GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2013

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2013 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

C1

1. (*a*) (i) Gradient of $BC = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $BC = -4$ (or equivalent) A1

(ii) A correct method for finding the equation of BC using candidate's gradient for BC M1

Equation of BC : $y - (-5) = -4(x - 6)$ (or equivalent)
(f.t. candidate's gradient of BC) A1

Equation of BC : $4x + y - 19 = 0$ (convincing) A1

(iii) Use of $m_{AD} \times m_{BC} = -1$ M1

A correct method for finding the equation of AD using candidate's gradient for AD (M1)

(to be awarded only if corresponding M1 is not awarded in part (ii))

Equation of AD : $y - 4 = {}^1/_4\,(x - 8)$ (or equivalent)
(f.t. candidate's gradient of BC) A1

Note: Total mark for part (*a*) is 7 marks

(*b*) An attempt to solve equations of BC and AD simultaneously M1

$x = 4$, $y = 3$ (convincing) (c.a.o.) A1

(*c*) A correct method for finding the length of BD M1

$BD = \sqrt{68}$ A1

(*d*) A correct method for finding E M1

$E(0, 2)$ A1

2. (*a*) $\frac{2 + 5\sqrt{7}}{4 + \sqrt{7}} = \frac{(2 + 5\sqrt{7})(4 - \sqrt{7})}{(4 + \sqrt{7})(4 - \sqrt{7})}$ M1

Numerator: $8 - 2\sqrt{7} + 20\sqrt{7} - 35$ A1

Denominator: $16 - 7$ A1

$\frac{2 + 5\sqrt{7}}{4 + \sqrt{7}} = -3 + 2\sqrt{7}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $4 + \sqrt{7}$

(*b*) $\sqrt{360} = 6\sqrt{10}$ B1

$\sqrt{2} \times (\sqrt{5})^3 = 5\sqrt{10}$ B1

$\frac{\sqrt{30} \times \sqrt{8}}{\sqrt{6}} = 2\sqrt{10}$ B1

$\sqrt{360} - \sqrt{2} \times (\sqrt{5})^3 - \frac{\sqrt{30} \times \sqrt{8}}{\sqrt{6}} = -\sqrt{10}$ (c.a.o.) B1

3. (*a*) $\frac{dy}{dx} = 4x - 10$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 2$ (c.a.o.) A1

Use of gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal at P: $y - (-5) = -{}^1/_2(x - 3)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and both m1's awarded) A1

(*b*) An attempt to put candidate's expression for $\frac{dy}{dx} = 0$ M1

x-coordinate of $Q = 2\cdot5$

(f.t. one error in candidate's expression for $\frac{dy}{dx}$) A1

4. (*a*) $2(x - 4)^2 - 40$ B1 B1 B1

(*b*) least value $= -20$ (f.t. candidate's value for c) B1

x-coordinate $= 4$ (f.t. candidate's value for b) B1

5. (*a*) $(1 + 2x)^7 = 1 + 14x + 84x^2 . . .$ B1 B1 B1

(*b*) $(1 - 4x)(1 + 2x)^7 = 1 - 4x + 14x - 56x^2 + 84x^2$

Constant term and terms in x B1

Terms in x^2 B1

(f.t. candidate's expression in (*a*))

$(1 - 4x)(1 + 2x)^7 = 1 + 10x + 28x^2$ (c.a.o.) B1

6. (*a*) (i) An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = (4k + 1)^2 - 4 \times (k + 1) \times (k - 5)$ A1

Putting $b^2 - 4ac = 0$ m1

$4k^2 + 8k + 7 = 0$ (convincing) A1

(ii) An expression for $b^2 - 4ac$, with at least two of a, b, c correct (M1)

(to be awarded only if corresponding M1 is not awarded in part (i))

$b^2 - 4ac = 64 - 112$ $(= -48)$ A1

$b^2 - 4ac < 0 \Rightarrow$ no real roots A1

Note: Total mark for part (*a*) is 6 marks

(*b*) Finding critical values $x = -{}^{3}/_{4}$, $x = 3$ B1

A statement (mathematical or otherwise) to the effect that

$x \le -{}^{3}/_{4}$ or $3 \le x$ (or equivalent)

(f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = 5(x + \delta x)^2 + 8(x + \delta x) - 11$ B1

Subtracting y from above to find δy M1

$\delta y = 10x\delta x + 5(\delta x)^2 + 8\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 10x + 8$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 6 \times \frac{2}{3} \times x^{-1/3} + 5 \times -2 \times x^{-3}$ (completely correct answer) B2

(**If B2 not awarded**, award B1 for at least one correct non-zero term)

8. Attempting to find $f(r) = 0$ for some value of r M1

$f(-1) = 0 \Rightarrow x + 1$ is a factor A1

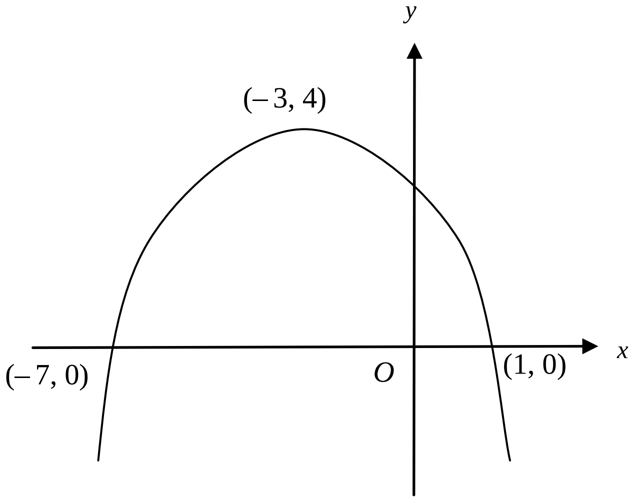
$f(x) = (x + 1)(8x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x + 1)(8x^2 - 10x + 3)$ A1

$f(x) = (x + 1)(2x - 1)(4x - 3)$ (f.t. only $8x^2 + 10x + 3$ in above line) A1

$x = -1, {}^{1}/_{2}, {}^{3}/_{4}$ (f.t. for factors $2x \pm 1$, $4x \pm 3$) A1

9. (*a*)

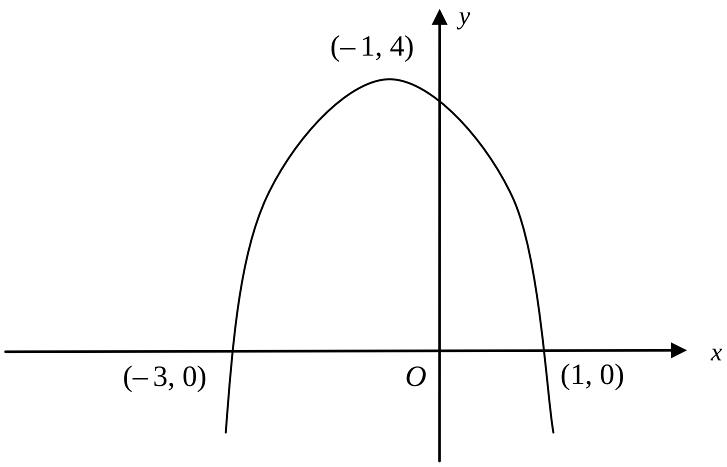


Concave down curve with y-coordinate of maximum = 4 B1

x-coordinate of maximum = – 3 B1

Both points of intersection with x-axis B1

(*b*)



Concave down curve with y-coordinate of maximum = 4 B1

x-coordinate of maximum = – 1 B1

Both points of intersection with x-axis B1

Note: A candidate who draws a curve with no changes to the original graph is awarded 0 marks (both parts)

10. (*a*) (i) $(2x \times x) + (2x \times x) + (2x \times y) + (2x \times y) + (x \times y) + (x \times y)$
$= 108$ M1

$6xy + 4x^2 = 108 \Rightarrow xy = 18 - \frac{2x^2}{3}$ (convincing) A1

(ii) $V = 2x \times x \times y = 2x(xy) \Rightarrow V = 36x - \frac{4x^3}{3}$ (convincing) B1

(*b*) $\frac{dV}{dx} = 36 - 3 \times \frac{4x^2}{3}$ B1

Putting derived $\frac{dV}{dx} = 0$ M1

$x = 3, (-3)$ (f.t. candidate's $\frac{dV}{dx}$) A1

Stationary value of V at $x = 3$ is 72 (c.a.o) A1

A correct method for finding nature of the stationary point yielding a maximum value (for $0 < x$) B1

C2

1.

0	0·5		
0·5	0·470588235		
1	0·333333333		
1·5	0·186046511		
2	0·1	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$I \approx \frac{0{\cdot}5}{2} \times \{0{\cdot}5 + 0{\cdot}1 + 2(0{\cdot}470588235 + 0{\cdot}333333333 + 0{\cdot}186046511)\}$

$I \approx 2{\cdot}579936152 \times 0{\cdot}5 \div 2$

$I \approx 0{\cdot}644984038$

$I \approx 0{\cdot}645$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}4$

0	0·5		
0·4	0·484496124		
0·8	0·398089172		
1·2	0·268240343		
1·6	0·164041994		
2	0·1	(all values correct)	B1

Correct formula with $h = 0{\cdot}4$ M1

$I \approx \frac{0{\cdot}4}{2} \times \{0{\cdot}5 + 0{\cdot}1 + 2(0{\cdot}484496124 + 0{\cdot}398089172 + 0{\cdot}268240343 + 0{\cdot}164041994)\}$

$I \approx 3{\cdot}229735266 \times 0{\cdot}4 \div 2$

$I \approx 0{\cdot}645947053$

$I \approx 0{\cdot}646$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) (i) Correct use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (o.e.) M1

Correct use of $\cos^2\theta = 1 - \sin^2\theta$ M1

$6(1 - \sin^2\theta) + 5\sin\theta = 0 \Rightarrow 6\sin^2\theta - 5\sin\theta - 6 = 0$

(convincing) A1

(ii) An attempt to solve given quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c = 6$ and $b \times d = -6$ M1

$6\sin^2\theta - 5\sin\theta - 6 = 0 \Rightarrow (3\sin\theta + 2)(2\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{2}{3}$, ($\sin\theta = {}^3/_2$) (c.a.o.) A1

$\theta = 221{\cdot}81°, 318{\cdot}19°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $3\sin\theta + 2 = 0$, ignore roots outside range.

$\sin\theta = -$, f.t. for 2 marks, $\sin\theta = +$, f.t. for 1 mark

(*b*) $2x - 60° = -38°, 38°, 322°$ (one value) B1

$x = 11°, 49°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) Either: $(x + 2)^2 = x^2 + (x - 2)^2 - 2 \times x \times (x - 2) \times \cos B\hat{A}C$

Or: $\cos B\hat{A}C = \dfrac{x^2 + (x - 2)^2 - (x + 2)^2}{2 \times x \times (x - 2)}$

(substituting the correct expressions in the correct places in the cos rule) M1

Either: $\cos B\hat{A}C = \dfrac{x^2 + x^2 - 4x + 4 - x^2 - 4x - 4}{2 \times x \times (x - 2)}$ (o.e.)

Or: $\cos B\hat{A}C = \dfrac{x^2 + x^2 - 4x + 4 - x^2 - 4x - 4}{2x^2 - 4x}$ (o.e.) A1

$\cos B\hat{A}C = \dfrac{x - 8}{2x - 4}$ (convincing) A1

(*b*) (i) $\dfrac{x - 8}{2x - 4} = -\dfrac{1}{2}$ M1

$x = 5$ A1

(ii) **Either:**

$\dfrac{\sin ABC}{3} = \dfrac{\sin 120°}{7}$

(substituting the correct values in the correct places in the sin rule, f.t. candidate's value for x, provided $x > 2$) M1

$ABC = 21{\cdot}8\,°$

(f.t. candidate's value for x, provided $x > 2$) A1

Or:

$3^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos ABC$

(substituting the correct values in the correct places in the cos rule, f.t. candidate's value for x, provided $x > 2$) M1

$ABC = 21{\cdot}8\,°$

(f.t. candidate's value for x, provided $x > 2$) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

(at least three terms, including those derived from the first pair and the last pair plus one other pair of terms)

Or:

$2S_n = [a + a + (n - 1)d] + \ldots$ (n times) M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \dfrac{n}{2}[2a + (n - 1)d]$ (convincing) A1 (*b*) **Either:**

$\frac{10}{2}(2a + 9d) = 115$ B1

$S_{14} = 115 + 130$ M1

$\frac{14}{2}(2a + 13d) = 245$ A1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$a = -2$, $d = 3$ (both values) (c.a.o.) A1

Or:

$\frac{10}{2}(2a + 9d) = 115$ B1

$(a + 10d) + (a + 11d) + (a + 12d) + (a + 13d) = 130$ M1

$4a + 46d = 130$ (seen or implied by later work) A1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$a = -2$, $d = 3$ (both values) (c.a.o.) A1

5. (*a*) $r = 0{\cdot}8$ B1

$S_{18} = \frac{100(1 - 0{\cdot}8^{18})}{1 - 0{\cdot}8}$ M1

$S_{18} \approx 490{\cdot}992 = 491$ (c.a.o.) A1

(*b*) (i) $ar = -20$ B1

$\frac{a}{1 - r} = 64$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$16r^2 - 16r - 5 = 0$ (convincing) A1

