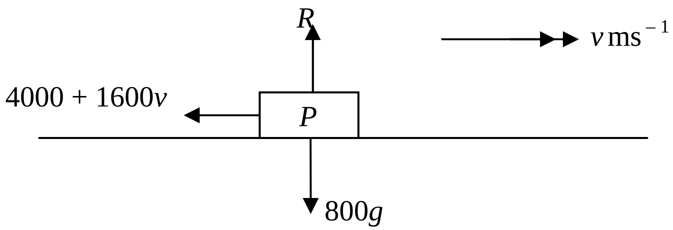
Question	Solution	Mark	Notes
4(a)			
	Constant speed $a = 0$	M1	si
	$T = 540$	A1	
	Power $P = T \times 60$	M1	any equivalent statement,T horizontal
	Power $=$ $\underline{32400\ (\text{W})}$ $=$ $\underline{32.4\ (\text{kW})}$	A1	
4(b)			
	$T = 32.4 \times 1000 \div 15 = (2160)$	M1	use of P/v
	N2L $T - F - 800g\sin\alpha = 800a$	M1 A2	dim correct 4 terms -1 mark for each error
	$a = \underline{1.4125\ (\text{ms}^{-2})}$	A1	cao, allow+/-

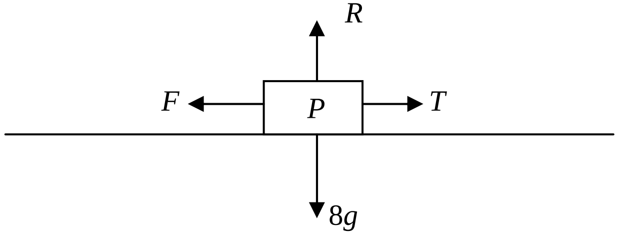
Question	Solution	Mark	Notes
5(a)	Hooke's Law $T = \frac{80 \times 0 \cdot 4}{1 \cdot 6}$ T = <u>20 (N)</u>	M1 A1	use of correct formula with at least 2 correct values
5(b)	Using ceiling as zero potential energy Initial energy = $-4 \times 9.8 \times 0.5$ = -19.6 (J) Energy when string is 2m = $-4 \times 9.8 \times 2 + 0.5 \times 4v^2 + \frac{1}{2} \times 80 \times \frac{0 \cdot 4^2}{1 \cdot 6}$ $2v^2 - 74.4 = -19.6$ v = <u>5.23 (ms^{-1})</u> <u>Alternative</u> $\frac{1}{2} \times 4 \times v^2 + \frac{80 \times 0 \cdot 4^2}{2 \times 1 \cdot 6} = 4 \times 9.8 \times 1.5$ $2v^2 + 4 = 58.8$ $v^2 = 27.4$ v = <u>5.23 (ms^{-1})</u>	M1 A1 M1 A1 B1 M1 A1 A1 B1 M1A1 M1A1 M1A1 A1	any correct use of potential energy correct value of PE, h=0.5/2/1.5 Use of EE formula with 80, 1.6 correct EE correct KE Energy equation with 3 types Correct equation, any form accept answers rounding to 5.23 cao KE EE PE correct equation cao

Question	Solution	Mark	Notes
6(a)	Initial vertical velocity = $6.5\sin\alpha$ = (2.5) Using $s = ut + 0.5 \times 9.8 \times t^2$ with $s = -100$, $u = 2.5$, $a = -9.8$ $-100 = 2.5t - 4.9t^2$ $4.9t^2 - 2.5t - 100 = 0$ t = <u>4.78 (s)</u>	B1 M1 A1 m1 A1	si allow +ve values, ft 2.5(c) attempt to solve by quadratic formula accept unrounded values, cao
6(b)	Initial horizontal velocity = $6.5\cos\alpha$ = (6) Required distance = 6×4.78 = <u>28.68</u>	B1 B1	 ft t and 2.5
6(c)	Using $v^2 = u^2 + 2as$ with $u = 2.5$, $s = -100$, $a = -9.8$ $v^2 = 2.5^2 + 2 \times (-9.8) \times (-100)$ $v = \pm 44.34$ Required speed = $\sqrt{6^2 + 44\cdot34^2}$ = <u>44.7465</u> $\theta = \tan^{-1}\left(\frac{44\cdot34}{6}\right)$ θ = <u>82.29°</u>	M1 A1 A1 M1 A1 M1 A1	oe. Accept +100,9.8. ft 6 ft t if appropriate ft t if appropriate ft candidate's velocities Accept 7.71, 8 or 82. ft candidate's velocities. Accept –ve values

