GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2014

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2014 examination in GCE MATHEMATICS C1-C4 & FP1-FP3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

C1

1. (*a*) (i) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^1/_2$ (or equivalent) A1

(ii) A correct method for finding the equation of AB using the candidate's value for the gradient of AB. M1

Equation of AB : $y - 3 = -{}^1/_2\,(x - 12)$ (or equivalent)

(f.t. the candidate's value for the gradient of AB) A1

(*b*) (i) Use of gradient $L \times$ gradient $AB = -1$ M1

Equation of L : $y = 2x - 1$ A1

(f.t. the candidate's value for the gradient of AB)

(ii) A correct method for finding the coordinates of D M1

$D(4, 7)$ (convincing) A1

(iii) A correct method for finding the length of $AD(BD)$ M1

$AD = \sqrt{45}$ A1

$BD = \sqrt{80}$ A1

(*c*) (i) A correct method for finding the coordinates of E M1

$E(8, 15)$ A1

(ii) $ACBE$ is a kite (c.a.o.) B1

2. (*a*) $\frac{3\sqrt{3} + 1}{5\sqrt{3} - 7} = \frac{(3\sqrt{3} + 1)(5\sqrt{3} + 7)}{(5\sqrt{3} - 7)(5\sqrt{3} + 7)}$ M1

Numerator: $45 + 21\sqrt{3} + 5\sqrt{3} + 7$ A1

Denominator: $75 - 49$ A1

$\frac{3\sqrt{3} + 1}{5\sqrt{3} - 7} = 2 + \sqrt{3}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5\sqrt{3} - 7$

(*b*) $\sqrt{12} \times \sqrt{24} = 12\sqrt{2}$ B1

$\frac{\sqrt{150}}{\sqrt{3}} = 5\sqrt{2}$ B1

$\frac{36}{\sqrt{2}} = 18\sqrt{2}$ B1

$(\sqrt{12} \times \sqrt{24}) + \frac{\sqrt{150}}{\sqrt{3}} - \frac{36}{\sqrt{2}} = -\sqrt{2}$ (c.a.o.) B1

3. (*a*) $\frac{dy}{dx} = 2x - 8$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 6$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 4$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - 2 = -{}^{1}/_{4}(x - 6)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and both m1's awarded) A1

(*b*) Putting candidate's expression for $\frac{dy}{dx} = 2$ M1

x-coordinate of $Q = 5$ A1

y-coordinate of $Q = -1$ A1

$c = -11$ A1

(f.t. candidate's expression for $\frac{dy}{dx}$ and at most one error in the enumeration of the coordinates of Q for all three A marks provided both M1's are awarded)

4. (*a*) $(1 + x)^6 = 1 + 6x + 15x^2 + 20x^3 + \ldots$

All terms correct B2

If B2 not awarded, award B1 for three correct terms

(*b*) An attempt to substitute $x = 0{\cdot}1$ in the expansion of part (*a*)

(f.t. candidate's coefficients from part (*a*)) M1

$1{\cdot}1^6 \approx 1 + 6 \times 0{\cdot}1 + 15 \times 0{\cdot}01 + 20 \times 0{\cdot}001$

(At least three terms correct, f.t. candidate's coefficients from part (*a*)) A1

$1{\cdot}1^6 \approx 1{\cdot}77$ (c.a.o.) A1

5. (*a*) $a = 4$ B1

$b = -1$ B1

$c = 7$ B1

(*b*) An attempt to substitute 1 for x in an appropriate quadratic expression

(f.t. candidate's value for b) M1

Greatest value of $\frac{1}{4x^2 - 8x + 29} = \frac{1}{25}$ (c.a.o.) A1

6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = (2k)^2 - 4 \times (k - 1) \times (7k - 4)$ A1

Putting $b^2 - 4ac < 0$ m1

$6k^2 - 11k + 4 > 0$ (convincing) A1

Finding critical values $k = {}^1/_2$, $k = {}^4/_3$ B1

A statement (mathematical or otherwise) to the effect that

$k < {}^1/_2$ or $k > {}^4/_3$ (or equivalent)

(f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = -3(x + \delta x)^2 + 8(x + \delta x) - 7$ B1

Subtracting y from above to find δy M1

$\delta y = -6x\delta x - 3(\delta x)^2 + 8\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -6x + 8$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 9 \times \frac{5}{4} \times x^{1/4} - 8 \times -\frac{1}{3} \times x^{-4/3}$ B1, B1

8. **Either:** showing that $f(2) = 0$

Or: trying to find $f(r)$ for at least two values of r M1

$f(2) = 0 \Rightarrow x - 2$ is a factor A1

$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x - 2)(6x^2 - x - 2)$ A1

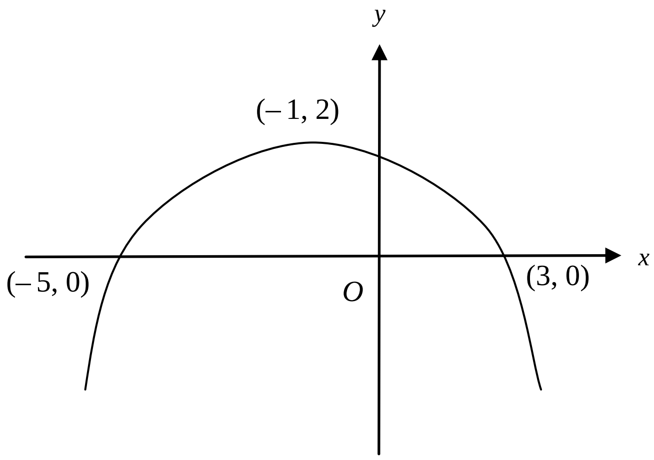
$f(x) = (x - 2)(3x - 2)(2x + 1)$ (f.t. only $6x^2 + x - 2$ in above line) A1