(ii) $r = -^1/_4$ (c.a.o.) B1

$|r| < 1$ E1

6. (*a*) $\dfrac{x^{5/4}}{5/4} + 2 \times \dfrac{x^{-4}}{-4} + c$ B1,B1

(– 1 if no constant present)

(*b*) (i) $x^2 + 3 = 4x$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 3$ m1

$(x - 1)(x - 3) = 0 \Rightarrow x = 1, x = 3$ (both values, c.a.o) A1

Note: Answer only with no working earns 0 marks

(ii) Area of small triangle = 2

(any method, f.t. candidate's value for x_A) B1

Use of integration to find the area under the curve M1

$\int x^2\,dx = (1/3)x^3$, $\int 3\,dx = 3x$ (correct integration) B1

Correct method of substitution of candidate's limits m1

$$\left[(1/3)x^3 + 3x\right]_1^3 = (9 + 9) - (1/3 + 3) = 44/3$$

Use of candidate's values for x_A and x_B as limits and trying to find total area by adding area under curve to area of triangle

m1

Shaded area = 44/3 + 2 = 50/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indices) B1

$\log_a xy = p + q$ (the relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) **Either:**

$(2 - 3x) \log_{10} 5 = \log_{10} 8$

(taking logs on both sides and using the power law) M1

$x = \dfrac{2 \log_{10} 5 - \log_{10} 8}{3 \log_{10} 5}$ A1

$x = 0{\cdot}236$ (f.t. one slip, see below) A1

Or:

$2 - 3x = \log_5 8$ (rewriting as a log equation) M1

$x = \dfrac{2 - \log_5 8}{3}$ A1

$x = 0{\cdot}236$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}236$ from $x = \dfrac{\log_{10} 8 - 2 \log_{10} 5}{3 \log_{10} 5}$

earns M1 A0 A1

an answer of $x = 1{\cdot}097$ from $x = \dfrac{2 \log_{10} 5 + \log_{10} 8}{3 \log_{10} 5}$

earns M1 A0 A1

an answer of $x = 0{\cdot}708$ from $x = \dfrac{2 \log_{10} 5 - \log_{10} 8}{\log_{10} 5}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) $\frac{1}{2}\log_a 144x^8 = \log_a 12x^4$ (power law) B1

$\log_a 90x^2 - \log_a\left(\frac{5}{x}\right) = \log_a\left(\frac{90x^2 \times x}{5}\right)$ (subtraction law) B1

$\frac{90x^2 \times x}{5} = 12x^4$ (removing logs, f.t one incorrect term) B1

$x = 1{\cdot}5$ (c.a.o.) B1

8. (*a*) $A(-1, 3)$ B1

A correct method for finding the radius M1

Radius = 5 A1

(*b*) (i) Showing that the coordinates of *A* do not satisfy the equation of *L* (f.t. candidate's coordinates for *A*) B1

(ii) An attempt to substitute $(9 - x)$ for y in the equation of C_1 M1

$x^2 - 5x + 6 = 0$ (or $2x^2 - 10x + 12 = 0$) A1

$x = 2, x = 3$

(correctly solving candidate's quadratic, both values) A1

Points of intersection are (2, 7), (3, 6) (c.a.o.) A1

(*c*) Distance between centres of C_1 and C_2 = 13

(f.t. candidate's coordinates for *A*) B1

Use of the fact that the shortest distance between the circles = distance between centres – sum of the radii M1

Shortest distance between the circles = 2

(f.t. candidate's coordinates for *A* and radius for C_1.) A1

9. (*a*) Substitution of values in area formula for triangle M1

Area = $\frac{1}{2} \times 7{\cdot}2^2 \times \sin 1{\cdot}1 = 23{\cdot}1 \text{ cm}^2$. A1

(*b*) Let $B\hat{O}C = \phi$ radians

$\frac{1}{2} \times 7{\cdot}2^2 \times \phi = 19{\cdot}44$ M1

$\phi = 0{\cdot}75$ (o.e.) A1

Length of arc $BC = 7{\cdot}2 \times 0{\cdot}75 = 5{\cdot}4$ cm

(f.t. candidate's value for ϕ) A1

C3

1. (*a*)

1	1·945910149
1·5	2·238046572
2	2·63905733
2·5	3·073850053
3	3·496507561

(5 values correct) B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$I \approx \frac{0·5}{3} \times \{1·945910149 + 3·496507561 + 4(2·238046572 + 3·073850053) + 2(2·63905733)\}$

$I \approx 31·96811887 \times 0·5 \div 3$

$I \approx 5·328019812$

$I \approx 5·328$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\int_1^3 \ln \sqrt{(x^3 + 6)}\, dx \approx 2·664$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $4(\text{cosec}^2\theta - 1) - 8 = 2\,\text{cosec}^2\theta - 5\,\text{cosec}\,\theta$

(correct use of $\cot^2\theta = \text{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\,\text{cosec}\,\theta + b)(c\,\text{cosec}\,\theta + d)$, with $a \times c$ = coefficient of $\text{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$2\,\text{cosec}^2\theta + 5\,\text{cosec}\,\theta - 12 = 0 \Rightarrow (2\,\text{cosec}\,\theta - 3)(\text{cosec}\,\theta + 4) = 0$

$\Rightarrow \text{cosec}\,\theta = \frac{3}{2}, \text{cosec}\,\theta = -4$

$\Rightarrow \sin\theta = \frac{2}{3}, \sin\theta = -\frac{1}{4}$ (c.a.o.) A1

$\theta = 41·81°, 138·19°$ B1

$\theta = 194·48°, 345·52°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks

$\sin\theta = +, +$, f.t. for 1 mark

(*b*) Correct use of $\sec\phi = \frac{1}{\cos\phi}$ and $\tan\phi = \frac{\sin\phi}{\cos\phi}$ (o.e.) M1

$\sin\phi = -\frac{1}{2}$ A1

$\phi = 210°, 330°$ (f.t. for $\sin\phi = -a$) A1

3. (*a*) Use of product formula yielding $x^3 \times 2y \times \frac{dy}{dx} + 3x^2 \times y^2$ B1 B1

$\frac{dy}{dx} = -\frac{3x^2y^2}{2x^3y}$ (c.a.o.) B1

(*b*) (i) Putting candidate's expression for $\frac{dy}{dx} = 3$ and an attempt to simplify M1

$-\frac{3a^2b^2}{2a^3b} = 3 \Rightarrow b = -2a$ (convincing) A1

(ii) Substituting a for x and $-2a$ for y in the equation for C M1

$a = 2, b = -4$ A1

4. (*a*) Differentiating $\ln t$ and $5t^4$ with respect to t, at least one correct M1

candidate's x-derivative $= \frac{1}{t}$,

candidate's y-derivative $= 20t^3$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = 20t^4$ (c.a.o.) A1

(*b*) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 80t^3$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = 80t^4$ (f.t. one slip) A1

$\frac{d^2y}{dx^2} = 0{\cdot}648 \Rightarrow t = 0{\cdot}3$ (c.a.o.) A1

5. (*a*) $\frac{dy}{dx} = 5 \times (7 - 9x^2)^4 \times f(x)$, $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = -90x \times (7 - 9x^2)^4$ A1

(*b*) $\frac{dy}{dx} = \boldsymbol{\frac{6}{1 + (6x)^2}}$ **or** $\frac{1}{1 + (6x)^2}$ **or** $\frac{6}{1 + 6x^2}$ M1

$\frac{dy}{dx} = \frac{6}{1 + 36x^2}$ A1

(*c*) $\frac{dy}{dx} = e^{4x} \times m \sec^2 2x + \tan 2x \times ke^{4x}$ ($m = 1$, **2**, $k = 1$, **4**) M1

$\frac{dy}{dx} = e^{4x} \times 2\sec^2 2x + \tan 2x \times 4e^{4x}$ (at least one correct term) B1

$\frac{dy}{dx} = e^{4x} \times 2\sec^2 2x + \tan 2x \times 4e^{4x}$ (c.a.o.) A1 (*d*) $\frac{dy}{dx} = \frac{(2 + \cos x) \times m \cos x - (3 + \sin x) \times k \sin x}{(2 + \cos x)^2}$ (m = **1**, – 1 k = 1, **– 1**) M1

$\frac{dy}{dx} = \frac{(2 + \cos x) \times (\cos x) - (3 + \sin x) \times (-\sin x)}{(2 + \cos x)^2}$ A1

$\frac{dy}{dx} = \frac{2 \cos x + 3 \sin x + 1}{(2 + \cos x)^2}$ A1

6. (*a*) (i) $\int \cos(3x + \pi/2)\, dx = k \times \sin(3x + \pi/2) + c$

(k = 1, 3, $\mathbf{^1/_3}$, – $^1/_3$) M1

$\int \cos(3x + \pi/2)\, dx = {}^1/_3 \times \sin(3x + \pi/2) + c$ A1

(ii) $\int e^{3-4x} dx = k \times e^{3-4x} + c$ (k = 1, – 4, $^1/_4$, **–** $\mathbf{^1/_4}$) M1

$\int e^{3-4x} dx = -{}^1/_4 \times e^{3-4x} + c$ A1

(iii) $\int \frac{7}{8x + 5}\, dx = 7 \times k \times \ln|8x + 5| + c$ (k = 1, 8, $\mathbf{^1/_8}$) M1

$\int \frac{7}{8x + 5}\, dx = 7 \times {}^1/_8 \times \ln|8x + 5| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (2x - 1)^{-4} dx = k \times \frac{(2x - 1)^{-3}}{-3}$ (k = 1, 2, $\mathbf{^1/_2}$) M1

$\int_1^2 9 \times (2x - 1)^{-4} dx = \left[9 \times {}^1/_2 \times \frac{(2x - 1)^{-3}}{-3} \right]_1^2$ A1

Correct method for substitution of limits in an expression of the form $m \times (2x - 1)^{-3}$
M1

$\int_1^2 9 \times (2x - 1)^{-4} dx = \frac{13}{9} = 1{\cdot}44$ (f.t. for k = 1, 2 only) A1

Note: Answer only with no working earns 0 marks

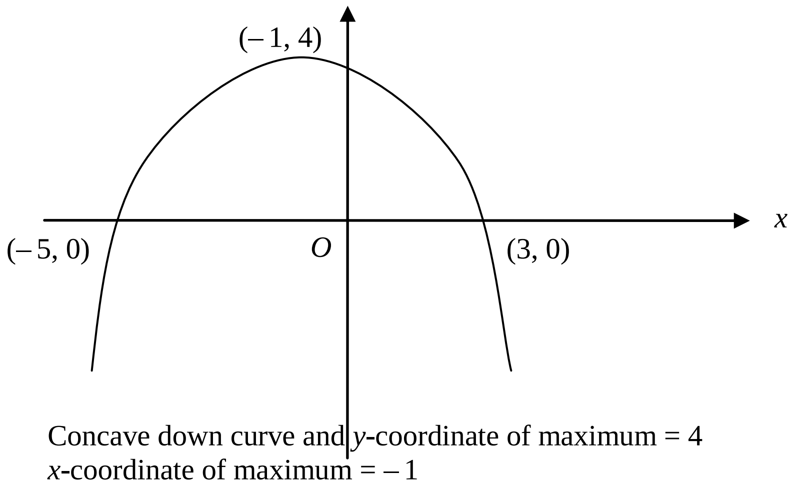
7. (*a*) Choice of $a \neq -1$ and $b = -a - 2$ M1
Correct verification that given equation is satisfied A1

(*b*) Trying to solve either $x^2 - 10 \leq 6$ or $x^2 - 10 \geq -6$ M1
$x^2 - 10 \leq 6 \Rightarrow x^2 \leq 16$
$x^2 - 10 \geq -6 \Rightarrow x^2 \geq 4$ (both inequalities) A1
At least one of: $2 \leq x \leq 4$, $-4 \leq x \leq -2$ (f.t. one slip) A1
Required range: $2 \leq x \leq 4$ or $-4 \leq x \leq -2$ (c.a.o.) A1

Alternative mark scheme
$(x^2 - 10)^2 \leq 36$ (forming and trying to solve quadratic in x^2)M1
Critical values $x^2 = 4$ and $x^2 = 16$ A1
At least one of: $2 \leq x \leq 4$, $-4 \leq x \leq -2$ (f.t. one slip) A1
Required range: $2 \leq x \leq 4$ or $-4 \leq x \leq -2$ (c.a.o.) A1

8. $x_0 = -1{\cdot}5$
$x_1 = -1{\cdot}666394263$ (x_1 correct, at least 5 places after the point) B1
$x_2 = -1{\cdot}676625462$
$x_3 = -1{\cdot}677198866$
$x_4 = -1{\cdot}677230823 = -1{\cdot}67723$ (x_4 correct to 5 decimal places) B1
Let $f(x) = x^2 + e^x - 3$
An attempt to check values or signs of $f(x)$ at $x = -1{\cdot}677225$, $x = -1{\cdot}677235$
M1
$f(-1{\cdot}677225) = -2{\cdot}44 \times 10^{-5} < 0$, $f(-1{\cdot}677235) = 7{\cdot}26 \times 10^{-6} > 0$ A1
Change of sign $\Rightarrow \alpha = -1{\cdot}67723$ correct to five decimal places A1

9.