Question	Solution	Mark	Notes
7.	$\boldsymbol{v}_A = 2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k}$	B1	si
	$\lvert v_A \rvert = \sqrt{2^2 + 6^2 + 9^2}$	M1	
	$= \underline{11}$	A1	cao
7(b)	$AB = (5 + 3t - 2 - 2t)\mathbf{i} + (-8 - 6t - 3 + 6t)\mathbf{j} + (10 + 7t - 1 - 9t)\mathbf{k}$	M1	allow BA
	$AB = (3 + t)\mathbf{i} + (-11)\mathbf{j} + (9 - 2t)\mathbf{k}$	A1	
	$AB^2 = (3 + t)^2 + (-11)^2 + (9 - 2t)^2$	M1	
	$AB^2 = 5t^2 - 30t + 211$	A1	correct intermediate step
	$\frac{dAB^2}{dt} = 2(3 + t) + 2(9 - 2t)(-2)$	M1	attempt to diff or complete sq
	$= 6 + 2t - 36 + 8t$		
	$= 10t - 30$		
	When closest $\frac{dAB^2}{dt} = 0$	M1	or $5(t-3)^2 + k$
	$10t = 30$		
	$t = \underline{3}$	A1	cao

Question	Solution	Mark	Notes
8.			
8(a)	Conservation of energy $0.5 \times 3 \times 4^2 = 0.5\,mv^2 + mg \times 0.4(1 - \cos\theta)$ $48 = 3v^2 + 6 \times 9.8 \times 0.4(1 - \cos\theta)$ $3v^2 = 48 - 23.52 + 23.52\cos\theta$ $v^2 = 8.16 + 7.84\cos\theta$	M1 A1A1 A1	 Ke correct, PE with correct h correct equation
8(b)	$T - mg\cos\theta = mv^2/r$ $T - 3 \times 9.8\cos\theta = 3(8.16 + 7.84\cos\theta)/0.4$ $T = 29.4\cos\theta + 61.2 + 58.8\cos\theta$ $T = 61.2 + 88.2\cos\theta$	M1A1 m1 A1	 cao
8(c)	Consider T when $\theta = 180°$ $T = 61.2 - 88.2 < 0$ Therefore *P* does not describe complete circles	M1 A1 A1	 ft $T = a + b\cos\theta$
8(d)	Consider v^2 when $\theta = 180°$ $v^2 = 8.16 - 7.84 > 0$. Therefore *P* does describe complete circles	M1 A1	 cao

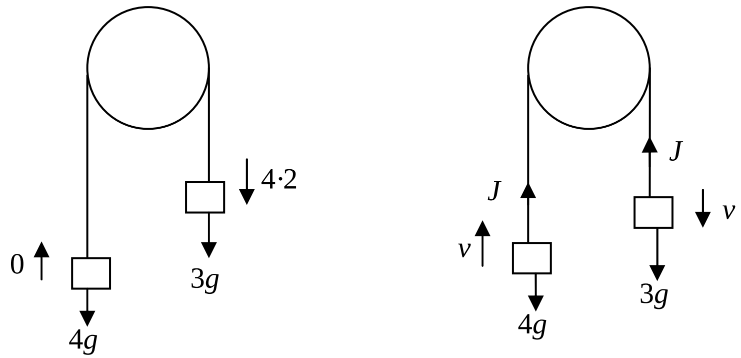
Question	Solution	Mark	Notes
1.			
1(a)	Apply N2L to P $-(4000 + 1600v) = 800a$	M1	
	Divide by 800 $\frac{dv}{dt} = -(5 + 2v)$	A1	
1(b)(i)	Separate variables	M1	Attempt at separating variables
	$\int \frac{dv}{5+2v} = -\int dt$	A1	correct equation
	$0.5 \ln \lvert 5 + 2v \rvert = -t + (C)$	A1	correct integration
	When $t = 0$, $v = 5$	m1	use of initial conditions
	$C = 0.5 \ln 15$	A1	ft 0.5 missing only
	$t = \frac{1}{2}\ln\left\lvert\frac{15}{5+2v}\right\rvert$		
	If P is at rest, $v = 0$	m1	
	$t = 0.5 \ln 3 = (0.55\text{s})$	A1	cao