$x = 2, {}^2/_3, -{}^1/_2$ (f.t. for factors $3x \pm 2$, $2x \pm 1$) A1

Special case

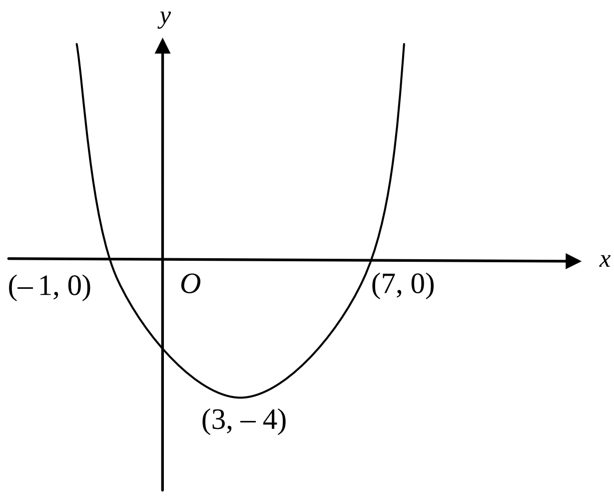
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks

9. (*a*) (i)



Concave down curve with y-coordinate of maximum = 2	B1
x-coordinate of maximum = – 1	B1
Both points of intersection with x-axis	B1

(ii)



Concave up curve with x-coordinate of minimum = 3	B1
y-coordinate of minimum = – 4	B1
Both points of intersection with x-axis	B1

(*b*) $x = 3$ (c.a.o.) B1

10. (*a*) $\frac{dy}{dx} = 3x^2 + 18x + 27$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$3(x + 3)^2 = 0 \Rightarrow x = -3$ (c.a.o) A1

$x = -3 \Rightarrow y = 4$ (c.a.o) A1

(*b*) **Either:**

An attempt to consider value of $\frac{dy}{dx}$ at $x = -3^-$ and $x = -3^+$ M1

$\frac{dy}{dx}$ has same sign at $x = -3^-$ and $x = -3^+ \Rightarrow (-3, 4)$ is a point of inflection A1

Or:

An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = -3$, $x = -3^-$ and $x = -3^+$ M1

$\frac{d^2y}{dx^2} = 0$ at $x = -3$ and $\frac{d^2y}{dx^2}$ has different signs at $x = -3^-$ and $x = -3^+$

$\Rightarrow (-3, 4)$ is a point of inflection A1

Or:

An attempt to find the value of y at $x = -3^-$ and $x = -3^+$ M1

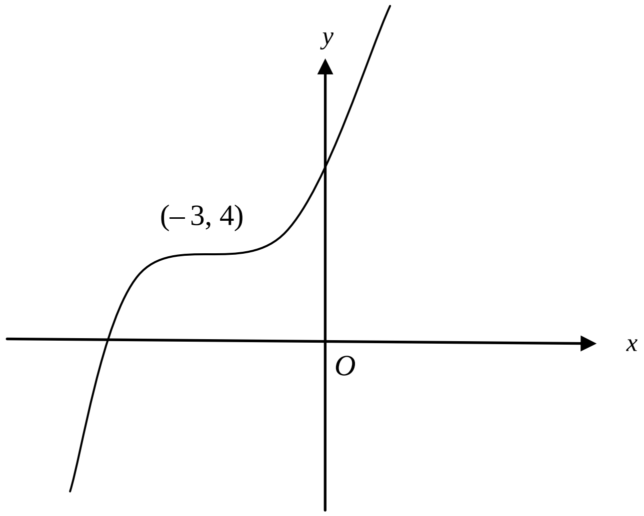
Value of y at $x = -3^- < 4$ and value of y at $x = -3^+ > 4 \Rightarrow (-3, 4)$ is a point of inflection A1

Or:

An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = -3$ M1

$\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = -3 \Rightarrow (-3, 4)$ is a point of inflection A1

(*c*)



G1

1. (*a*)

1	0·301029995		
1·5	0·544068044		
2	0·698970004		
2·5	0·812913356		
3	0·903089987	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$$I \approx \frac{0·5}{2} \times \{0·301029995 + 0·903089987 + 2(0·544068044 + 0·698970004 + 0·812913356)\}$$

$I \approx 5·31602279 \times 0·5 \div 2$

$I \approx 1·329005698$

$I \approx 1·329$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

Special case for candidates who put $h = 0·4$

1	0·301029995		
1·4	0·505149978		
1·8	0·643452676		
2·2	0·748188027		
2·6	0·832508912		
3	0·903089987	(all values correct)	B1

Correct formula with $h = 0·4$ M1

$$I \approx \frac{0·4}{2} \times\{0·301029995 + 0·903089987 + 2(0·505149978 + 0·643452676 + 0·748188027 +0·832508912))$$

$I \approx 6·662719168 \times 0·4 \div 2$

$I \approx 1·332543834$

$I \approx 1·333$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\int_1^3 \log_{10}(3x - 1)^2 \,dx \approx 2·658$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $4\cos^2\theta + 1 = 4(1 - \cos^2\theta) - 2\cos\theta$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$8\cos^2\theta + 2\cos\theta - 3 = 0 \Rightarrow (2\cos\theta - 1)(4\cos\theta + 3) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$, $\cos\theta = -\frac{3}{4}$ (c.a.o.) A1

$\theta = 60°, 300°$ B1

$\theta = 138{\cdot}59°, 221{\cdot}41°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $\alpha + 40° = 45°, 135°, \Rightarrow \alpha = 5°, 95°$ (at least one value of α) B1