Concave down curve and y-coordinate of maximum = 4 B1
x-coordinate of maximum = – 1 B1
Both points of intersection with x-axis B1

10. (*a*) $y - 6 = e^{5 - x/2}$. B1

An attempt to express equation as a logarithmic equation and to isolate x M1

$x = 2\,[5 - \ln (y - 6)]$ (c.a.o.) A1

$f^{-1}(x) = 2\,[5 - \ln (x - 6)]$

(f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = [7, \infty)$ B1 B1

11. (*a*) (i) $D(fg) = (0, \pi/4]$ B1

(ii) $R(fg) = (-\infty, 0]$ B1 B1

(*b*) (i) $fg(x) = -0{\cdot}4 \Rightarrow \tan x = e^{-0{\cdot}4}$ M1

$x = 0{\cdot}59$ A1

(ii) Equation has solution only if $k \in R(fg)$.

$\therefore$ choose any $k \notin R(fg)$ (f.t. candidate's $R(fg)$) B1

C4

1. (*a*) $f(x) \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(x+2)}$ (correct form) M1

$6 + x - 9x^2 \equiv A(x+2) + Bx(x+2) + Cx^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3$, $C = -8$, $B = -1$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) (i) $f'(x) = \frac{-6}{x^3} + \frac{1}{x^2} + \frac{8}{(x+2)^2}$ (o.e.)

(f.t. candidate's values for A, B, C)

(first term) B1

(at least one of last two terms) B1

(ii) $f'(2) = 0 \Rightarrow$ stationary value when $x = 2$ (c.a.o.) B1

2. $3x^2 - 2x \times 2y\frac{dy}{dx} - 2y^2 + 3y^2\frac{dy}{dx} = 0$ $\left(-2x \times 2y\frac{dy}{dx} - 2y^2\right)$ B1

$\left(3x^2, 3y^2\frac{dy}{dx}\right)$ B1

Either $\frac{dy}{dx} = \frac{2y^2 - 3x^2}{3y^2 - 4xy}$ **or** $\frac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 1 = -\frac{1}{2}(x - 2)$ $\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

3. (*a*) $8(2\cos^2\theta - 1) + 6 = \cos^2\theta + \cos\theta$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - \cos\theta - 2 = 0 \Rightarrow (5\cos\theta - 2)(3\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{5}$, $\cos\theta = -\frac{1}{3}$, (c.a.o.) A1

$\theta = 66{\cdot}42°, 293{\cdot}58°$ B1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks

$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $R = 4$ B1

Correctly expanding $\cos(\theta + \alpha)$, correctly comparing coefficients and using either $4\cos\alpha = \sqrt{15}$ **or** $4\sin\alpha = 1$ **or** $\tan\alpha = \frac{1}{\sqrt{15}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 14{\cdot}48°$ (c.a.o.) A1

$\cos(\theta + 14{\cdot}48°) = \frac{3}{4} = 0{\cdot}75$

(f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1

$\theta + 14{\cdot}48° = 41{\cdot}41°, 318{\cdot}59°$

(at least one value, f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1

$\theta = 26{\cdot}93°, 304{\cdot}11°$ (c.a.o.) B1

4.

Volume $= \pi \int_{\pi/6}^{\pi/2} \sin^2 2x \, dx$ B1

$\sin^2 2x = \frac{(1 - \cos 4x)}{2}$ B1

$\int (a + b\cos 4x)\, dx = ax + \frac{1}{4} b \sin 4x$, $a \neq 0, b \neq 0$ B1

Correct substitution of candidate's limits in candidate's integrated expression of form $mx + n\sin 4x$ $m \neq 0, n \neq 0$ M1

Volume = $1{\cdot}985$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. (*a*) (i) $(1 + 6x)^{1/3} = 1 + 2x - 4x^2$ $(1 + 2x)$ B1
$(-4x^2)$ B1

(ii) $|x| < \frac{1}{6}$ or $-\frac{1}{6} < x < \frac{1}{6}$ B1

(*b*) $2 + 4x - 8x^2 = 2x^2 - 15x \Rightarrow 10x^2 - 19x - 2 = 0$ M1

(An attempt to set up and use a correct method to solve quadratic using candidate's expansion for $(1 + 6x)^{1/3}$)

$x = -0{\cdot}1$ (f.t. only candidate's range for x in (*a*)) A1

6. (*a*) candidate's x-derivative = a

candidate's y-derivative = $-\dfrac{b}{t^2}$ (at least one term correct) B1

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = -\dfrac{b}{at^2}$ (c.a.o.) A1

Tangent at P: $y - \dfrac{b}{p} = -\dfrac{b}{ap^2}(x - ap)$ (o.e.)

(f.t. candidate's expression for $\dfrac{dy}{dx}$) M1

$ap^2y - abp = -bx + abp$

$ap^2y + bx - 2abp = 0.$ (convincing) A1

(*b*) $y = 0 \Rightarrow x = 2ap$ (o.e.) B1

$x = 0 \Rightarrow y = 2b/p$ (o.e.) B1

Area of triangle $AOB = 2ab$ (c.a.o.) B1

(*c*) $p^2 - 2p + 2 = 0$ $(abp^2 - 2abp + 2ab = 0)$ B1

Attempting **either** to use the formula to solve the candidate's quadratic in p **or** to find the discriminant of the candidate's quadratic **or** to complete the square M1

Either discriminant < 0 ($\Rightarrow$ no real roots) $\Rightarrow$ no such P can exist **or** $(p - 1)^2 + 1 = 0$ ($\Rightarrow (p - 1)^2 = -1$)) $\Rightarrow$ no such P can exist

(c.a.o.) A1

7. (*a*) $u = 3x - 1 \Rightarrow du = 3dx$ (o.e.) B1

$dv = \cos 2x\,dx \Rightarrow v = \dfrac{1}{2}\sin 2x$ (o.e.) B1

$\int (3x - 1)\cos 2x\,dx = (3x - 1) \times \dfrac{1}{2}\sin 2x - \int \dfrac{1}{2}\sin 2x \times 3dx$ M1

$\int (3x - 1)\cos 2x\,dx = \dfrac{1}{2}(3x - 1)\sin 2x + \dfrac{3}{4}\cos 2x + c$ (c.a.o.) A1 (*b*) $\displaystyle\int \frac{x}{(2x+1)^3}\,dx = \int \frac{f(u)}{u^3} \times k\,du$

($f(u) = pu + q$, $p \neq 0$, $q \neq 0$ and $k = {}^1/_2$ or 2) M1

$\displaystyle\int \frac{x}{(2x+1)^3}\,dx = \int \frac{(u-1)}{2} \times \frac{1}{u^3} \times \frac{du}{2}$ A1

$\displaystyle\int (au^{-2} + bu^{-3})\,du = \frac{au^{-1}}{-1} + \frac{bu^{-2}}{-2}$ ($a \neq 0$, $b \neq 0$) B1

Either: Correctly inserting limits of 1, 3 in candidate's $cu^{-1} + du^{-2}$
($c \neq 0$, $d \neq 0$)

or: Correctly inserting limits of 0, 1 in candidate's
$c(2x+1)^{-1} + d(2x+1)^{-2}$ ($c \neq 0$, $d \neq 0$) m1

$\displaystyle\int_0^1 \frac{x}{(2x+1)^3}\,dx = \frac{1}{18}$ (= 0·055 . . .) (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $\dfrac{dA}{dt} = k\sqrt{A}$ B1

(*b*) $\displaystyle\int \frac{dA}{\sqrt{A}} = \int k\,dt$ M1

$\dfrac{A^{1/2}}{{}^1/_2} = kt + c$ A1

Substituting 64 for A and 3 for t and 196 for A and 5·5 for t in candidate's derived equation m1

$16 = 3k + c$, $28 = 5{\cdot}5k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c m1

$A = (2{\cdot}4t + 0{\cdot}8)^2$ (o.e.) (c.a.o.) A1

9. (*a*) $\mathbf{AB} = 8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$ B1

(*b*) $\mathbf{OC} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + {}^3/_4(8\mathbf{i} - 4\mathbf{j} + 12\mathbf{k})$ (o.e.) M1

$\mathbf{OC} = 5\mathbf{i} + 2\mathbf{k}$ A1

(*c*) (i) Use of $\mathbf{OA} + \mu(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ on r.h.s. M1

$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \mu(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ (all correct) A1

(ii) $-1 + \lambda \times (-4) = 7$
(an equation in λ from one set of coefficients) M1

$\lambda = -2$ A1

$1 + (-2) \times 1 = -1$

$11 + (-2) \times 3 = 5$ (both verifications) A1

An attempt to evaluate $\mathbf{AB}.(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ M1

$\mathbf{AB}.(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 0$ (convincing) A1

B lies on L, AB is perpendicular to L and thus B is the foot of the perpendicular from A to L (c.a.o.) A1

10. Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2.$$

$25x^2 - 30x + 9 + 1 < 9x^2 - 6x + 1 \Rightarrow 16x^2 - 24x + 9 < 0$ B1

$(4x - 3)^2 < 0$ B1

This contradicts the fact that x is real and thus $(5x - 3)^2 + 1 \geq (3x - 1)^2$. B1

Ques	Solution	Mark	Notes
1	$S_n = \sum_{r=1}^{n}(2r-1)^2 = \sum_{r=1}^{n}4r^2 - \sum_{r=1}^{n}4r + \sum_{r=1}^{n}1$	M1A1	M1A0 for 2 correct terms
	$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$	A1A1	Award A1 for 2 correct
	$= \frac{n}{6}\left(8n^2 + 12n + 4 - 12n - 12 + 6\right)$	A1	FT line above if at least 2 terms present
	$= \frac{4n^3}{3} - \frac{n}{3}$ cao	A1	Penalise 1 mark if n used as dummy variable in summations
2(a)	EITHER $\frac{1}{w} = \frac{1}{1-\mathrm{i}} + \frac{1}{1+2\mathrm{i}}$		
	$= \frac{1+2\mathrm{i}+1-\mathrm{i}}{(1-\mathrm{i})(1+2\mathrm{i})}$	M1A1	
	$= \frac{2+\mathrm{i}}{3+\mathrm{i}}$	A1	
	$w = \frac{3+\mathrm{i}}{2+\mathrm{i}} \times \frac{2-\mathrm{i}}{2-\mathrm{i}}$	M1	
	$= \frac{7-\mathrm{i}}{5}$	A1A1	1 each for num and denom
	OR $\frac{1}{1-\mathrm{i}} = \frac{1+\mathrm{i}}{1-\mathrm{i}^2} = \frac{1+\mathrm{i}}{2}$	M1A1	
	$\frac{1}{1+2\mathrm{i}} = \frac{1-2\mathrm{i}}{1-4\mathrm{i}^2} = \frac{1-2\mathrm{i}}{5}$	A1	
	$\frac{1}{w} = \frac{5+5\mathrm{i}+2-4\mathrm{i}}{10} = \frac{7+\mathrm{i}}{10}$	A1	
	$w = \frac{10}{7+\mathrm{i}} \times \frac{7-\mathrm{i}}{7-\mathrm{i}}$	M1	
	$= \frac{7-\mathrm{i}}{5}$	A1	1 each for num and denom
(b)	$\mathrm{Mod}(w) = \frac{\sqrt{50}}{5}$ $(\sqrt{2})$	B1	FT on their w
	$\mathrm{Arg}(w) = -0.142$ $(-8.13°)$	B1	Accept 351.9° or 6.14 Do not FT arg if in 1st quadrant

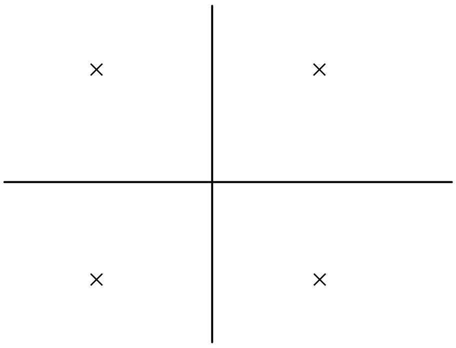
3(a)	$\alpha+\beta+\gamma=2, \beta\gamma+\gamma\alpha+\alpha\beta=2, \alpha\beta\gamma=-1$	**B1**	
	$\frac{\beta\gamma}{\alpha}+\frac{\gamma\alpha}{\beta}+\frac{\alpha\beta}{\gamma}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha\beta\gamma}$	**M1**	
	$=\frac{(\beta\gamma+\gamma\alpha+\alpha\beta)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$	**A1**	
	$=\frac{(2)^2-2\times(-1)\times 2}{-1}=-8$	**A1**	Convincing
(b)	Consider		
	$\frac{\gamma\alpha}{\beta}\times\frac{\alpha\beta}{\gamma}+\frac{\alpha\beta}{\gamma}\times\frac{\beta\gamma}{\alpha}+\frac{\beta\gamma}{\alpha}\times\frac{\gamma\alpha}{\beta}$	**M1**	
	$=\alpha^2+\beta^2+\gamma^2$	**A1**	
	$=(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)$	**A1**	
	$= 4 - 2 \times 2 = 0$	**A1**	
	Consider		
	$\frac{\beta\gamma}{\alpha}\times\frac{\gamma\alpha}{\beta}\times\frac{\alpha\beta}{\gamma}=\alpha\beta\gamma=-1$	**M1A1**	
	The required equation is		
	$x^3+8x^2+1=0$	**B1**	FT their coefficients