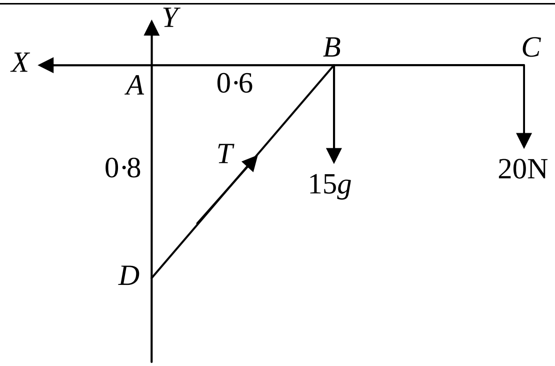
Question	Solution	Mark	Notes
1(b)(ii)	$e^{2t} = \dfrac{15}{5+2v}$	M1	Inversion, ft expressions of correct form.
	$5 + 2v = 15\,e^{-2t}$		
	$v = 0.5(15e^{-2t} - 5)$		
	$v = 2.5(3e^{-2t} - 1)$	A1	cao
2(a)			
	Apply N2L to particle $T - F = ma$	M1	T and F opposing
	$4v - (4 - 16t) = 8a$	A1	
	Divide by 4 $2\dfrac{d^2x}{dt^2} - \dfrac{dx}{dt} = 4t - 1$	A1	

Question	Solution	Mark	Notes
2(b)	Auxilliary equation $2m^2 - m = 0$ $m(2m - 1) = 0$ $m = 0, 0.5$ Complementary function is $A + Be^{0.5t}$ For particular integral, try $x = at^2 + bt$ $\frac{dx}{dt} = 2at + b$ $\frac{d^2x}{dt^2} = 2a$ $4a - (2at + b) = 4t - 1$ Comparing coefficients $-2a = 4$ $a = -2, b = -7$ general solution is $x = A + Be^{0.5t} - 2t^2 - 7t$ $t = 0, x = 0$ $0 = A + B$ $\frac{dx}{dt} = \frac{1}{2}Be^{0.5t} - 4t - 7$ $t = 0, \frac{dx}{dt} = 3$ $3 = 0.5B - 7$ $B = 20, A = -20$ $x = 20e^{0.5t} - 2t^2 - 7t - 20$	B1 B1 B1 M1 A1 m1 A1 B1 M1 B1 m1 A1	 both values ft solution of auxiliary equation allow $at^2 + bt + c$ both values required, cao follow through CF and PI use of initial conditions ft similar expressions both values, cao

Question	Solution	Mark	Notes
3(a).	Consider the position when the piston has moved a distance x m $T = \frac{\lambda x}{l} = \frac{3 \cdot 2x}{0 \cdot 5}$ $T = 6.4x$ N2L applied to piston $0.1a = -6.4x$ $\frac{d^2x}{dt^2} = -64x = -(8)^2x$ Therefore the motion is Simple Harmonic with $\omega = 8$. Centre is at O Period $= 2\pi/8 = \pi/4$ s.	M1 A1 M1 A1 B1 A1	used, accept ± depends on both M's both
3(b)	Maximum velocity $= 0.8$ ms^{-1} $A\omega = 0.8$ $A = \underline{0.1\text{ (m)}}$	M1 A1	
3(c)	Using $v^2 = \omega^2(A^2 - x^2)$ with $\omega = 8$, $A = 0.1$, $x = 0.08$ $v^2 = 8^2(0.1^2 - 0.08^2)$ $v = \underline{0.48\text{ (ms}^{-1})}$	M1 A1 A1	 cao
3(d)	maximum aceleration = $\omega^2 A = 8^2 \times 0.1$ $= \underline{6.4\text{ (ms}^{-2})}$	M1 A1	
3(e)	$x = 0.1\sin(8t)$ $0.05 = 0.1\sin(8t)$ $t = 0.125\sin^{-1}(0.5) = \underline{\pi/48} = \underline{0.065\text{ (s)}}$	M1 m1 A1	allow cos cao