$\alpha - 35° = 60°, 120°, \Rightarrow \alpha = 95°, 155°$ (at least one value of α) B1

$\alpha = 95°$ (c.a.o.) B1

(*c*) Correct use of $\frac{\sin\phi}{\cos\phi} = \tan\phi$ (o.e.) M1

$\tan\phi = \frac{10}{7}$ A1

$\phi = 55°, 235°$ (f.t $\tan\phi = a$) B1

3. (*a*) $\frac{y}{4/5} = \frac{x}{8/17}$ (o.e.) (correct use of sine rule) M1

$y = 1{\cdot}7x$ (convincing) A1

(*b*) $10{\cdot}5^2 = x^2 + y^2 - 2 \times x \times y \times (^{-13}/_{85})$

(correct use of the cosine rule) M1

Substituting $1{\cdot}7x$ for y in candidate's equation of form

$10{\cdot}5^2 = x^2 + y^2 \pm 2 \times x \times y \times {}^{13}/_{85}$ M1

$10{\cdot}5^2 = x^2 + 2{\cdot}89x^2 + 0{\cdot}52x^2$ (o.e.) A1

$x = 5$

(f.t. candidate's equation for x^2 provided both M's awarded) A1

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ n times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $\frac{n}{2}[2 \times 3 + (n - 1) \times 2] = 360$ M1

Rewriting above equation in a form ready to be solved

$2n^2 + 4n - 720 = 0$ or $n^2 + 2n - 360 = 0$ or $n(n + 2) = 360$ A1

$n = 18$ (c.a.o.) A1

(*c*) $a + 9d = 7 \times (a + 2d)$ B1

$a + 7d + a + 8d = 80$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = -5$, $d = 6$ (both values) (c.a.o.) A1

5. (*a*) $ar + ar^2 = -216$ B1

$ar^4 + ar^5 = 8$ B1

A correct method for solving the candidate's equations simultaneously

e,g multiplying the first equation by r^3 and subtracting

or eliminating a and $(1 + r)$ M1

$-216r^3 = 8$ (o.e.) A1

$r = -{}^1/_3$ (convincing) A1

(*b*) $a \times (-{}^1/_3) \times (1 - {}^1/_3) = -216 \Rightarrow a = 972$ B1

$S_\infty = \frac{972}{1 - (-{}^1/_3)}$ (correct use of formula for S_∞, f.t. candidate's derived value for a) M1

$S_\infty = 729$ (f.t. candidate's derived value for a) A1

6. (*a*) $5 \times \frac{x^{1/4}}{1/4} - 7 \times \frac{x^{3/2}}{3/2} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $16 - x^2 = x + 10$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b$ = candidate's constant m1

$(x - 2)(x + 3) = 0 \Rightarrow x = 2, -3$ (both values, c.a.o.) A1

$y = 12, y = 7$ (both values, f.t. candidate's x-values) A1

(ii) Use of integration to find the area under the curve M1

$\int 16\,dx = 16x$, $\int x^2\,dx = (1/3)x^3$, (correct integration) B1

Correct method of substitution of candidate's limits m1

$[16x - (1/3)x^3]_{-3}^{2} = (32 - 8/3) - (-48 - (-9)) = 205/3$

Use of a correct method to find the area of the trapezium (f.t. candidate's coordinates for A, B) M1

Use of candidate's values for x_A and x_B as limits and trying to find total area by subtracting area of trapezium from area under curve m1

Shaded area = 205/3 – 95/2 = 125/6 (c.a.o.) A1

7. (*a*) **Either:**

$(5x/4 - 2)\log_{10}3 = \log_{10}7$

(taking logs on both sides and using the power law) M1

$\frac{5x}{4} = \frac{(\log_{10}7 + 2\log_{10}3)}{\log_{10}3}$ A1

$x = 3{\cdot}017$ (f.t. one slip, see below) A1

Or:

$5x/4 - 2 = \log_3 7$ (rewriting as a log equation) M1

$5x/4 = \log_3 7 + 2$ A1

$x = 3{\cdot}017$ (f.t. one slip, see below) A1

Note: an answer of $x = -0{\cdot}183$ from $\frac{5x}{4} = \frac{(\log_{10}7 - 2\log_{10}3)}{\log_{10}3}$ earns M1 A0 A1

an answer of $x = 0{\cdot}183$ from $\frac{5x}{4} = \frac{(2\log_{10}3 - \log_{10}7)}{\log_{10}3}$ earns M1 A0 A1

Note: Answer only with no working earns 0 marks

(*b*) (i) $b = a^5$ (relationship between log and power) B1

(ii) $a = b^{1/5}$ (the laws of indices) B1

$\log_b a = 1/5$ (relationship between log and power) B1

8. (*a*) (i) A correct method for finding the length of *AB* M1

$AB = 20$ A1

Sum of radii = distance between centres,

$\therefore$ circles touch A1

(ii) Gradient *AP*(*BP*)(*AB*) = $\frac{\text{inc in } y}{\text{inc in } x}$ M1

Gradient $AP = \frac{9-5}{-2-1} = -\frac{4}{3}$ (o.e) A1

Use of $m_{\tan} \times m_{\text{rad}} = -1$ M1

Equation of common tangent is:

$y - 5 = \frac{3}{4}(x - 1)$ (o.e)

(f.t. one slip provided both M's are awarded) A1

(*b*) **Either:**

An attempt to rewrite the equation of *C* with l.h.s. in the form

$(x - a)^2 + (y - b)^2$ M1

$(x + 2)^2 + (y - 3)^2 = -7$ A1

Impossible, since r.h.s. must be positive ($= r^2$) A1

Or:

$g = 2$, $f = -3$, $c = 20$ and an attempt to use $r^2 = g^2 + f^2 - c$ M1

$r^2 = -7$ A1

Impossible, since r^2 must be positive A1

9. (*a*) (i) Area of sector $POQ = \frac{1}{2} \times r^2 \times 0{\cdot}9$ B1

(ii) Length of $PS = r \times \tan(0{\cdot}9)$ B1

(iii) Area of triangle $POS = \frac{1}{2} \times r \times r \times \tan(0{\cdot}9)$

(f.t. candidate's expression in *r* for the length of *PS*) B1

(*b*) $\frac{1}{2} \times r \times r \times \tan(0{\cdot}9) - \frac{1}{2} \times r^2 \times 0{\cdot}9 = 95{\cdot}22$

(f.t. candidate's expressions for area of sector and area of triangle, at least one correct) M1

$r^2 = \frac{2 \times 95{\cdot}22}{(1{\cdot}26 - 0{\cdot}9)}$ (o.e.) (c.a.o.) A1

$r = 23$ (f.t. one numerical slip) A1

1. (*a*)

0	2·197224577		
0·75	2·314217179		
1·5	2·524262696		
2·25	2·861499826		
3	3·335254744	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·75$ M1

$I \approx \frac{0·75}{3} \times \{2·197224577 + 3·335254744 + 4(2·314217179 + 2·861499826) + 2(2·524262696)\}$

$I \approx 31·28387273 \times 0·75 \div 3$

$I \approx 7·820968183$

$I \approx 7·82$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^3 \ln(16 + 2e^x)\,dx = \int_0^3 \ln(8 + e^x)\,dx + \int_0^3 \ln 2\,dx$ M1

$\int_0^3 \ln(16 + 2e^x)\,dx = 7·82 + 2·08 = 9·90$

(f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. $8(\sec^2\theta - 1) - 5\sec^2\theta = 7 + 4\sec\theta.$ (correct use of $\tan^2\theta = \sec^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant m1

$3\sec^2\theta - 4\sec\theta - 15 = 0 \Rightarrow (3\sec\theta + 5)(\sec\theta - 3) = 0$

$\Rightarrow \sec\theta = -\frac{5}{3}, \sec\theta = 3$

$\Rightarrow \cos\theta = -\frac{3}{5}, \cos\theta = \frac{1}{3}$ (c.a.o.) A1

$\theta = 126·87°, 233·13°$ B1 B1

$\theta = 70·53°, 289·47°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{d}{dx}(8xy^2) = (8x)(2y)\frac{dy}{dx} + 8y^2$ B1

$\frac{d}{dx}(2x^2) = 4x,\ \frac{d}{dx}(9) = 0$ B1

$\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$ (convincing) (c.a.o.) B1

(*b*) $\frac{dy}{dx} = 0 \Rightarrow x = 2y^2$ B1

Substitute $2y^2$ for x in equation of C M1

$9y^4 + 9 = 0$ (o.e.) (c.a.o.) A1

$9y^4 + 9 > 0$ for any real y (o.e.) and thus no such point exists A1

4. candidate's x-derivative = $2e^t$ B1

candidate's y-derivative = $-8e^{-t} + 3e^t$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{-8e^{-t} + 3e^t}{2e^t}$ (o.e.) (c.a.o.) A1

Putting candidate's $\frac{dy}{dx} = -1$, rearranging and obtaining either an equation in **both** e^t and e^{-t}, or an equation in e^{2t}, or an equation in e^{-2t}. M1

Either $e^{2t} = \frac{8}{5}$ or $e^{-2t} = \frac{5}{8}$

(f.t. one numerical slip in candidate's derived expression for $\frac{dy}{dx}$) A1

$t = 0{\cdot}235$ (c.a.o.) A1

5. (*a*) $\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{ax + b}{3x^2 - 2x - 1}$ (including $a = 0$, $b = 1$) M1

$\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{6x - 2}{3x^2 - 2x - 1}$ A1

$6x - 2 = 8x(3x^2 - 2x - 1)$ (o.e.) (f.t. candidate's a, b) A1

$12x^3 - 8x^2 - 7x + 1 = 0$ (convincing) A1

(*b*) $x_0 = -0{\cdot}6$

$x_1 = -0{\cdot}578232165$ (x_1 correct, at least 4 places after the point) B1

$x_2 = -0{\cdot}582586354$

$x_3 = -0{\cdot}581770386$

$x_4 = -0{\cdot}581925366 = -0{\cdot}5819$ (x_4 correct to 4 decimal places) B1

Let $g(x) = 12x^3 - 8x^2 - 7x + 1$

An attempt to check values or signs of $g(x)$ at $x = -0{\cdot}58185$, $x = -0{\cdot}58195$ M1

$g(-0{\cdot}58185) = 7{\cdot}35 \times 10^{-4}$, $g(-0{\cdot}58195) = -7{\cdot}15 \times 10^{-4}$ A1

Change of sign $\Rightarrow \alpha = -0{\cdot}5819$ correct to four decimal places A1

6. (*a*) (i) $\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times f(x)$ ($f(x) \neq 1$) M1