4(a)	Rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Translation matrix = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Ref matrix in $y = x$ = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$	**M1**	
	$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	**A1**	
	$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$		
(b)	Fixed points satisfy $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	**M1**	
	$x = x + 1$, ($y = -y + 2$)	**A1**	
	These equations are not consistent so there are no fixed points.	**A1**	Accept equivalent reason
5	Putting n = 1, the formula gives 6 which is divisible by 6 so the result is true for n = 1	**B1**	
	Assume formula is true for $n = k$, ie $7^k - 1$ is divisible by 6 or $7^k = 6N + 1$	**M1**	
	Consider, for $n = k + 1$, $7^{k+1} - 1 = 7.7^k - 1$	**M1**	
	$= 7(6N + 1) - 1$	**A1**	
	$= 42N + 6$	**A1**	
	This is divisible by 6 therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**A1**	

6(a)(i)	Det(**A**) = 7 – $4\lambda + \lambda(5\lambda - 14) + 3(8-5)$ $= 5\lambda^2 - 18\lambda + 16$	**M1** **A1**	
(ii)	Putting $\lambda = 2$, det = 20 – 36 + 16 = 0 So **A** is singular. Putting det(**A**) = 0, product of roots is 16/5 So the other root is 8/5	**B1** **B1**	
(b)(i)	$x + 2y + 3z = 2$ $2x + y + 2z = 1$ $5x + 4y + 7z = 4$		
	Attempting to use row operations, $x + 2y + 3z = 2$ $3y + 4z = 3$ $6y + 8z = 6$ Since the 3rd equation is twice the 2nd equation, it follows that the equations are consistent.	**M1** **A1** **A1** **A1**	Or because the next step gives a row of zeroes
(ii)	Let $z = \alpha$ $y = 1 - \frac{4}{3}\alpha$ $x = -\frac{1}{3}\alpha$ (or equivalent)	**M1** **A1** **A1**	
(c)(i)	**A** = $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ 5 & 4 & 7 \end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix} 3 & -9 & 3 \\ 5 & -8 & 1 \\ -2 & 5 & -1 \end{bmatrix}$ si	**M1A1**	Award M1 if at least 5 correct elements
	Adjugate matrix = $\begin{bmatrix} 3 & 5 & -2 \\ -9 & -8 & 5 \\ 3 & 1 & -1 \end{bmatrix}$	**A1**	No FT from incorrect cofactor matrix
(ii)	Determinant = 3	**B1**	
	Inverse matrix = $\frac{1}{3}\begin{bmatrix} 3 & 5 & -2 \\ -9 & -8 & 5 \\ 3 & 1 & -1 \end{bmatrix}$	**B1**	FT from incorrect adjugate
(iii)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 & 5 & -2 \\ -9 & -8 & 5 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$	**M1** **A1**	FT from inverse matrix

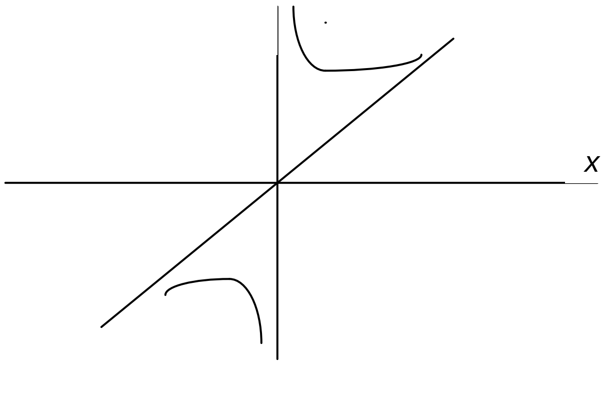
	$= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$		
7	Taking logs, $\ln f(x) = \ln\sqrt{1+\sin x} - \ln(1+\tan x)^2$ $= \frac{1}{2}\ln(1+\sin x) - 2\ln(1+\tan x)$ Differentiating, $\frac{f'(x)}{f(x)} = \frac{\cos x}{2(1+\sin x)} - \frac{2\sec^2 x}{(1+\tan x)}$ Putting $x = \pi/4$, $f'(\pi/4) = -0.586$ cao	**M1A1** **A1** **B3** **M1** **A2**	B1 for each correct term
8(a) **(b)**	$u + \mathrm{i}v = (x + \mathrm{i}y)^2$ $= x^2 - y^2 + 2\mathrm{i}xy$ Equating real and imaginary parts, $u = x^2 - y^2$ $v = 2xy$ Substituting for y, $u = x^2 - (2x^2 + 1) = -1 - x^2$ $v^2 = 4x^2(2x^2 + 1)$ Eliminating x, $x^2 = -(u+1)$ So that $v^2 = 4(u+1)(2u+1)$ cao	**M1** **A1** **M1** **A1** **M1** **A1** **A1** **M1** **A1**	 FT their expressions from (a)

Ques	Solution	Mark	Notes
1	$u = x^2 \Rightarrow \mathrm{d}u = 2x\mathrm{d}x$,	B1	
	$[1,2] \to [1, 4]$	B1	
	$I = \frac{1}{2}\int_1^4 \frac{\mathrm{d}u}{\sqrt{25-u^2}}$	M1	
	$= \frac{1}{2}\left[\sin^{-1}(\frac{u}{5})\right]_1^4$	A1	
	$= 0.363$ cao	A1	
2(a)	Substituting $t = \tan(\theta/2)$		
	$\frac{2t}{1+t^2} + \frac{3(1-t^2)}{1+t^2} = 2$	M1A1	
	$2t + 3 - 3t^2 = 2 + 2t^2$		
	$5t^2 - 2t - 1 = 0$	A1	Convincing.
(b)	$t = \frac{2 \pm \sqrt{24}}{10} = 0.68989..,-0.28989..$	M1A1	
	t = 0.68989.. giving $\theta/2$ = 0.6039..	B1	FT their roots from (a)
	The general solution is θ = 1.21 + $2n\pi$	B1	
	$t = -0.28989..$giving $\theta/2 = -0.2821..$	B1	Accept 2.859…
	The general solution is θ = – 0.564 + $2n\pi$	B1	Accept 5.72 + $2n\pi$

3(a)	$-1 = \cos\pi + i\sin\pi$	**B1**	
	$\sqrt[4]{-1} = \cos\pi/4 + i\sin\pi/4 = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	**M1A1**	
	$\text{Root2} = \cos 3\pi/4 + i\sin 3\pi/4 = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$	**A1**	
	$\text{Root3} = \cos 5\pi/4 + i\sin 5\pi/4 = -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	**A1**	Special case : Award 2/6 if they misread –1 as 1.
	$\text{Root4} = \cos 7\pi/4 + i\sin 7\pi/4 = \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$	**A1**	
(b)(i)		**B1**	FT their roots if possible
(ii)	Length of side = $\frac{2}{\sqrt{2}}$	**B1**	
	Area of square = 2	**B1**	

4(a)	$f'(x) = \frac{2(x-1)-(2x+3)}{(x-1)^2}$ $= -\frac{5}{(x-1)^2}$ This is negative for all $x > 1$ therefore f is strictly decreasing.	**M1** **A1** **A1**	
(b)(i)	$f(4) = 11/3$, $f(5) = 13/4$ $f(S) = [13/4, 11/3]$	**M1** **A1**	A0 if wrong way around but penalise only once.
(ii)	EITHER $y = \frac{2x+3}{x-1} \Rightarrow x = \frac{y+3}{y-2}$ $f^{-1}(4) = 7/2,\ f^{-1}(5) = 8/3$ $f^{-1}(S) = [8/3, 7/2]$ OR $\frac{2x+3}{x-1} = 4 \to x = \frac{7}{2}$ $\frac{2x+3}{x-1} = 5 \to x = \frac{8}{3}$ $f^{-1}(S) = [8/3, 7/2]$	**M1A1** **A1** **A1** **M1A1** **A1** **A1**	A0 if wrong way around. M1A1 for the first and then A1 for the second. A0 if wrong way around.
5(a)(i)	Completing the square, $(x-2)^2 + 2(y+1)^2 = 4$ The centre is therefore (2, –1)	**M1A1** **A1**	
(ii)	In standard form, the equation is $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{2} = 1$ so $a = 2$, $b = \sqrt{2}$ si $e = \sqrt{\frac{4-2}{4}} = \frac{1}{\sqrt{2}}$	**B1** **M1A1**	FT their equation in (ii), (iii) and (iv)
(iii)	The foci are $(2+\sqrt{2}, -1)$ and $(2-\sqrt{2}, -1)$	**B1B1**	
(iv)	The equations of the directrices are $x = 2 \pm 2\sqrt{2}$	**B1**	
(b)(i)	EITHER Putting $x = 0$, $(y+1)^2 = 0$ This has a repeated root, hence $x = 0$ is a tangent OR Semi-major axis = 2 = x-coordinate of centre This equality shows that $x = 0$ is a tangent	**M1** **A1** **M1** **A1**	
(ii)	Substituting $y = mx$, $x^2(1+2m^2) - x(4-4m) + 2 = 0$ Use of the condition for tangency, ie $'b^2 = 4ac'$ $16(1-m)^2 = 8(1+2m^2)$ $2 - 4m + 2m^2 = 1 + 2m^2 \Rightarrow m = \frac{1}{4}$	**M1** **A1** **M1** **A1** **A1**	

6(a)	Let $$\frac{4x^2-2x+9}{x(x^2+3)} \equiv \frac{A}{x}+\frac{Bx+C}{x^2+3}$$ $$=\frac{A(x^2+3)+x(Bx+C)}{x(x^2+3)} \text{ (oe)}$$	**M1**	
	$x = 0$ gives $A = 3$	**A1**	
	Coeff of x^2 gives $A + B = 4$, $B = 1$	**A1**	
	Coeff of x gives $C = -2$	**A1**	
(b)	$$\int_1^3 \frac{4x^2-2x+9}{x(x^2+3)}\mathrm{d}x = \int_1^3\left(\frac{3}{x}+\frac{x}{x^2+3}-\frac{2}{x^2+3}\right)\mathrm{d}x$$	**M1**	
	$$=\left[3\ln x+\frac{1}{2}\ln(x^2+3)-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right]_1^3$$	**B3**	B1 each term
	$$=3\ln 3+\frac{1}{2}\ln 12-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{3}{\sqrt{3}}\right)$$ $$-3\ln 1-\frac{1}{2}\ln 4+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$	**A1**	
	= 3.24 cao	**A1**	

7(a)	Consider $f(-x)=\dfrac{(2(-x)^2+1)^2}{(-x)^3}=-f(x)$ Therefore f is odd	**M1A1** **A1**	
(b)	EITHER Differentiating, $f'(x)=\dfrac{2(2x^2+1).4x.x^3-3x^2(2x^2+1)^2}{x^6}$ At a stationary point, putting $f'(x)=0$, $8x^2=3(2x^2+1)$ $x=\pm\sqrt{\dfrac{3}{2}}$ OR Consider $f(x)=4x+\dfrac{4}{x}+\dfrac{1}{x^3}$ $f'(x)=4-\dfrac{4}{x^2}-\dfrac{3}{x^4}$ At a stationary point, putting $f'(x)=0$, $4x^4-4x^2-3=0$ $x=\pm\sqrt{\dfrac{3}{2}}$	**M1A1** **m1** **A1** **M1** **A1** **m1** **A1**	Condone the cancellation of $x^2(2x^2+1)$
(c)	The asymptotes are $x=0$ $y=4x$	**B1** **B1**	
(d)		**G1** **G1**	

8	EITHER Consider $\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$ Expanding and taking real parts, $\cos 5\theta = \cos^5\theta + 10\cos^3\theta(\mathrm{i}\sin\theta)^2$ $+ 5\cos\theta(\mathrm{i}\sin\theta)^4$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta$ $-10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**M1** **m1A1** **A1** **A1** **A1**	
	OR Let $z = \cos\theta + \mathrm{i}\sin\theta$ So that $z + \frac{1}{z} = 2\cos\theta$ and $z^n + \frac{1}{z^n} = 2\cos n\theta$ Consider $\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^3 + 10z + \frac{10}{z} + \frac{5}{z^3} + \frac{1}{z^5}$ $32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ $\cos 5\theta = 16\cos^5\theta - 5\cos 3\theta - 10\cos\theta$ $= 16\cos^5\theta - 5(4\cos^3\theta - 3\cos\theta) - 10\cos\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**M1** **A1** **A1** **A1** **A1** **A1**	