Question	Solution	Mark	Notes
4.	$a = -\frac{9}{2x^2}$	M1	
	$v\frac{dv}{dx} = -\frac{9}{2x^2}$	m1	
	$\int 2v\ dv = -9\int x^{-2}\ dx$	M1	separate variables attempted
	$v^2 = 9x^{-1} + (C)$	A1A1	
	When $x = 0.75$, $v = 3$	m1	
	$9 = 9 / 0.75 + C$		
	$C = -3$	A1	cao
	$v^2 = \frac{9}{x} - 3$		
	When $x = 2$	m1	
	$v^2 = 4.5 – 3 = 1.5$		
	$v = 1.22$	A1	cao
	Speed of P when $x = 2$ is <u>1.22ms^{-1}</u>		
	When P comes to rest, $v = 0$		
	$\frac{9}{x} = 3$		
	$x = \underline{3}$		
	P is at rest when $x = 3$.	A1	

Question	Solution	Mark	Notes
5(a).	Using $v^2 = u^2 + 2as$ with $u = 0$, $a = 9.8$, $s = 0.9$ (downwards positive) $v^2 = 0 + 2 \times 9.8 \times 0.9$ $v = \underline{4.2\ (\text{ms}^{-1})}$	M1 A1 A1	allow -9.8, s = - 0.9 correct equation cao
5(b)	Before After 		
	$J = 3(4.2 – v)$ $J = 4v$	M1A1 M1A1	dimensionally correct dimensionally correct
	$12.6 – 3v = 4v$ $7v = 12.6$	m1	attempt to solve simultaneously
	$v = \underline{1.8\ \text{ms}^{-1}}$	A1	cao
	$J = 4v$ $J = \underline{7.2\ (\text{Ns})}$	A1	cao

Question	Solution	Mark	Notes
6.			
6(a)	Moments about *A* $T \times 0.6\sin\theta = 15g \times 0.6 + 20 \times 1.2$ $T = \underline{233.75\ (N)}$	M1 A3 A1	about point, one cor term, dim cor eq. -1 each error cao
6(b)	Resolve vertically $Y + T\sin\theta = 15g + 20$ $Y = -20$ (N)	M1 A1	no extra forces, no left out forces cao
	Resolve horizontally $X = T\cos\theta$ $X = 140.25$ (N)	M1 A1	no extra forces, no left out forces cao

Question	Solution	Mark	Notes
6(b).	Therefore $R = \sqrt{140 \cdot 25^2 + 20^2}$ R = 141.67 (N) $\alpha = \tan^{-1}\left(\frac{20}{140 \cdot 67}\right)$ α = 8.1° below the horizontal	M1 A1 M1 A1	 ft if both M's awarded ft if both M's awarded

1. (a) Prob = $\frac{5}{9}\times\frac{3}{8}\times\frac{1}{7}\times 6$ or $\binom{5}{1}\times\binom{3}{1}\times\binom{1}{1}\div\binom{9}{3}$ M1A1

$= \frac{5}{28}$ (0.179) A1

(b) Prob = $\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}$ or $\binom{6}{3}\div\binom{9}{3}=\frac{5}{21}$ (0.238) M1A1

(c) P(All red) = $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}$ or $\binom{5}{3}\div\binom{9}{3}$ $\left(\frac{5}{42}\right)$ B1

P(All green) = $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}$ or $\binom{3}{3}\div\binom{9}{3}$ $\left(\frac{1}{84}\right)$ B1

P(Same colour) = $\frac{5}{42}+\frac{1}{84}=\frac{11}{84}$ (0.131) B1

[FT their two probs found in (c)]