$\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times (-20x^4)$

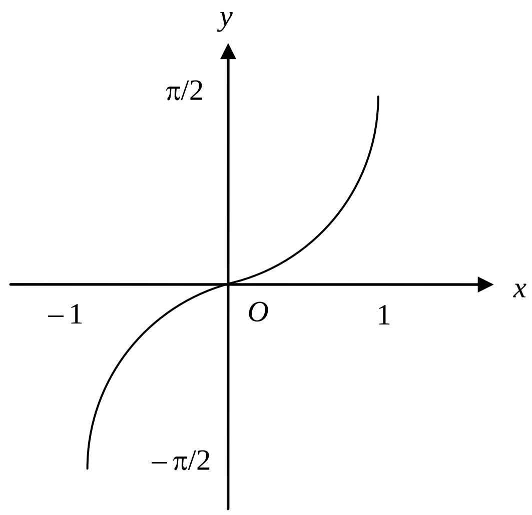
$\frac{dy}{dx} = 5x^4 \times (9 - 4x^5)^{-5/4}$ A1

(ii) $\frac{dy}{dx} = \frac{(7 - x^3) \times f(x) - (3 + 2x^3) \times g(x)}{(7 - x^3)^2}$ ($f(x), g(x) \neq 1$) M1

$\frac{dy}{dx} = \frac{(7 - x^3) \times 6x^2 - (3 + 2x^3) \times (-3x^2)}{(7 - x^3)^2}$ A1

$\frac{dy}{dx} = \frac{51x^2}{(7 - x^3)^2}$ (c.a.o.) A1

(*b*) (i)



G1

(ii) $x = \sin y \Rightarrow \frac{dx}{dy} = \cos y$ B1

$\frac{dx}{dy} = \pm\sqrt{(1 - \sin^2 y)}$ B1

The +ive sign is chosen because the graph shows the gradient to be positive E1

$\frac{dx}{dy} = \sqrt{(1 - x^2)}$ B1

$\frac{dy}{dx} = \frac{1}{\sqrt{(1 - x^2)}}$ B1

7. (*a*) (i) $\int \cos(2-5x)\,dx = k \times \sin(2-5x) + c$ ($k = 1, {}^1/_5, -5, -{}^1/_5$) M1

$\int \cos(2-5x)\,dx = -{}^1/_5 \times \sin(2-5x) + c$ A1

(ii) $\int \frac{4}{e^{3x-2}}\,dx = k \times 4 \times e^{2-3x} + c$ ($k = 1, -3, {}^1/_3\ -{}^1/_3$) M1

$\int \frac{4}{e^{3x-2}}\,dx = -{}^4/_3 \times e^{2-3x} + c$ A1

(iii) $\int \frac{5}{{}^1/_6 x - 3}\,dx = k \times 5 \times \ln|{}^1/_6 x - 3| + c$ ($k = 1, {}^1/_6, \mathbf{6}$) M1

$\int \frac{5}{{}^1/_6 x - 3}\,dx = 30 \times \ln|{}^1/_6 x - 3| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (4x+1)^{1/2}\,dx = k \times \frac{(4x+1)^{3/2}}{3/2}$ ($k = 1, 4, {}^1/_4$) M1

$\int_2^6 (4x+1)^{1/2}\,dx = \left[{}^1/_4 \times \frac{(4x+1)^{3/2}}{3/2}\right]_2^6$ A1

A correct method for substitution of limits in an expression of the form $m \times (4x+1)^{3/2}$ M1

$\int_2^6 (4x+1)^{1/2}\,dx = \frac{125}{6} - \frac{27}{6} = \frac{98}{6} = 16\cdot33$

(f.t. only for solutions of $\frac{392}{6}$ and $\frac{1568}{6}$ from $k = 1, 4$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) Choice of *a*, *b*, with one positive and one negative and one side correctly evaluated M1

Both sides of identity evaluated correctly A1

(*b*) Trying to solve $3x - 2 = 7x$ M1

Trying to solve $3x - 2 = -7x$ M1

$x = -0\cdot5$, $x = 0\cdot2$ (both values) (c.a.o.) A1

Alternative mark scheme

$(3x-2)^2 = 7^2 \times x^2$ (squaring both sides) M1

$40x^2 + 12x - 4 = 0$ (o.e.) (c.a.o.) A1

$x = -0\cdot5$, $x = 0\cdot2$ (both values, f.t. one slip in quadratic) A1

9. (*a*) $f(x) = (x - 4)^2 - 9$ B1

(*b*) $y = (x - 4)^2 - 9$ and an attempt to isolate x
(f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) M1
$x = (\pm)\sqrt{(y + 9)} + 4$
(f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
$x = -\sqrt{(y + 9)} + 4$ (o.e.) (c.a.o.) A1
$f^{-1}(x) = -\sqrt{(x + 9)} + 4$ (o.e.)
(f.t. only incorrect choice of sign in front of the √ sign and candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1

10. (*a*) $R(g) = [2k - 4, \infty)$ B1

(*b*) (i) $2k - 4 \geq -2$ M1
$k \geq 1$ ($\Rightarrow$ least value of k is 1)
(f.t. candidate's $R(g)$ provided it is of form $[a, \infty)$) A1

(ii) $fg(x) = (kx - 4)^2 + k(kx - 4) - 8$ B1

(iii) $(3k - 4)^2 + k(3k - 4) - 8 = 0$
(substituting 3 for x in candidate's expression for $fg(x)$ and putting equal to 0) M1
Either $12k^2 - 28k + 8 = 0$ or $6k^2 - 14k + 4 = 0$
or $3k^2 - 7k + 2 = 0$ (c.a.o.) A1
$k = {}^1/_3, 2$ (f.t. candidate's quadratic in k) A1
$k = 2$ (c.a.o.) A1

1. $9x^2 - 5x \times 2y\frac{dy}{dx} - 5y^2 + 8y^3\frac{dy}{dx} = 0$ $\left(-5x \times 2y\frac{dy}{dx} - 5y^2\right)$ B1