Ques	Solution	Mark	Notes
1	Using $\cosh 2x = 2\cosh^2 x - 1$, the eqn becomes $2\cosh^2 x - 7\cosh x + 6 = 0$ Solving the quadratic equation, $\cosh x = 2,\ 1.5$ The positive roots are therefore $x = \cosh^{-1} 2 = 1.32$ and $x = \cosh^{-1}(1.5) = 0.96$	M1 A1 M1 A1 A1 A1	 FT their roots
2(a)(i)	The Newton-Raphson iteration is $x_{n+1} = x_n - \dfrac{(x_n{}^3 - a)}{3x_n^2}$ $= \dfrac{2x_n^3 + a}{3x_n^2}$	 M1 A1	 Convincing
(ii)	$x_0 = 2$ $x_1 = 2.166666667$ $x_2 = 2.154503616$ $x_3 = 2.154434692$ $x_4 = 2.15443469$ $\sqrt[3]{10} = 2.1544$ correct to 4 decimal places.	 M1A1 A1	
(b)	Consider $\dfrac{d}{dx}\left(\dfrac{a}{x^2}\right) = -\dfrac{2a}{x^3}$ $= -2$ when $x = \sqrt[3]{a}$ The sequence diverges because this exceeds 1 in modulus.	 M1A1 A1 A1	 M0 if a =10
3(a)	$f'(x) = \dfrac{2e^x}{2e^x - 1}$ $f''(x) = \dfrac{2e^x(2e^x - 1) - 2e^x.2e^x}{(2e^x - 1)^2}$ $= \dfrac{-2e^x}{(2e^x - 1)^2}$	B1 M1 A1	 convincing
(b)	$f'''(x) = \dfrac{-2e^x(2e^x - 1)^2 + 2e^x.2e^x 2(2e^x - 1)}{(2e^x - 1)^4}$ $f(0) = 0,\ f'(0) = 2,\ f''(0) = -2,\ f'''(0) = 6$ The Maclaurin series is $2x - x^2 + x^3 + \ldots$	M1A1 B2 M1A1	 Award B1 for 2 correct values FT on their values of $f^{(n)}(0)$

4	Completing the square, $3+2x-x^2=4-(x-1)^2$ so $I=\int_1^2\sqrt{4-(x-1)^2}\,dx$ Put $x-1=2\sin\theta$ $dx = 2\cos\theta d\theta$, [1,2] $\to$ [0,π/6] $I=\int_0^{\pi/6}\sqrt{4-4\sin^2\theta}\,.2\cos\theta d\theta$ $= 4\int_0^{\pi/6}\cos^2\theta\,d\theta$ $= 2\int_0^{\pi/6}(1+\cos 2\theta)\,d\theta$ $= 2\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi/6}$ = 1.91	M1A1 M1 A1A1 m1 A1 A1 A1 A1	Allow $x-1=2\cos\theta$
5(a) **(b)**	$I_n=\left[x^n\cosh x\right]_0^1-n\int_0^1 x^{n-1}\cosh x\,dx$ $= \cosh 1-n\int_0^1 x^{n-1}\cosh x\,dx$ $=\cosh 1-\left[nx^{n-1}\sinh x\right]_0^1+n(n-1)I_{n-2}$ $= \cosh 1-n\sinh 1+n(n-1)I_{n-2}$ $I_0=\int_0^1\sinh x\,dx=\left[\cosh x\right]_0^1=\cosh 1-1$ $I_4=\cosh 1-4\sinh 1+12I_2$ $= \cosh 1-4\sinh 1+12(\cosh 1-2\sinh 1+2I_0)$ = 13cosh1 – 28sinh1 + 24(cosh1 – 1) = 37cosh1– 28sinh1 – 24 cao	M1A1 A1 M1A1 M1A1 M1 A1 A1	 M1A1 for evaluating I_0 at any stage FT their I_0 if substituted here

6(a)	Consider $x = r\cos\theta$ $= \sin^2\theta\cos\theta$ $\frac{dx}{d\theta} = 2\sin\theta\cos^2\theta - \sin^3\theta$ The tangent is perpendicular to the initial line where $\frac{dx}{d\theta} = 2\sin\theta\cos^2\theta - \sin^3\theta = 0$ $\tan^2\theta = 2$ $\theta = \tan^{-1}\sqrt{2} = 0.955$ $r = 0.667$	 M1 A1 M1A1 M1 A1 A1 A1	 Do not penalise the removal of the factor $\sin\theta$
(b)	Area = $\frac{1}{2}\int r^2 d\theta$ $= \frac{1}{2}\int_0^{\pi/2}(1-\sin\theta)^2 d\theta$ $= \frac{1}{2}\int_0^{\pi/2}(1-2\sin\theta+\sin^2\theta)d\theta$ $= \frac{1}{4}\int_0^{\pi/2}(3-4\sin\theta-\cos 2\theta)d\theta$ $= \frac{1}{4}\left[3\theta+4\cos\theta-\frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$ $= \frac{3\pi-8}{8}$ (0.178) cao	M1 A1 A1 A1 A1 A1	

7(a)(i)	$D(\text{cosech}x) = D\left(\dfrac{1}{\sinh x}\right)$	M1	
	$= \dfrac{-1}{\sinh^2 x} \times \cosh x$	A1	
	$= -\text{cosech}x\coth x$		
	$D(\coth x) = D\left(\dfrac{\cosh x}{\sinh x}\right)$	M1	
	$= \dfrac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$	A1	
	$= -\text{cosech}^2 x$		
(ii)	$D\ln(\text{cosech}x + \coth x)$		
	$= \dfrac{-(\text{cosech}x\coth x + \text{cosech}^2 x)}{(\text{cosech}x + \coth x)}$	M1	
	$= -\text{cosech}x$	A1	convincing
(b)(i)	$L = \int \sqrt{1+\left(\dfrac{\text{d}y}{\text{d}x}\right)^2}\,\text{d}x$	M1	
	$= \int_1^{\text{e}} \sqrt{1+\left(\dfrac{1}{x}\right)^2}\,\text{d}x$	A1	
	$= \int_1^{\text{e}} \dfrac{\sqrt{1+x^2}}{x}\,\text{d}x$		
(ii)	Putting $x = \sinh u$,		
	$\text{d}x = \cosh u\text{d}u$, $[1,\text{e}] \to [\sinh^{-1}1, \sinh^{-1}\text{e}]$ $([\alpha,\beta])$	B1B1	
	Arc length $= \int_\alpha^\beta \dfrac{\sqrt{1+\sinh^2 u}}{\sinh u}.\cosh u\text{d}u$	M1	
	$= \int_\alpha^\beta \dfrac{\cosh^2 u}{\sinh u}\text{d}u$	A1	
	$= \int_\alpha^\beta \dfrac{1+\sinh^2 u}{\sinh u}\text{d}u$	A1	
	$= \int_\alpha^\beta (\text{cosech}u + \sinh u)\text{d}u$		
(iii)		M1A1	
	$= \left[-\ln(\text{cosech}u + \coth u) + \cosh u\right]_\alpha^\beta$	A2	
	$= 2.00$		

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GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3
AS/Advanced

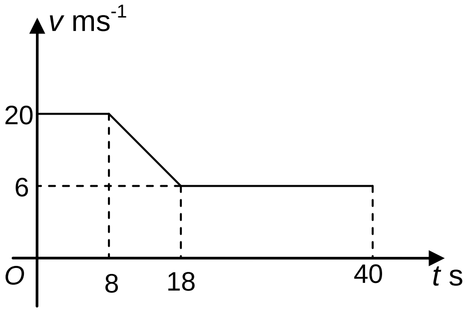
SUMMER 2013

INTRODUCTION

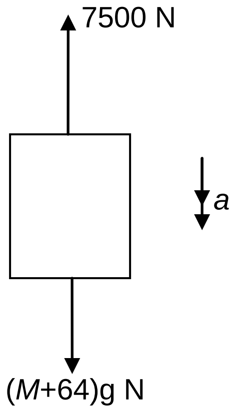
The marking schemes which follow were those used by WJEC for the Summer 2013 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

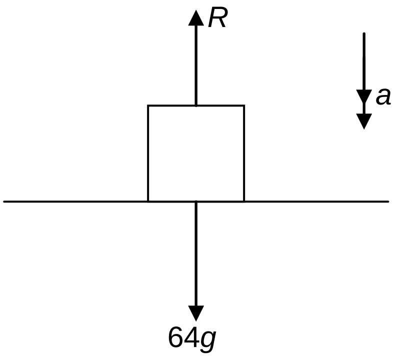
WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Q	Solution	Mark	Notes
1(a)		B1	(0, 20) to (8, 20) Or (18, 6) to (40, 6)
		B1	(8, 20) to (18, 6)
		B1	completely correct with all units and labels.
1(b)	Deceleration = gradient of graph	M1	any correct method
	$D = \dfrac{20-6}{18-8}$	A1	ft graph +/-
	D = <u>1.4 ms^{-2}</u>	A1	cao
	OR		
	Use of v = u + at, v=6, u=20, t=10	M1	
	6 = 20 + 10a	A1	allow –a
	a = -1.4 ms^{-2}		
	Magnitude of acceleration = <u>1.4 ms^{-2}</u>	A1	cao
1(c)	Distance AB = Area under graph	M1	used. Oe
	= (8 × 20) + 0.5(20 + 6)×10 + (22 × 6)	B1	any correct area, ft graph
	= 160 + 130 + 132		
	= <u>422 m</u>	A1	cao

Q	Solution	Mark	Notes



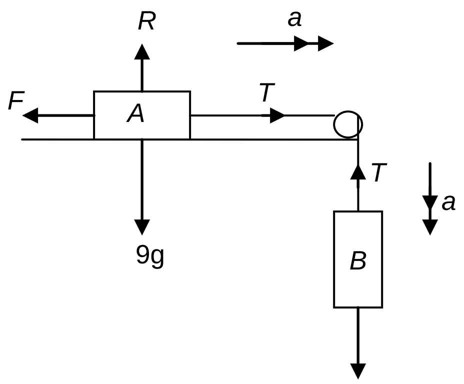
2(a)			
	N2L applied to lift and person	M1	dim correct equation, forces opposing
	$(M + 64)g - 7500 = (M+64) \times 0.425$	A1	correct equation
	$M = \underline{736}$	A1	



2(b)			
	N2L applied to person	M1	64g and R opposing Dim correct equation
	$64g - R = 64a$	A1	correct equation
	$R = 64 \times 9.8 - 64 \times 0.425$		
	$R = \underline{600 \text{ N}}$	A1	

Q	Solution	Mark	Notes
3(a)	$v^2 = u^2 + 2as$, v=0, a=(±)9.8, s=18.225	M1	oe used
	$0 = u^2 - 2 \times 9.8 \times 18.225$	A1	
	u = 18.9	A1	convincing
3(b)	Use of $s = ut + 0.5at^2$, s=(±)2.8, a=(±)9.8, u=18.9	M1	oe
	$-2.8 = 18.9t + 0.5 \times (-9.8)t^2$	A1	
	$4.9t^2 - 18.9t - 2.8 = 0$		
	$7t^2 - 27t - 4 = 0$		
	$(7t + 1)(t - 4) = 0$	m1	correct method for solving quad equ seen
	t = 4s	A1	cao

Q	Solution	Mark	Notes



4

5
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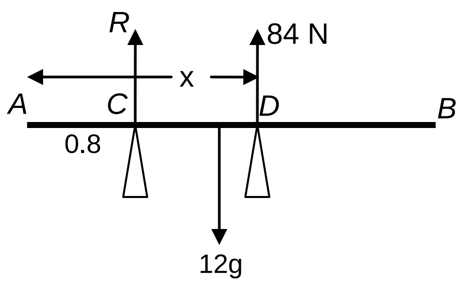
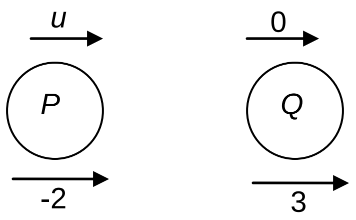
Q	Solution	Mark	Notes
4(a)	N2L applied to B		
	$5g - T = 5a$	M1	dim correct equation 5g and T opposing.
	$T = 5 \times 9.8 - 5 \times 1.61$	A1	
	$T =$ $\underline{40.95 \text{ N}}$	A1	cao
	$R = 9g = (88.2 \text{ N})$	B1	si
	$F = 9\mu g = (88.2\mu)$	B1	si
	N2L applied to A	M1	dim correct equation T and F opposing
	$T - F = 9a$	A1	
	$T - 88.2\mu = 9 \times 1.61$		
	$\mu = \underline{0.3}$	A1	cao
4(b)	limiting friction = $9\mu g = 9 \times 0.6g = 5.4g$	B1	
	Limiting friction > $5g$		
	Particle will remain at rest	R1	oe
	$T = 5g = \underline{49 \text{ N}}$	B1	

Q	Solution	Mark	Notes
5			
5(a)(i)	Resolve vertically	M1	all forces, no extras
	$R + 84 = 12g$	A1	
	$R = \underline{33.6}$	A1	cao
5(a)(ii)	Moments about C	M1	equation, no extra force oe
	$12g \times 0.2 = 84(x - 0.8)$	B1	any correct moment
		A1	correct equation
	$84x = 12g \times 0.2 + 84 \times 0.8$		
	$x = \underline{1.08}$	A1	cao
5(b)	When about to tilt about C, $R_D = 0$	M1	si
	Moments about C	m1	equation, no extra force
	$Mg \times 0.8 = 12g \times 0.2$		
	$M = \underline{3}$	A1	

Q	Solution	Mark	Notes



6.