2. (a) $E(Y) = 4a + b = 16$ M1A1

$Var(Y) = 4a^2 = 16$ M1A1

$a = 2$ cao A1

$b = 8$ cao A1

(b) Because Y cannot take all appropriate values, eg 0. B1

3. (a) $P(A\cup B) = 1 - P(A'\cap B')$ M1

$= 0.55$ A1

Not mutually exclusive because $P(A)+P(B)\neq P(A\cup B)$ A1

(b) EITHER $P(A\cap B) = P(A)+P(B)-P(A\cup B)$ M1

$= 0.1$ A1

Use of $P(A\cap B) = P(A)\times P(B) = 0.1$ m1

A and B are independent. A1

OR

$P(A') = 0.75, P(B') = 0.6$ M1A1

Use of $P(A'\cap B') = P(A')\times P(B') = 0.45$ m1

A and B are independent. A1

[Accept correct use of these arguments in reverse]

4. (a)(i) X is Poi(12). si B1

$P(X = 10) = e^{-12}\times\frac{12^{10}}{10!}$ M1

$= 0.105$ (FT their mean] A1

[Award M0 if answer only given]

(ii) Y is Poi(6). si B1

$P(Y > 5) = 1 - 0.4457$ M1

$= 0.5543$ (FT their mean] A1

(b) $p_0 = e^{-0.2t} = 0.03$ M1A1

$-0.2t\log e = \log 0.03$ m1

$t = 17.5$ cao A1

5. (a) $k(1+4+9+16)=1$ M1A1

$k = 1/30$

(b) $E(X)=\frac{1}{30}(1\times1+2\times4+3\times9+4\times16)$ M1

$=\frac{10}{3}$ A1

$E(X^2)=\frac{1}{30}(1\times1+4\times4+9\times9+16\times16)$

$=\frac{59}{5}$ B1

$\text{Var}(X) = \frac{59}{5}-\left(\frac{10}{3}\right)^2$ M1

$=\frac{31}{45}$ (0.688) cao A1

(c) Possibilities are 1,3 ; 3,1 ; 2,2 si B1

[Accept 1,3 ; 2,2]

Prob = $\frac{1}{30^2}(1\times9+9\times1+4\times4)$ M1A1

= 0.038 A1

6. (a) If the fair coin is chosen, P(3 heads = 1/8) si B1

P(3 heads) = $\frac{1}{3}\times1+\frac{2}{3}\times\frac{1}{8}$ M1A1

$=\frac{5}{12}$ A1

(b) Reqd prob = $\frac{1/3}{5/12}$ (FT the denominator from (a)) B1B1

$=\frac{4}{5}$ cao B1

(c) P(Head) = $\frac{4}{5}\times1+\frac{1}{5}\times\frac{1}{2}=\frac{9}{10}$ M1A1

[FT their probability from (b)]

7. (a) Independent trials. B1

Constant probability of success. B1

(b)(i) P(X = 8) = $\binom{20}{8}\times0.4^8\times0.6^{12}$ M1

= 0.180 A1

[or 0.5956 – 0.4159 or 0.5841 – 0.4044]

(ii) $P(6\le X\le10)=0.8725-0.1256$ or 0.8744 – 0.1275 B1B1

= 0.747 cao B1

[Award M0 if answer only given in (i) or (ii)]

(c) The number of hits, Y, is approx Poi(4). si B1

$P(Y<5)$ = 0.6288 M1A1

8. (a)(i) $E(X) = \int_0^1 12x.x^2(1-x)\,dx$ (No limits required here) M1

$= \left[\frac{12x^4}{4} - \frac{12x^5}{5}\right]_0^1$ A1

$= 0.6$ A1

(ii) $E(1/X) = \int_0^1 \frac{12}{x} x^2(1-x)\,dx$ (No limits required here) M1

$= \left[\frac{12x^2}{2} - \frac{12x^3}{3}\right]_0^1$ A1

$= 2$ A1

(iii) EITHER

$P(0.2 \le X \le 0.5) = \int_{0.2}^{0.5} 12x^2(1-x)dx$ M1

$= \left[\frac{12x^3}{3} - \frac{12x^4}{4}\right]_{0.2}^{0.5}$ A1

$= 0.285$ A1

OR

$F(x) = 4x^3 - 3x^4$ B1

Required prob = $F(0.5) - F(0.2)$ M1

$= 0.285$ A1

(b) $a + b = 0$ M1

$2a + 4b = 1$ A1

[Award M1A0 for 1 correct equation]

Solving,

$a = -\frac{1}{2}$, $b = \frac{1}{2}$ A1A1

S2

1. (a) (i) $z = \frac{30-28}{2} = 1.0$ M1A1

Prob = 0.1587 cao A1

[Award full marks for answer only]

(ii) Distribution of $\bar{X}$ is N(28, 4/5) M1A1

[Award M1A0 for N and 1 correct parameter]

$z = \frac{30-28}{\sqrt{4/5}} = 2.24$ m1A1

Prob = 0.987 cao A1

[Award m0A0A0 for answer only]

(b) Let *A*,*B* denote the times taken by Alan, Brenda.