$\left(9x^2 + 8y^3\frac{dy}{dx}\right)$ B1

Either $\frac{dy}{dx} = \frac{9x^2 - 5y^2}{10xy - 8y^3}$ **or** $\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 2$ in candidate's expression **and** the use of $grad_{normal} \times grad_{tangent} = -1$ M1

Equation of normal: $y - 2 = -4(x - 1)$

$\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

2. (*a*) $f(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-4)}$ (correct form) M1

$5x^2 + 7x + 17 \equiv A(x - 4) + B(x + 1)(x - 4) + C(x + 1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3$, $C = 5$, $B = 0$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \frac{2}{(x+1)^2}$ M1

$\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{-1}{(x+1)^2} + \frac{5}{(x-4)}$

(f.t. candidates values for A, B, C) A1

3. (*a*) $\dfrac{2\tan x}{1-\tan^2 x} = 3\cot x$ (correct use of formula for $\tan 2x$) M1

$\dfrac{2\tan x}{1-\tan^2 x} = \dfrac{3}{\tan x}$ (correct use of $\cot x = \dfrac{1}{\tan x}$) M1

$\tan^2 x = {}^3/_5$ (o.e.) A1

$x = 37{\cdot}76°, 142{\cdot}24°$ (both values)

(f.t. $a\tan^2 x = b$ provided both M1's are awarded) A1

(*b*) (i) $R = 29$ B1

Correctly expanding $\sin(\theta - \alpha)$ and using either $29\cos\alpha = 21$ **or** $29\sin\alpha = 20$ **or** $\tan\alpha = \dfrac{20}{21}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 43{\cdot}6°$ (c.a.o) A1

(ii) Greatest value of $\dfrac{1}{21\sin\theta - 20\cos\theta + 31} = \dfrac{1}{29\times(\pm1)+31}$

(f.t. candidate's value for R) M1

Greatest value $= \dfrac{1}{2}$ (f.t. candidate's value for R) A1

Corresponding value for $\theta = 313{\cdot}6°$ (o.e.)

(f.t. candidate's value for α) A1

4.

Volume $= \pi\displaystyle\int_0^{\pi/4}(3 + 2\sin x)^2\,dx$ B1

Correct use of $\sin^2 x = \dfrac{(1-\cos 2x)}{2}$ M1

Integrand $= (9 + 2 + 12\sin x - 2\cos 2x)$ (c.a.o.) A1

$\displaystyle\int(a + b\sin x + c\cos 2x)\,dx = (ax - b\cos x + \frac{c}{2}\sin 2x)$ ($a \neq 0, b \neq 0, c \neq 0$) B1

Correct substitution of correct limits in candidate's integrated expression of form $(ax - b\cos x + \dfrac{c}{2}\sin 2x)$ ($a \neq 0, c \neq 0$) M1

Volume = 35 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. $(1-2x)^{1/2} = 1 + (1/2)\times(-2x) + \dfrac{(1/2)\times(1/2-1)\times(-2x)^2}{1\times 2} + \ldots$

(– 1 each incorrect term) B2

$\dfrac{1}{1+4x} = 1 + (-1)\times(4x) + \dfrac{(-1)\times(-2)\times(4x)^2}{1\times 2} + \ldots$

(– 1 each incorrect term) B2

$6\sqrt{1-2x} - \dfrac{1}{1+4x} = 5 - 2x - 19x^2 + \ldots$

(– 1 each incorrect term) B2

Expansion valid for $|x| < 1/4$ (o.e.) B1

6. (*a*) candidate's *x*-derivative = 2

candidate's *y*-derivative = $15t^2$ (at least one term correct)

and use of

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{15}{2}t^2$ (o.e.) (c.a.o.) A1

Equation of tangent at *P*: $y - 5p^3 = \dfrac{15}{2}p^2(x-2p)$

(f.t. candidate's expression for $\dfrac{dy}{dx}$) m1

$2y = 15p^2x - 20p^3$ (convincing) A1

(*b*) Substituting $p = 1$, $x = 2q$, $y = 5q^3$ in equation of tangent M1

$q^3 - 3q + 2 = 0$ (convincing) A1

Putting $f(q) = q^3 - 3q + 2$

Either $f(q) = (q-1)(q^2 + q - 2)$ **or** $f(q) = (q+2)(q^2 - 2q + 1)$ M1

Either $f(q) = (q-1)(q-1)(q+2)$ **or** $q = 1$, $q = -2$ A1

$q = -2$ A1

7. (*a*) $u = \ln 2x \Rightarrow du = 2 \times \frac{1}{2x}\,dx$ (o.e.) B1

$dv = x^4\,dx \Rightarrow v = \frac{1}{5}x^5$ (o.e.) B1

$\int x^4 \ln 2x\,dx = \ln 2x \times \frac{1}{5}x^5 - \int \frac{1}{5}x^5 \times \frac{1}{x}\,dx$ (o.e.) M1

$\int x^4 \ln 2x\,dx = \ln 2x \times \frac{1}{5}x^5 - \frac{1}{25}x^5 + c$ (c.a.o.) A1

(*b*) $\int \sqrt{(10\cos x - 1)}\sin x\,dx = \int k \times u^{1/2}\,du$ ($k = -\frac{1}{10}, \frac{1}{10}$ or ± 10) M1

$\int a \times u^{1/2}\,du = a \times \frac{u^{3/2}}{3/2}$ B1

$\int_0^{\pi/3} \sqrt{(10\cos x - 1)}\sin x\,dx = k\left[\frac{u^{3/2}}{3/2}\right]_9^4$ or $k\left[\frac{(10\cos x - 1)^{3/2}}{3/2}\right]_0^{\pi/3}$ B1

$\int_0^{\pi/3} \sqrt{(10\cos x - 1)}\sin x\,dx = \frac{19}{15} = 1{\cdot}27$ (c.a.o.) A1