6(a)	Conservation of momentum	M1	equation required, only 1 sign error.
	$2u + 5 \times 0 = 2 \times (-2) + 5 \times 3$	A1	correct equation
	$u = \underline{5.5}$	A1	
6(b)	Restitution	M1	only 1 sign error
	$3 - (-2) = -e(0 - 5.5)$	A1	ft u
	$e = \frac{10}{11} = 0.909$	A1	cao
6(c)	Impulse = change of momentum	M1	for P or Q
	$I = 5(3 - 0)$		
	$I = \underline{15\ (\text{Ns})}$	A1	+ required
6(d)	$v' = ev$	M1	used
	$v' = 0.25 \times 3$		
	$v' = \underline{0.75\ \text{ms}^{-1}}$	A1	+ required

Q	Solution	Mark	Notes
7.(a)	Resolve	M1	attempted
	$X = 85 - 40 + 75\cos\alpha$	B1	any correct resolution
	$X = 85 - 40 + 75 \times 0.8$	A1	all correct accept cos36.9
	$X = 105$		
	Resolve	M1	attempted
	$Y = 60 - 75\sin\alpha$		
	$Y = 60 - 75 \times 0.6$	A1	all correct, accept sin 36.9
	$Y = 15$		
	$R = \sqrt{105^2 + 15^2}$	M1	
	$R = 75\sqrt{2} = \underline{106.066\text{ N}}$	A1	cao
	$\theta = \tan^{-1}\left(\frac{15}{105}\right)$	M1	allow reciprocal
	$\theta = \underline{8.13°}$	A1	cao
7(b)	N2L applied to particle	M1	dim correct equation
	$75\sqrt{2} = 5a$		
	$a = 15\sqrt{2} = \underline{21.21\text{ ms}^{-2}}$	A1	ft *R* if first 2 M's gained.

Q	Solution				Mark	Notes
8.		Area	from *AD*	from *AB*		
	APCD	48	3	4	B1	
	PBC	24	8	8/3	B1	
	Circle	4π	3	3	B1	
	Lamina	$(72-4\pi)$	x	y	B1	areas
8(a)	Moments about *AD*				M1	equation
	$48 \times 3 + 24 \times 8 = 4\pi \times 3 + (72 - 4\pi)x$				A1	ft table
	x = <u>5.02 cm</u>				A1	cao
	Moments about *AB*				M1	equation
	$48 \times 4 + 24 \times 8/3 = 4\pi \times 3 + (72 - 4\pi)y$				A1	ft table
	y = <u>3.67 cm</u>				A1	cao
8(b)	*AQ* = <u>3.67 cm</u>				B1	ft y

Q	Solution	Mark	Notes
1(a)	Loss in KE $= 0.5mv^2$		
	$= 0.5 \times 8 \times 7^2$	M1	Corr use of KE formula
	$=$ <u>196J</u>	A1	
1(b)	Work energy principle	M1	correct use
	$196 = F \times 15$	A1	ft loss in KE
	$F = \mu R$		
	$= 8g\mu = (78.4\mu)$	B1	
	Therefore $196 = 78.4\mu \times 15$		
	$\mu = \frac{1}{6}$	A1	ft loss in KE. Isw
	OR		
	Use of $v^2=u^2+2as$		
	$0=7^2 + 2a\times15$	(M1)	
	$a = -1.633$		
	Use $F = ma$		
	$-F = 8\times -1.633$	(M1)	
	$F = 8\mu g$	(B1)	
	$\mu = \frac{13\cdot067}{8g} = \frac{1}{6}$	(A1)p	

Q	Solution	Mark	Notes
2(a)	$\mathbf{r} = \int v\,\mathrm{d}t$	M1	use of integration
	$\mathbf{r} = \int (13t - 3)\mathbf{i} + (2 + 3t^2)\mathbf{j}\,\mathrm{d}t$		
	$\mathbf{r} = \left(\frac{13}{2}t^2 - 3t\right)\mathrm{i} + (2t + t^3)\mathrm{j}$ + (<u>c)</u>	A1 A1	one for each coefficient
	When t = 0,	m1	use of initial conditions
	$\mathbf{c} = 2\mathbf{i} + 7\mathbf{j}$	A1	ft **r**
	$\mathbf{r} = (6.5t^2 - 3t + 2)\mathbf{i} + (2t + t^3 + 7)\mathbf{j}$		
2(b)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$		M1 use of differentiation
	$= 13\mathbf{i} + 6t\mathbf{j}$	A1	
2(c)	We require $\mathbf{v}.(\mathbf{i} - 2\mathbf{j}) = 0$	M1	used
	$\mathbf{v}.(\mathbf{i} - 2\mathbf{j}) = (13t - 3) - 2(2 + 3t^2)$	M1	allow sign errors
	$= -6t^2 + 13t - 7$	A1	any form
	$6t^2 - 13t + 7 = 0$		
	$(6t - 7)(t - 1) = 0$	m1	method for quad equation Depends on both M's
	t = <u>1, 7/6</u>	A1	

Q	Solution	Mark	Notes
3(a)(i)	Initial horizontal speed = $15\cos\alpha$	B1	
	$= 15 \times 0.8$		
	$= 12\ \text{ms}^{-1}$		
	Time of flight = 9/12	M1	
	= <u>0.75s</u>	A1	any correct form
3(a)(ii)	Initial vertical speed = $15\sin\alpha$	B1	
	$= 15 \times 0.6$		
	$= 9\ \text{ms}^{-1}$		
	Use of $s = ut + 0.5at^2$, u=9(c), a=(±)9.8, t=0.75(c)	M1	
	$s = 9 \times 0.75 - 0.5 \times 9.8 \times 0.75^2$		
	$s = 3.99375$ m	A1	si
	Height of B above ground = <u>4.99375 m</u>	A1	ft s
3(b)	use of $v^2 = u^2 + 2as$, u=9, a=(±)9.8, s=-1	M1	allow sign errors
	$v^2 = 9^2 + 2(-9.8)(-1)$	A1	
	$v^2 = 100.6$		
	$u_H = 12$	B1	ft candidate's value
	Speed = $\sqrt{12^2 + 100.6}$	m1	
	Speed = <u>15.64 ms^{-1}</u>	A1	cao

Q	Solution	Mark	Notes
4(a)	Resolve vertically	M1	dim correct
	$R\sin\theta = Mg$	A1	
	$\sin\theta = \frac{3}{5}$	B1	
	$R = Mg \times \frac{5}{3}$		
	$R = 5Mg/3$	A1	answer given, convincing.
4(b)	N2L towards centre	M1	dim correct
	$R\cos\theta = Ma$	A1	
	$\frac{5Mg}{3} \times \frac{4}{5} = M \times \frac{8g}{3r}$		
	$CP = r = 2$	A1	
	$\frac{\text{Height}}{r} = \frac{4}{3}$	M1	use of similar triangles
	Height = $\frac{8}{3}$ m	A1	ft candidate's r if first M1 given.

Q	Solution	Mark	Notes
5(a)	$0 < t < 6$	B1 B1	
5(b)	Distance t = 6 to t = 9 = $\int_6^9 2t^2 - 12t \, dt$	M1	use of integration Limits not required
	Distance = $[\, 2t^3/3 – 6t^2]_6^9$ = 72	A1	correct integration
	Distance t = 0 to t = 6 = $-\int_0^6 2t^2 - 12t \, dt$ Distance = $-[\, 2t^3/3 – 6t^2]_0^6$ = -[-72] = 72	A1	or for the other integral
	Required distance = 72 + 72	m1	
	= <u>144</u>	A1	cao

Q	Solution	Mark	Notes
6(a)	T = P/v	M1	used
	$T = \dfrac{60\times1000}{20}$		
	T = <u>3000 N</u>	A1	
6(b)	Apply N2L to car and trailer	M1	dim correct equation All forces present
	$T - (1500+500)g\sin\alpha - (170+30) = 2000a$	A2	-1 each error
	$3000 - 2000 \times 9.8 \times \dfrac{1}{14} - 200 = 2000a$		
	a = <u>0.7 ms^{-2}</u>	A1	convincing
6(c)	N2L applied to trailer	M1	dim correct, all forces
	$T - 500g\sin\alpha - 30 = 500a$	A2	-1 each error
	$T = 500 \times 9.8 \times \dfrac{1}{14} + 30 + 500 \times 0.7$		
	T = <u>730 N</u>	A1	
	OR		
	N2L applied to car	(M1)	dim correct, all forces
	$3000 - 1500g\sin\alpha - 170 - T = 1500 \times 0.7$	(A2)	-1 each error
	$T = 3000 - 1500\times9.8\times\dfrac{1}{14} - 170 - 1500 \times 0.7$		
	T = <u>730 N</u>	(A1)	

Q	Solution	Mark	Notes
7(a)	PE at start = $- 2 \times 9.8 \times 0.7$	M1	mgh used
		A1	allow 0.7, (1.2+x), (0.5+x), 1.2, 0.5, x.
	$= -13.72$ J		
	PE at end = $-2 \times 9.8 \times (1.2 + x)$		
	$= -23.52 - 19.6x$		
	EE at end = $\frac{1}{2} \times \frac{360}{1 \cdot 2} x^2$	M1	use of formula
		A1	
	EE at end $= 150x^2$		
	Conservation of energy	M1	equation, all energies
	$150x^2 - 19.6x - 23.52 = -13.72$	A1	correct equation any form
	$150x^2 - 19.6x - 9.8 = 0$		
	$x = \underline{0.33}$	A1	cao
7(b)	KE at end $= 0.5 \times 2v^2$	B1	
	$= v^2$		
	PE at end $= -2 \times 9.8 \times 1.2$		
	$= -23.52$		
	Conservation of energy	M1	equation, no EE
	$v^2 - 23.52 = -13.72$	A1	correct equation, any form
	$v^2 = 9.8$		
	$v = \underline{3.13 \text{ ms}^{-1}}$	A1	

Q	Solution	Mark	Notes
8(a)	Conservation of energy	M1	equation required
	$0.5mu^2 + mgr\cos\alpha = 0.5mv^2 + mgr\cos\theta$	A1	KE
		A1	PE
	$0.5 \times 3 \times 5^2 + 3 \times 9.8 \times 4 \times 0.8 =$		
	$0.5 \times 3 \times v^2 + 3 \times 9.8 \times 4 \times \cos\theta$		
	$75 + 188.16 = 3v^2 + 235.2\cos\theta$		
	$v^2 = 87.72 – 78.4\cos\theta$		
	$v = \sqrt{(87.72 – 78.4\cos\theta)}$	A1	cao
8(b)	N2L towards centre	M1	dim correct, all forces
	$mg\cos\theta - R = ma$	A1	
	$R = 3 \times 9.8\cos\theta - \frac{3}{4}(87.72 – 78.4\cos\theta)$	m1	substitute, v^2/r
	$R = 29.4\cos\theta - 65.79 + 58.8\cos\theta$		
	$R = \underline{88.2\cos\theta - 65.79}$		

Q	Solution	Mark	Notes
1(a)(i)	Apply N2L to particle	M1	dim correct equation
	ma = -mg – 3v	A1	
	$2\frac{dv}{dt} = -19.6 - v$		
1(a)(ii)	$\int \frac{2dv}{19.6+v} = -\int dt$	M1	sep. of variables
	$2\ln\lvert 19.6+v\rvert = -t + (C)$	A1	correct integration
	$t = 0,\ v = 24.5$	m1	use of initial conditions
	$C = 2\ln\lvert 44.1\rvert$	A1	ft no 2,1/2.
	$-t = 2\ln\left\lvert\frac{19.6+v}{44.1}\right\rvert$		
	$e^{-t/2} = \frac{19.6+v}{44.1}$	m1	inversion ln to e
	$\underline{v = 44.1\,e^{-t/2} - 19.6}$	A1	cao
1(b)	At maximum height, $v = 0$	M1	si
	$t = -2\ln\left\lvert\frac{19.6}{44.1}\right\rvert$		
	$= \underline{2\ln(2.25) = 1.62\text{ s}}$	A1	ft similar expression
1(c)	$\frac{dx}{dt} = 44.1\,e^{-t/2} - 19.6$	M1	$v = \frac{dx}{dt}$ used
	$x = -88.2\,e^{-t/2} - 19.6t\ (+ C)$	A1	ft correct integration
	When $t = 0,\ x = 0$	m1	use of initial conditions
	C = 88.2		
	$x = \underline{88.2 - 88.2\,e^{-t/2} - 19.6t}$	A1	ft one slip

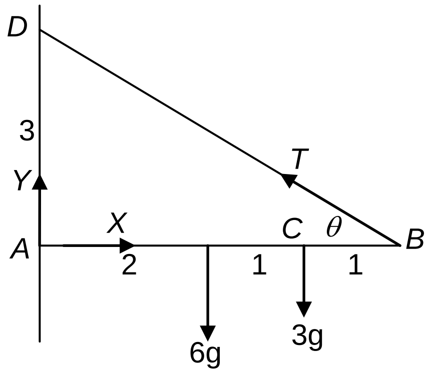
Q	Solution	Mark	Notes
2(a)	Amplitude $a = 0.5$	B1	
2(b)	Period $= \frac{2\pi}{\omega} = 2$	M1	si
	$\omega = \pi$	A1	
	Maximum acceleration $= a\omega^2 = 0.5 \times \pi^2$	B1	ft amplitude a.
	Occurs at end points of motion	B1	
2(c)	Let $x = a\cos(\omega t)$	M1	
	$-0.25 = 0.5\cos(\pi t)$	m1	
	$\cos(\pi t) = -0.5$		
	$\pi t = \frac{2\pi}{3}$		
	$t = \frac{2}{3}$	A1	cao
2(d)	$v^2 = \omega^2(a^2 - x^2)$, $x = 0.3$, $\omega = \pi$	M1	
	$v^2 = \pi^2(0.5^2 - 0.3^2)$	A1	ft
	$v^2 = \pi^2 \times 0.4^2$		
	$v = (\pm)0.4\pi$		
	speed $= 0.4\pi$	A1	cao