Then $A - B$ is N(3,13). M1A1

[Award M1A0 for N and 1 correct parameter]

We require $P(B > A) = P(A - B < 0)$

$z = \frac{0-3}{\sqrt{13}} = -0.83$ [Accept +0.83] m1A1

Prob = 0.2033 cao A1

[Award m0A0A0 for answer only]

2. (a) $\bar{x} = \frac{1290}{60}$ (= 21.5) B1

SE of $\bar{X} = \frac{0.5}{\sqrt{60}}$ (= 0.0645…) B1

95% conf limits are

$21.5 \pm 1.96 \times 0.0645$ M1A1

[M1 correct form, A1 1.96]

giving [21.37, 21.63] cao A1

(b) We solve

$3.92 \times \frac{0.5}{\sqrt{n}} < 0.1$ M1A1

$n > 384.16$ A1

[Award M1A0A0 for 1.96 in place of 3.92]

Minimum sample size is 385. B1

[Award B1 for rounding up their *n*]

3. (a) $H_0 : \mu = 0.5; \; H_1 : \mu < 0.5$ B1

(b) Under H_0, mean = 15 B1

p-value = $P(X \le 12 | \mu = 15)$ M1

= 0.2676 cao A1

Insufficient evidence to reject H_0. B1

[FT their p-value]

(c) X is now Po(100) which is approx N(100,100) si B1

$$z = \frac{80.5 - 100}{\sqrt{100}}$$ M1A1

[Award M1A0 for incorrect continuity correction]

= – 1.95 A1

[80 gives z = – 2, p = 0.02275;79.5 gives z = – 2.05,p = 0.02018]

p-value = 0.0256 A1

Strong evidence to accept H_1. B1

[FT their p-value]

4. (a) $H_0 : \mu_x = \mu_y ; H_1 : \mu_x \ne \mu_y$ B1

(b) $\bar{x} = \frac{114.8}{8} (= 14.35)$ B1

$\bar{y} = \frac{98.0}{7} = (14.0)$ B1

$$\text{SE } (\bar{X} - \bar{Y}) = \sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{7}} \; (= 0.2587..)$$ M1A1

$$z = \frac{14.35 - 14.0}{0.2587..} = 1.35$$ M1A1

Prob from tables = 0.0885 A1

p-value = 0.177 B1

Insufficient evidence to reject her belief (at the 5% level). B1

[FT their p-value, conclusion must refer to her belief]

5. (a) $f(u) = \frac{1}{b - a}, \; a \le u \le b$, (= 0 otherwise) B1

$$E(U^2) = \frac{1}{(b-a)} \int_a^b u^2 du$$ (Limits not required here) M1

$$= \frac{1}{(b-a)} \left[\frac{u^3}{3} \right]_a^b$$ A1

$$= \frac{1}{(b-a)} \frac{(b^3 - a^3)}{3}$$ A1

$$= \frac{1}{(b-a)} \frac{(b-a)(a^2 + ab + b^2)}{3}$$ A1

$$= \frac{a^2 + ab + b^2}{3}$$

(b)(i) $E(X) = 3$, $Var(X) = 3$ B1B1

(ii) $Y = 12 - X$ B1

$E(XY) = E(12X - X^2)$ M1

$= 12 \times 3 - \frac{36}{3}$ A1

[FT their values from (i)]

$= 24$ A1

(iii) Let T denote the total length.