8. (*a*) $\frac{dV}{dt} = kV$ B1

(*b*) $\int \frac{dV}{V} = \int k\,dt$ M1

$\ln V = kt + c$ A1

$V = e^{kt + c} = Ae^{kt}$ (convincing) A1

(*c*) (i) $292 = Ae^{2k}$

$637 = Ae^{28k}$ (both values) B1

Dividing to eliminate A M1

$\frac{637}{292} = e^{26k}$ A1

$k = \frac{1}{26}\ln\left[\frac{637}{292}\right] = 0{\cdot}03$ A1

(ii) $A = 275$ B1

(iii) When $t = 0$, initial value of investment = £275

(f.t. candidate's derived value for A) B1

9. (*a*) $\mathbf{p.q} = -18$ B1

$|\mathbf{p}| = \sqrt{14}$, $|\mathbf{q}| = \sqrt{105}$ (at least one correct) B1

Correctly substituting candidate's derived values in the formula

$\mathbf{p.q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos\theta$ M1

$\theta = 118°$ (c.a.o.) A1

(*b*) (i) Use of $\mathbf{CD} = \mathbf{CO} + \mathbf{OD}$ and the fact that $\mathbf{OC} = \frac{1}{2}\mathbf{b}$ and

$\mathbf{OD} = 2\mathbf{a}$, leading to printed answer $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$

(convincing) B1

Use of $\frac{1}{2}\mathbf{b} + \lambda\mathbf{CD}$ (o.e.) to find vector equation of *CD* M1

Vector equation of *CD*: $\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1-\lambda)\mathbf{b}$ (convincing) A1

(ii) **Either:**

Either substituting $\frac{1}{3}$ for λ in the vector equation of *CD*

or substituting 2 for μ in the vector equation of *L* M1

At least one of these position vectors $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1

Both position vectors $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \Rightarrow$ this must be the position vector of the point of intersection *E* A1

Or:

$2\lambda = \frac{\mu}{3}$

$\frac{1}{2}(1-\lambda) = \frac{1}{3}(\mu - 1)$

(comparing candidate's coefficients of **a** and **b** and an attempt to solve) M1

$\lambda = \frac{1}{3}$ or $\mu = 2$ A1

$\mathbf{OE} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (convincing) A1

(iii) **Either**: *E* lies on *AB* and is such that *AE* : *EB* = 1 : 2 (o.e.)

Or: *E* is the point of intersection of *AB* and *CD* B1

10. Squaring both sides we have

$1 + 2\sin\theta\cos\theta > 2$ B1

$\sin 2\theta > 1$ B1

Contradiction, since the sine of any angle ≤ 1 B1

FP1

Ques	Solution	Mark	Notes
1(a)	$f(x+h)-f(x)=\frac{1}{(x+h)^2}-\frac{1}{x^2}$	M1A1	
	$=\frac{x^2-(x+h)^2}{x^2(x+h)^2}$	A1	
	$=\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}$		
	$=\frac{-2xh-h^2}{x^2(x+h)^2}$	A1	
	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	M1	
	$=\lim_{h\to 0}\frac{-2xh-h^2}{hx^2(x+h)^2}=-\frac{2}{x^3}$	A1	
(b)	$\ln f(x)=x\ln\sec x$	B1	
	$\frac{f'(x)}{f(x)}=\ln\sec x+\frac{x\sec x\tan x}{\sec x}$	B1B1	B1 each side
	$f'(x)=(\sec x)^x(\ln\sec x+x\tan x)$	B1	
2(a)	$S_n=\sum_{r=1}^{n}r(r+3)=\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}3r$	M1	
	$=\frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}$	A1	
	$=\frac{n(n+1)}{6}(2n+1+9)$	m1	
	$=\frac{n(n+1)(n+5)}{3}$ or $\frac{n^3+6n^2+5n}{3}$ oe	A1	
(b)	$T_n=S_n-S_{n-1}$	M1	
	$=n(n+3)-(n-1)(n+2)$	A1	
	$=n^2+3n-(n^2+n-2)$		
	$=2(n+1)$	A1	

Ques	Solution	Mark	Notes
3(a)	$x + 2y + 4z = 3$ $x - y + 2z = 4$ $4x - y + 10z = k$		
	Attempting to use row operations,	M1	
	$x + 2y + 4z = 3$ $3y + 2z = -1$	A1	
	$9y + 6z = 12 - k$	A1	
	Since the 3rd equation is three times the 2nd equation, it follows that	M1	
	$12 - k = -3$; $k = 15$	A1	
(b)	Let $z = \alpha$	M1	
	$y = -\dfrac{(1+2\alpha)}{3}$	A1	
	$x = \dfrac{11-8\alpha}{3}$ (or equivalent)	A1	
4	EITHER $z = \dfrac{1+2i}{1-i} \times \dfrac{1+i}{1+i}$	M1	
	$= \dfrac{1+2i+i+2i^2}{1-i+i-i^2)}$	A1	
	$= \dfrac{-1+3i}{2}$	A1	
	Mod(z) = $\dfrac{\sqrt{10}}{2}$ $(\sqrt{2.5}, 1.58)$	B1	FT their z
	Arg(z) = $\tan^{-1}(-3) + \pi$ $= 1.89$ (108°)	M1A1	Award M1A0 for $\tan^{-1}(-3)$ (– 1.25 or – 72°)
	OR		
	Mod(1 + 2i) $= \sqrt{5}$	B1	
	Mod(1 – i) = $\sqrt{2}$	B1	
	Mod$\left(\dfrac{1+2i}{1-i}\right) = \sqrt{\dfrac{5}{2}}$	B1	
	Arg(1 + 2i) = $\tan^{-1} 2 = 1.107..$	B1	FT one incorrect mod
	Arg(1 – i) = $\tan^{-1}(-1) = -0.785..$	B1	
	Arg$\left(\dfrac{1+2i}{1-i}\right) = 1.107.. + 0.785..$ $= 1.89$ (108°)	B1	FT one incorrect arg