Q	Solution	Mark	Notes
3(a)(i)	Apply N2L to *P*	M1	
	$2a = -8x - 10v$	A1	
	$\frac{d^2x}{dt^2} = -4x - 5\frac{dx}{dt}$		
3(a)(ii)	$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 0$		
	Auxiliary equation $m^2 + 5m + 4 = 0$	B1	
	$(m + 4)(m + 1) = 0$		
	$m = -4, -1$	B1	
	CF $x = Ae^{-t} + Be^{-4t}$	B1	ft values of roots
	When $t = 0$, $x = 2$, $\frac{dx}{dt} = 3$	M1	use of initial conditions
	$2 = A + B$		
	$\frac{dx}{dt} = -Ae^{-t} - 4Be^{-4t}$	B1	
	$3 = -A - 4B$	A1	both equations correct
	Adding	m1	solving simultaneously
	$5 = -3B$		
	$B = -\frac{5}{3}$		
	$A = 2 + -\frac{5}{3} = \frac{11}{3}$		
	$x = \frac{11}{3}e^{-t} - \frac{5}{3}e^{-4t}$	A1	cao
3(b)	Try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$5a + 4(at + b) = 12t - 3$	A1	
	$4a = 12$	m1	comparing coefficients
	$a = 3$		
	$5a + 4b = -3$		
	$15 + 4b = -3$		
	$4b = -18$		
	$b = -\frac{9}{2}$		
	General solution $x = Ae^{-t} + Be^{-4t} + 3t - \frac{9}{2}$	A1	cao

Q	Solution	Mark	Notes
4	Initial speed of A just before impact = v		
	$v^2 = u^2 + 2as$, u=0, a=(±)9.8, s =(1.8-0.2)	M1	
	$v^2 = 0 + 2 \times 9.8 \times 1.6$	A1	
	v = $\underline{5.6 \text{ ms}^{-1}}$	A1	cao
	Impulse = Change in momentum	M1	used
	Applied to B		
	J = 3v	B1	
	Applied to A		
	J = $5 \times 5.6 - 5v$	A1	ft c's answer in (a)
	Solving		
	3v = 28 – 5v	m1	
	8v = 28		
	v = $\underline{3.5 \text{ ms}^{-1}}$	A1	cao
	J = $\underline{10.5 \text{ Ns}}$	A1	cao

Q	Solution	Mark	Notes
5(a)	N2L applied to particle		
	$0.25\,a = \dfrac{5}{2x+1}$	M1	
	$v\dfrac{dv}{dx} = \dfrac{20}{2x+1}$	M1	$a = v\dfrac{dv}{dx}$
	$\int v\,dv = 10\int \dfrac{2}{2x+1}dx$	M1	separating variables
	$\frac{1}{2}v^2 = 10\ln\lvert 2x+1\rvert + C$	A1	correct integration ln
		A1	LHS correct
	When $x = 0$, $v = 4$	m1	use of boundary cond. All 3 M's awarded
	$8 = 10\ln(1) + C$		
	$C = 8$		
	$v^2 = 20\ln\lvert 2x+1\rvert + 16$		
	$\ln\lvert 2x+1\rvert = \frac{1}{20}(v^2 - 16)$		
	$2x + 1 = e^{0.05(v^2-16)}$	m1	inversion, 3 M's awarded
	$x = 0.5(e^{0.05(v^2-16)} - 1)$	A1	cao any equivalent exp.
5(b)	$v = 6$		
	$x = 0.5(e^{0.05(36-16)} - 1)$	M1	exp. with v^2 needed
	$x = 0.5(e - 1)$		
	$x = \underline{0.86\text{ m}}$	A1	cao
5(c)	$a = 5$		
	$\dfrac{20}{2x+1} = 5$	M1	
	$20 = 10x + 5$		
	$x = 1.5$	A1	
	$v^2 = 20\ln(3 + 1) + 16$	m1	substitution in expression with v^2.
	$= 20\ln4 + 16$		
	$v = \underline{6.61\text{ ms}^{-1}}$	A1	cao

Q	Solution	Mark	Notes



Q	Solution	Mark	Notes
6			
6(a)	Moments about *A*	M1	equation, no extra forces No missing forces
	$6g \times 2 + 3g \times 3 = T \times 4\sin\theta$	A2	-1 each error
	$4 \times \frac{3}{5}T = 21g$		
	$T = \frac{35}{4}g = 85.75$ N	A1	cao
6(b)	Resolve vertically	M1	equation, all forces, no extra force
	$T\sin\theta + Y = 9g$	A1	
	$Y = 9g - \frac{35}{4}g \times \frac{3}{5}$		
	$Y = \frac{15}{4}g = 36.75$ N	A1	cao
	Resolve horizontally		
	$T\cos\theta = X$	M1	equation, all forces, No extra force
	$X = \frac{35}{4}g \times \frac{4}{5}$		
	$X = 7g = 68.6$ N	A1	cao
6(b)(i)	Magnitude of reaction at wall		
	$= \sqrt{68\cdot6^2 + 36\cdot75^2}$	M1	
	= 77.82 N	A1	ft *X* and *Y*
6(b)(ii)	$\mu = \frac{Y}{X}$	M1	used
	$\mu = \frac{15}{4\times7} = \frac{15}{28}$	A1	ft *X* and *Y* if answer<1.

Ques	Solution	Mark	Notes
1(a) **(b)**	P(A∪B) = P(A) + P(B) P(B) = 0.4 – 0.25 = 0.15 P(A∪B) = P(A) + P(B) – P(A)P(B) 0.4 = 0.25 + P(B) – 0.25P(B) P(B) = 0.15/0.75 = 0.2	**M1** **A1** **M1** **A1** **A1**	Award M1 for using formula Award M1 for using formula
2(a) **(b)** **(c)**	P(1 of each) = $\frac{5}{10}\times\frac{3}{9}\times\frac{2}{8}\times 6$ or $\binom{5}{1}\times\binom{3}{1}\times\binom{2}{1}\div\binom{10}{3}$ $= \frac{1}{4}$ P(3 war) = $\frac{5}{10}\times\frac{4}{9}\times\frac{3}{8}$ or $\binom{5}{3}\div\binom{10}{3}$ $= \frac{1}{12}$ P(3 cowboy) = $\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}$ or $\binom{3}{3}\div\binom{10}{3}$ $= \frac{1}{120}$ P(3 the same) = $\frac{1}{12}+\frac{1}{120}=\frac{11}{120}$	 **M1A1** **A1** **M1** **A1** **B1** **M1A1**	M1A0A0 if 6 omitted Special case : if they use an incorrect total, eg 9 or 11, FT their incorrect total but subtract 2 marks at the end FT previous values
3	$E(X) = 20$ Var(*X*) = 4 (SD = 2) $E(Y) = 20a + b = 65$ Var(*Y*) = $4a^2 = 36$ $a = 3$ $b = 5$	**B1** **B1** **B1** **B1** **B1** **B1**	 Accept SD(*Y*) = 2*a* = 6 Must be justified by solving the two equations
4(a)(i) **(ii)** **(iii)** **(b)(i)** **(ii)**	B(20,0.25) $P(3 \le X \le 9) = 0.9087 - 0.0139$ or $0.9861 - 0.0913$ = 0.8948 $P(X = 6) = \binom{20}{6}\times 0.25^6 \times 0.75^{14}$ = 0.169 Let *Y* denote the number of throws giving ‘8’ Then *Y* is B(160,0.0625) ≈ Poi(10). $P(Y = 12) = e^{-10}\times\frac{10^{12}}{12!}$ = 0.0948 $P(6 \le Y \le 14) = 0.9165 - 0.0671$ or $0.9329 - 0.0835$ = 0.8494 cao	**B1** **B1B1** **B1** **M1** **A1** **B1** **M1** **A1** **B1B1** **B1**	B must be mentioned and the parameters *n* and *p* must be seen or implied somewhere in the question FT an incorrect *p* except for the last three marks M0 if no working seen M0 if no working seen Accept the use of tables Correct values only (no FT)

5(a)	P(1) = $\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}$	**M1A1**	M1 Use of Law of Total Prob (Accept tree diagram)
	$= \frac{13}{36}$ (0.361)	**A1**	
(b)	P(A|1) = $\frac{1/12}{13/36}$	**B1B1**	FT denominator from (a) B1 num, B1 denom
	$= \frac{3}{13}$ cao (0.231)	**B1**	
6(a)	The sequence is MMMH si	**B1**	
	Prob = 0.3×0.3×0.3×0.7 = 0.0189	**M1A1**	
(b)	The sequence is MHH or HMH si	**B1**	
	Prob = 0.3×0.7×0.7 + 0.7×0.3×0.7 = 0.294	**M1A1**	Award B1 for 0.147
7(a)	$\sum p_x = k\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right) = 1$	**M1**	
	$k\left(\frac{8+4+2+1}{8}\right)=1 \to k=\frac{8}{15}$	**A1**	Convincing
(b)	$E(X)=\frac{8}{15}\times1+\frac{4}{15}\times2+\frac{2}{15}\times4+\frac{1}{15}\times8$	**M1**	
	$= \frac{32}{15}$ (2.13)	**A1**	
	$E(X^2)=\frac{8}{15}\times1+\frac{4}{15}\times4+\frac{2}{15}\times16+\frac{1}{15}\times64$ (8)	**M1A1**	
	Var(X) = $8-\left(\frac{32}{15}\right)^2=3.45$ (776/225)	**A1**	Accept 3.46
(c)(i)	The possibilities are (1,1); (2,2); (4,4); (8,8) si	**B1**	
	$P(X_1=X_2)=\left(\frac{8}{15}\right)^2+\left(\frac{4}{15}\right)^2+\left(\frac{2}{15}\right)^2+\left(\frac{1}{15}\right)^2$	**M1**	
	$= \frac{17}{45}$ (0.378)	**A1**	
(ii)	It follows that $P(X_1 \neq X_2)=\frac{28}{45}$	**M1**	FT their answer from (c)(i)
	And therefore by symmetry $P(X_1 > X_2)=\frac{14}{45}$	**A1**	Do not accept any other method.

8(a)	Let X denote the number of calls between 9am and 10 am so that X is Po(5)	**B1**	
	$P(X = 7) = \dfrac{e^{-5} \times 5^7}{7!}$	**M1**	M0 no working
	$= 0.104$	**A1**	
(b)	We require P(calls betw 9 and 10=7|calls betw 9 and 11=10)		
	$= \dfrac{P(\text{c b 9 and 10} = 7 \text{ AND c b 9 and 11} = 10)}{P(\text{calls between 9 and 11} = 10)}$	**M1**	
	$= \dfrac{P(\text{c b 9 and 10} = 7) \times P(\text{c b 10 and 11} = 3)}{P(\text{calls between 9 and 11} = 10)}$	**A1**	
	$= \dfrac{e^{-5} \times 5^7}{7!} \times \dfrac{e^{-5} \times 5^3}{3!} \div \dfrac{e^{-10} \times 10^{10}}{10!}$ (denom = 0.125)	**A1A1**	A1 numerator, A1 denominator The denominator A1 can be awarded if the M1 is awarded
	$= 0.117$	**A1**	
9(a)	$\int_0^2 k\left(1 - \dfrac{x^2}{4}\right)dx = 1$	**M1**	M1 for $\int f(x)dx$, limits not required until next line
	$k\left[x - \dfrac{x^3}{12}\right]_0^2 = 1$	**A1**	
	$k\left(2 - \dfrac{8}{12}\right) = 1$	**A1**	
	$k = \dfrac{3}{4}$		
(b)	$E(X) = \int_0^2 x\left(\dfrac{3}{4} - \dfrac{3x^2}{16}\right)dx$	**M1A1**	M1 for the integral of $xf(x)$, A1 for completely correct although limits may be left until 2nd line.
	$= \left[\dfrac{3x^2}{8} - \dfrac{3x^4}{64}\right]_0^2$	**A1**	
	$= 0.75$	**A1**	
(c)(i)	$F(x) = \int_0^x \left(\dfrac{3}{4} - \dfrac{3t^2}{16}\right)dt$	**M1**	M1 for $\int f(x)dx$
	$= \left[\dfrac{3t}{4} - \dfrac{t^3}{16}\right]_0^x$	**A1**	A1 for performing the integration
	$= \dfrac{3x}{4} - \dfrac{x^3}{16}$	**A1**	A1 for dealing with the limits
(ii)	$P(0.5 \le X \le 1.5) = F(1.5) - F(0.5)$	**M1**	FT their F(x)
	$= 0.547$	**A1**	