Then T is approx N(300,300). M1A1

[Award M1A0 for N and 1 correct parameter]

$z = \frac{280 - 300}{\sqrt{300}} = -1.15$ m1A1

Prob = 0.8749 A1

[Award m1A0A1 for use of continuity correction giving $z = -1.13$, p = 0.8708 or z = nm – 1.18, p = 0.8810]

6. (a)(i) X is B(20,0.3) si B1

P(Accept H_1 | H_0 true) = $P(X \ge 9) \mid p = 0.3)$ M1

= 0.1133 A1

(ii) X is B(20,0.6) B1

P(Accept H_0 | H_1 true = $P(X \le 8 \mid p = 0.6)$ M1

The number of tails, T, is B(20,0.4) m1

Required prob = $P(T \ge 12 \mid p = 0.4)$ A1

= 0.0565 A1

(b)(i) Y is B(80,0.3) which is approx N(24,16.80) B1

P(Accept H_1 | H_0 true) = $P(Y \ge 36 \mid H_0)$ M1

$z = \frac{35.5 - 24}{\sqrt{16.8}} = 2.81$ m1

Required prob = 0.00248 A1

[Award m1A0 for incorrect continuity correction]

(ii) Y is B(80,0.6) which is approx N(48,19.2) B1

P(Accept H_0 | H_1 true) = $P(Y \le 35 \mid H_0)$ M1

$z = \frac{35.5 - 48}{\sqrt{19.2}} = 2.85$ m1

Required prob = 0.00219 A1

[Award m1A0 for incorrect continuity correction]

S3

1. (a) $\hat{p} = 0.67$ B1

(b) ESE = $\sqrt{\dfrac{0.67 \times 0.33}{100}}$ = 0.04702.. si M1A1

(c) 95% confidence limits are

$0.67 \pm 1.96 \times 0.04702..$ [FT from (b)] M1A1

giving [0.58,0.76] cao A1

(d) Accept Bill's claim because 0.75 lies in the interval. B1

[FT the conclusion]

2. $\bar{x} = \dfrac{149.1}{100} = 1.491$ B1

$s^2 = \dfrac{222.9}{99} - \dfrac{149.1^2}{99 \times 100}$ = 0.0059787… B1

[Accept division by 100 giving 0.005919]

Test stat = $\dfrac{1.491 - 1.5}{\sqrt{0.0059787/100}}$ M1A1

= –1.16 (–1.17) cao A1

Value from tables = 0.1230 (0.1210) cao A1

p-value = 0.246 (0.242) (FT from line above) B1

The manufacturer's claim is supported OR mean lifetime is 1500 hrs B1

3. (a) The possibilities are

Numbers drawn	Sum
1 1 2	4
1 1 3	5
1 1 4	6
1 2 3	6
1 2 4	7
1 3 4	8
2 3 4	9

M1A1A1

[M1A1 possibilities, A1 sum ; M1A0A1 if 1 row omitted]

The sampling distribution of the sum is

Sum	4	5	6	7	8	9
Prob	1/10	1/10	3/10	2/10	2/10	1/10

M1A1

(b) The sampling distribution of the largest number is

Largest	2	3	4
Prob	1/10	3/10	6/10

M1A1

Expected value = 3.5 A1

4. (a) UE of μ = 279/12 = 23.25 B1

$\Sigma x = 279\,; \Sigma x^2 = 6503.64$ (seen or implied in next line) B1

UE of $\sigma^2 = \dfrac{6503.64}{11} - \dfrac{279^2}{11\times 12}$ M1

= 1.5354… A1

[Award M0 if no working shown for variance estimate]

(b) DF = 11 si B1

At the 90% confidence level, critical value = 1.796 B1

[FT if critical value is 1.363 leading to (22.8, 23.7)]

The 90% confidence limits are

$23.25 \pm 1.796\sqrt{\dfrac{1.5354..}{12}}$ M1A1

giving [22.6, 23.9] A1

[Award M0 if normal distribution used]

5. (a) $H_0 : \mu_x = \mu_y\,; H_1 : \mu_x < \mu_y$ B1

(b) $\bar{x} = 24.75; \bar{y} = 26.0$ B1B1

$s_x^2 = \dfrac{37364}{59} - \dfrac{1485^2}{59\times 60} = 10.3432...$ B1

$s_y^2 = \dfrac{41221}{59} - \dfrac{1560^2}{59\times 60} = 11.2033...$ B1

[Accept division by 60 giving 10.1708… and 11.0166..]