Ques	Solution	Mark	Notes
1(a)(i)	$z = \dfrac{10.5-10}{2} = 0.25$ $P(X \le 10.5) = 0.5987$	**M1A1** **A1**	M0 for 2^2 or $\sqrt{2}$ M1A0 for – 0.25 if final answer incorrect M0 no working
(ii)	$x = \dfrac{x-\mu}{\sigma} = 1.282$ $= 12.564$	**M1** **A1**	M1 for 2.326, 1.645, 2.576 Accept 12.6
(b)(i)	$E(X + 2Y) = 34$ $\text{Var}(X + 2Y) = \text{Var}(X) + 4\text{Var}(Y)$ $= 40$ We require $P(X + 2Y < 36)$ $z = \dfrac{36-34}{\sqrt{40}} = 0.32$ Prob = 0.6255	**B1** **B1** **M1A1** **A1**	FT their mean and variance M0 no working
(ii)	Consider $U = X_1 + X_2 + X_3 - Y_1 - Y_2$ $E(U) = 3 \times 10 - 2 \times 12 = 6$ $\text{Var}(U) = 3 \times 4 + 2 \times 9 = 30$ We require P(U < 0) $z = \dfrac{0-6}{\sqrt{30}} = -1.10$ Prob = 0.136	**B1** **M1A1** **m1A1** **A1**	Do not FT their mean and variance
2(a)	$\bar{x} = \dfrac{9980}{50}$ (= 199.6) SE of $\bar{X} = \dfrac{4}{\sqrt{50}}$ (= 0.5656…) 95% conf limits are 199.6 ± 1.96 × 0.5656… giving [198.5, 200.7] cao	**B1** **B1** **M1A1** **A1**	M1 correct form, A1 correct z. M0 no working
(b)	Width of 95% CI = $3.92 \times \dfrac{4}{\sqrt{n}}$ si We require $3.92 \times \dfrac{4}{\sqrt{n}} < 1$ $n > 245.86\ldots$ Minimum n = 246	**B1** **M1** **A1** **A1**	FT their z from (a) Award M1A0A0 for 1.96 instead of 3.92 FT from line above if n > 50

3(a)	$H_0 : \mu_B = \mu_G ; H_1 : \mu_B \neq \mu_G$	**B1**	
(b)	$\bar{x}_B = \frac{482}{8} = 60.25; \bar{x}_G = \frac{430}{8} = 53.75$	**B1B1**	
	SE of diff of means= $\sqrt{\frac{7.5^2}{8} + \frac{7.5^2}{8}}$ (3.75)	**M1A1**	
	Test statistic (z) = $\frac{60.25 - 53.75}{3.75}$	**m1A1**	
	= 1.73	**A1**	
	Prob from tables = 0.0418	**A1**	FT their z if M marks gained
	p-value = 0.0836	**B1**	FT on line above
	Insufficient evidence to conclude that there is a difference in performance between boys and girls.	**B1**	FT their p-value
4(a)	$H_0 : p = 0.4; H_1 : p > 0.4$	**B1**	
(b)	Let X = No. supporting politician so that		
	X is B(50,0.4) (under H_0) si	**B1**	
	p-value = P($X \geq 25 \| X$ is B(50,0.4))	**M1**	M0 for P(X = 25) or P(X > 25)
	= 0.0978	**A1**	M0 normal or Poisson approx
	Insufficient evidence to conclude that the support is greater than 40%.	**B1**	FT on p-value
(c)	X is now B(400,0.4) (under H_0) ≈ N(160,96)	**B1**	
	p-value = P($X \geq 181 \| X$ is N(160,96))	**M1**	
	$z = \frac{180.5 - 160}{\sqrt{96}}$	**m1A1**	Award m1A0A1A1 for incorrect or no continuity correction
	= 2.09	**A1**	$181.5 \rightarrow z = 2.19 \rightarrow p = 0.01426$
	p-value = 0.0183	**A1**	$181 \rightarrow z = 2.14 \rightarrow p = 0.01618$
	Strong evidence to conclude that the support is greater than 40%.	**B1**	FT on p-value
5(a)	H_0:μ = 1.2 : H_1:μ < 1.2	**B1**	Must be μ
(b)(i)	Let X = number of accidents in 60 days		
	Then X is Poi(72) (under H_0) ≈ N(72,72) si	**B1**	
	Sig level = P($X \leq 58 \|$ H_0)	**M1**	
	$z = \frac{58.5 - 72}{\sqrt{72}}$	**m1A1**	Award m1A0A1A1 for incorrect or no continuity correction
	= – 1.59	**A1**	$57.5 \rightarrow z = -1.71 \rightarrow p = 0.0436$
	Sig level = 0.0559	**A1**	$58 \rightarrow z = -1.65 \rightarrow p = 0.0495$
(ii)	X is now Poi(48) which is approx N(48,48) si	**B1**	
	P(wrong conclusion) = P($X \geq 59 \|$ μ = 0.8)	**M1**	
	$z = \frac{58.5 - 48}{\sqrt{48}}$	**m1A1**	Award m1A0A1A1 for incorrect or no continuity correction
	= 1.52	**A1**	$59.5 \rightarrow z = 1.66 \rightarrow p = 0.0485$
	P(wrong conclusion) = 0.0643	**A1**	$59 \rightarrow z = 1.59 \rightarrow p = 0.0559$

6(a)(i)	$E(C) = 2\pi E(R)$ $= 2\pi \times 7 = 14\pi$ (43.98) $\text{Var}(C) = 4\pi^2 \text{Var}(R)$ $= \dfrac{4\pi^2}{3}$ (13.16)	**M1** **A1** **M1** **A1**	Accept the use of integration, M1 for a correct integral and A1 for the correct answer
(ii)	$P(C \le 45) = P(R \le 45/2\pi)$ $= \dfrac{(45/2\pi - 6)}{8-6}$ $= 0.581$	**M1** **A1** **A1**	
(b)(i)	$A = \pi R^2$ $P(A \ge 150) = P\left(R \ge \sqrt{150/\pi}\right)$ $= \dfrac{8 - \sqrt{150/\pi}}{8-6}$ $= 0.545$	**M1A1** **A1** **A1**	
(ii)	EITHER $E(A) = \int_6^8 \pi r^2 \times \frac{1}{2}\,\mathrm{d}r$ $= \dfrac{\pi}{6}\left[r^3\right]_6^8$ $= \dfrac{148\pi}{3}$ (155) OR $E(A) = \pi E(R^2) = \pi\left(\text{var}(R) + (E(R))^2\right)$ $= \pi\left(\frac{1}{3} + 7^2\right)$ $= \dfrac{148\pi}{3}$ (155)	**M1** **A1** **A1** **M1** **A1** **A1**	

<table>
<tr><th>Ques</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>1</td><td>$\hat{p} = 0.29$ si
ESE = $\sqrt{\frac{0.29 \times 0.71}{300}}$ (= 0.02619..) si
95% confidence limits are
$0.29 \pm 1.96 \times 0.02619..$
giving [0.24,0.34]</td><td>B1
M1A1
m1A1
A1</td><td>m1 correct form, A1 1.96</td></tr>
<tr><td>2</td><td>The possibilities are
<u>3 red, 1 blue for which |X – Y| = 2</u>
Therefore,
$P(X - Y = 2) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7} \times 4$ OR $\frac{\binom{3}{3} \times \binom{7}{1}}{\binom{10}{4}}$
$= \frac{1}{30}$
<u>2 red, 2 blue for which |X – Y| = 0</u>
$P(X - Y = 0) = \frac{3}{10} \times \frac{2}{9} \times \frac{7}{8} \times \frac{6}{7} \times 6$ OR $\frac{\binom{3}{2} \times \binom{7}{2}}{\binom{10}{4}}$
$= \frac{3}{10}$
<u>1 red, 3 blue for which |X – Y |= 2</u>
$P(X - Y = -2) = \frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times 4$ OR $\frac{\binom{3}{1} \times \binom{7}{3}}{\binom{10}{4}}$
$= \frac{1}{2}$
<u>0 red, 4 blue for which |X – Y| = 4</u>
$P(X - Y = -4) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7}$ OR $\frac{\binom{7}{4}}{\binom{10}{4}} = \frac{1}{6}$
The distribution of |X – Y| is therefore
<table><tr><td>|X – Y|</td><td>0</td><td>2</td><td>4</td></tr><tr><td>Prob</td><td>3/10</td><td>8/15</td><td>1/6</td></tr></table></td><td>M1A1
A1
B1
B1
B1
M1A1</td><td>FT if found as 1 - Σprobs
FT their probabilities</td></tr>
</table>

3(a)	UE of μ = 34.3 $\Sigma x^2 = 10609.43$ UE of $\sigma^2 = \frac{10609.43}{8} - \frac{9 \times 34.3^2}{8}$ = 2.6275	**B1** **B1** **M1** **A1**	No working need be seen M0 division by 9 Answer only no marks
(b)	DF = 8 si t-value = 1.86 90% confidence limits are $34.3 \pm 1.86\sqrt{\frac{2.6275}{9}}$ giving [33.3,35.3] cao	**B1** **B1** **M1A1** **A1**	 Answer only no marks
(c)	EITHER Width of interval = $2t\sqrt{\frac{2.6275}{9}} = 3.2$ So t = 2.96 For a 99% confidence interval, t = 3.355 Since 2.96 < 3.355, the confidence level is less than 99% OR For 99% confidence interval, t = 3.355 99% confidence limits are $34.3 \pm 3.355\sqrt{\frac{2.6275}{9}}$ giving [32.5,36.1] The given confidence interval is narrower than this therefore its confidence level is less than 99%	 **M1** **A1** **B1** **A1** **B1** **M1** **A1** **A1**	
4(a)	The 5% critical value = 2000 + 1.645 × $\sqrt{\frac{2554}{120}}$ = 2007.6 The 10% critical value = 2000 + 1.282 × $\sqrt{\frac{2554}{120}}$ = 2005.9 The required range is therefore (2005.9,2007.6)	**M1** **A1** **M1** **A1** **A1**	M1A0 for – M1A0 for –
(b)	No because of the Central Limit Theorem AND THEN EITHER which ensures the normality of the sample mean OR which can be used because the sample is large	**B1** **B1**	

5(a) **(b)**	$H_0 : \mu_A = \mu_B ; H_1 : \mu_A \neq \mu_B$ $\bar{x} = 55.25; \bar{y} = 55.75$ si $s_x^2 = \frac{183345}{59} - \frac{3315^2}{59 \times 60} = 3.2415...$ $s_y^2 = \frac{186651}{59} - \frac{3345^2}{59 \times 60} = 2.8347...$ [Accept division by 60 giving 3.1875 and 2.7875] SE = $\sqrt{\frac{3.2415..}{60} + \frac{2.8347..}{60}}$ = (0.3182.., 0.3155..) si Test stat = $\frac{55.75 - 55.25}{0.3182..}$ = 1.57 (1.58) p-value = 0.116 (0.114) cao Insufficient evidence for believing that the mean weights are unequal.	**B1** **B1** **M1A1** **A1** **M1** **A1** **m1** **A1** **A1** **B1**	 FT 1 error in the means Answer only no marks FT their p-value
6(a) **(b)**	$\sum x = 175, \sum x^2 = 5075, \sum y = 118.1, \sum xy = 3170$ $S_{xy} = 3170 - 175 \times 118.1 / 7 = 217.5$ $S_{xx} = 5075 - 175^2 / 7 = 700$ $b = \frac{217.5}{700} = 0.311$ $a = \frac{118.1 - 175 \times 0.311..}{7} = 9.10$ SE of a = $\sqrt{\frac{0.1^2 \times 5075}{7 \times 700}}$ (0.1017…) 95% confidence limits for α are $9.10 \pm 1.96 \times 0.1017...$ giving [8.9,9.3]	**B2** **B1** **B1** **M1** **A1** **M1** **A1** **M1A1** **m1A1** **A1**	Minus 1 each error FT 1 error in sums FT their value of a M1 correct form, A1 1.96

7(a)	$\mathrm{E}(\hat{p}) = \frac{E(X)}{n} = \frac{np}{n} = p$	**M1** **A1**	
	Therefore unbiased.		This line need not be seen
	$\mathrm{SE}(\hat{p}) = \sqrt{\frac{\mathrm{Var}(X)}{n^2}} = \sqrt{\frac{np(1-p)}{n^2}} = \sqrt{\frac{p(1-p)}{n}}$	**M1** **A1**	Accept q for $1-p$
(b)(i)	$\mathrm{E}(\hat{p}^2) = \frac{\mathrm{E}(X^2)}{n^2}$	**M1**	
	$= \frac{\mathrm{Var}(X) + [\mathrm{E}(X)]^2}{n^2}$	**m1**	
	$= \frac{np(1-p) + n^2p^2}{n^2}$	**A1**	
	$(= p^2 + \frac{p(1-p)}{n})$		This line need not be seen
	$\neq p^2$ therefore not unbiased	**A1**	
(ii)	$\mathrm{E}[X(X-1)] = \mathrm{E}(X^2) - \mathrm{E}(X)$	**M1** **A1**	
	$= np(1-p) + n^2p^2 - np$		
	$= n(n-1)p^2$	**A1**	
	It follows that $\frac{X(X-1)}{n(n-1)}$	**A1**	
	is an unbiased estimator for p^2.		
(c)(i)	EITHER By reversing the interpretation of success and failure, it follows that	**M1**	
	$\frac{(n-X)(n-X-1)}{n(n-1)}$	**A1**	
	is an unbiased estimator for q^2. OR		
	$q^2 = (1-p)^2 = 1 - 2p + p^2$	**M1**	
	Therefore an unbiased estimator for q^2 is		
	$1 - \frac{2X}{n} + \frac{X(X-1)}{n(n-1)}$	**A1**	This expression need not be simplified
(ii)	Since $pq = p(1-p) = p - p^2$	**M1**	
	It follows that an unbiased estimator for pq		
	$= \frac{X}{n} - \frac{X(X-1)}{n(n-1)}$	**A1**	
	$= \frac{X(n-X)}{n(n-1)}$	**A1**	

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