SE = $\sqrt{\dfrac{10.3432..}{60} + \dfrac{11.2033}{60}}$ M1

= 0.5992.. (0.5942..) A1

Test stat = $\dfrac{26.0 - 24.75}{0.5992}$ M1

= 2.09 (2.10) A1

[FT their z-value]

EITHER

p-value = 0.0183 (0.0179) B1

OR

Critical value = 1.645 B1

Strong evidence to accept the managing director’s belief (at the 5% significance level). B1

[Accept the use of a confidence interval except for the final M1A1]

6. (a) $\sum x = 90, \sum x^2 = 1420, \sum y = 169.2, \sum xy = 2626.2$

$S_{xy} = 2626.2 - 90 \times 169.2/6 = 88.2$ B1

$S_{xx} = 1420 - 90^2/6 = 70$ B1

$b = \frac{88.2}{70} = 1.26$ cao M1A1

$a = \frac{169.2 - 90 \times 1.26}{6}$ M1

$= 9.3$ cao A1

[Award M0, M0 for answers only with no working]

(b) [FT from (a) where possible.]

Est solubility at 17°C = 9.3 + 1.26 × 17 = 30.72 M1A1

SError = $0.15\sqrt{\frac{1}{6} + \frac{(17-15)^2}{70}} = 0.07096..$ M1A1

The 99% confidence interval for solubility at 17°C is given by

30.72 ± 2.576 × 0.0710 M1A1

[FT from their est solubility and stand error if M marks awarded]

ie (30.5,30.9) cao A1

7. (a)(i) E(X) = $\int_{-1}^{1} x(\frac{1}{2} + \theta x)\,\mathrm{d}x$ M1

[must see limits either here or next line]

$= \left[\frac{x^2}{4} + \theta\frac{x^3}{3}\right]_{-1}^{1}$ A1

$= \frac{2\theta}{3}$ A1

$\mathrm{E}(X^2) = \int_{-1}^{1} x^2\left(\frac{1}{2} + \theta x\right)\mathrm{d}x$ M1

[must see limits either here or next line]

$= \left[\frac{x^3}{6} + \frac{\theta x^4}{4}\right]_{-1}^{1}$

$= \frac{1}{3}$ A1

Var(X) = $\frac{1}{3} - \frac{4\theta^2}{9}$ A1

$= \frac{3 - 4\theta^2}{9}$

(ii) $P(X > 0) = \int_0^1 \left(\frac{1}{2} + \theta x\right) \mathrm{d}x$ M1

[must see limits either here or next line]

$= \left[\frac{x}{2} + \frac{\theta x^2}{2}\right]_0^1$ A1

$= \frac{1+\theta}{2}$

(b) $\mathrm{E}(U) = \frac{3}{2}\mathrm{E}(\overline{X})$

$= \frac{3}{2}\mathrm{E}(X)$ M1

$= \frac{3}{2} \times \frac{2\theta}{3} = \theta$ A1

[Award M0 if E omitted]

$\mathrm{Var}(U) = \frac{9}{4}\mathrm{Var}(\overline{X})$ M1

$= \frac{9}{4} \times \frac{(3-4\theta^2)}{9n}$

$= \frac{3-4\theta^2}{4n}$ A1

(c) $\mathrm{E}(V) = \frac{2}{n}\mathrm{E}(Y) - 1$ M1

$= \frac{2}{n} \times \frac{n(1+\theta)}{2} - 1$ A1

$= \theta$ A1

[Award M0 if E omitted]

$\mathrm{Var}(V) = \frac{4}{n^2}\mathrm{Var}(Y)$ M1

$= \frac{4}{n^2} \times n\left(\frac{1+\theta}{2}\right)\left(\frac{1-\theta}{2}\right)$

$= \frac{1-\theta^2}{n}$ A1

(d) $\mathrm{Var}(V) - \mathrm{Var}(U) = \frac{1-\theta^2}{n} - \frac{3-4\theta^2}{4n}$

$= \frac{4-4\theta^2-3+4\theta^2}{4n}$ B1

$= \frac{1}{4n}$

Since Var(*U*) < Var(*V*), *U* is the better estimator. B1

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