Ques	Solution	Mark	Notes
5(a)	$\alpha+\beta+\gamma=-2,\ \beta\gamma+\gamma\alpha+\alpha\beta=2, \alpha\beta\gamma=-3$	B1	
	$\beta\gamma\times\gamma\alpha+\beta\gamma\times\alpha\beta+\gamma\alpha\times\alpha\beta=\alpha\beta\gamma(\alpha+\beta+\gamma)$	M1	FT their first line if one error
	$=-3\times-2=6$	A1	
	$\beta\gamma\times\gamma\alpha\times\alpha\beta=(\alpha\beta\gamma)^2=9$	M1A1	
	The required equation is		
	$x^3-2x^2+6x-9=0$	B1	FT previous values
(b)	$\alpha^2+\beta^2+\gamma^2$		
	$=(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)$	M1	
	= 4 – 2 × 2 = 0 (convincing)	A1	
	The equation has 1 real root	B1	
	Any valid reason, eg cubic equations have either 1 or 3 real roots and since $\alpha^2+\beta^2+\gamma^2=0$, not all roots are real	B1	
6(a)	Det(A) = $\lambda(2-\lambda)+2\times4+3(-\lambda-2)$	M1	
	$=-\lambda^2-\lambda+2$	A1	
	A is singular when $-\lambda^2-\lambda+2=0$	M1	
	$\lambda=1,-2$	A1	
(b)(i)	A = $\begin{bmatrix}-1 & 2 & 3\\-1 & 1 & 1\\2 & -1 & 2\end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix}3 & 4 & -1\\-7 & -8 & 3\\-1 & -2 & 1\end{bmatrix}$ si	M1A1	Award M1 if at least 5 cofactors are correct
	Adjugate matrix = $\begin{bmatrix}3 & -7 & -1\\4 & -8 & -2\\-1 & 3 & 1\end{bmatrix}$	A1	No FT on cofactor matrix
(ii)	Determinant = 2	B1	
	Inverse matrix = $\frac{1}{2}\begin{bmatrix}3 & -7 & -1\\4 & -8 & -2\\-1 & 3 & 1\end{bmatrix}$	B1	FT the adjugate or determinant

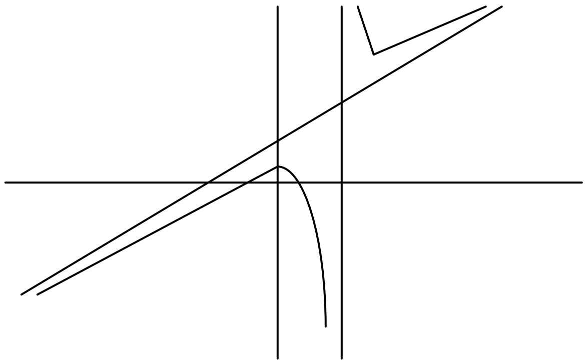
Ques	Solution	Mark	Notes
7(a)	Rotation matrix = $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	Translation matrix = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	Ref matrix in *y*-axis = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	
	$T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$	M1	
	$\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	A1	
	$= \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$		
(b)	EITHER The general point on the line is given by $(\lambda, 2\lambda + 1)$	M1	
	Consider $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \lambda \\ 2\lambda+1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2\lambda-2 \\ -\lambda+2 \\ 1 \end{bmatrix}$	m1	
	$x = -2\lambda - 2; y = -\lambda + 2$	A1	
	Eliminating λ, $x - 2y + 6 = 0$ oe	A1	
	OR Consider $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$	M1	
	$-y - 1 = X, -x + 2 = Y$	A1	
	$y = -1 - X, x = 2 - Y$	A1	
	$y = 2x + 1$ leading to $x - 2y + 6 = 0$	A1	

Ques	Solution	Mark	Notes
8	Putting n = 1, the formula gives 1 which is the first term of the series so the result is true for n = 1. Assume formula is true for $n = k$, ie $\left(\sum_{r=1}^{k} r \times 2^{r-1} = 1 + 2^k(k-1) \right)$ Consider, for $n = k + 1$, $\sum_{r=1}^{k+1} r \times 2^{r-1} = \sum_{r=1}^{k} r \times 2^{r-1} + 2^k(k+1)$ $= 1 + 2^k(k-1) + 2^k(k+1)$ $= 1 + 2^{k+1}k$ Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	B1 M1 M1 A1 A1 A1 A1	Award the final A1 only if a correct conclusion is made and the proof is correctly laid out
9(a) (b)	$u + \mathrm{i}v = (x + \mathrm{i}y)(x - 1 + \mathrm{i}y)$ $= x(x-1) - y^2 + \mathrm{i}(xy + xy - y)$ Equating real and imaginary parts, $u = x(x-1) - y^2$ $v = y(2x-1)$ Putting $y = -x$, $u = x(x-1) - x^2 = -x$ $v = -x(2x-1)$ Eliminating x, $v = u(-2u-1)$ cao (oe)	M1 A1 m1 A1 M1 A1 A1 m1 A1	 FT their expressions from (a)

Ques	Solution	Mark	Notes
1(a)	$f(-x) = \dfrac{((-x)^2+1)}{-x((-x)^2+2)} = -f(x)$	**M1A1**	
	Therefore f is odd.	**A1**	
(b)	Let		
	$\dfrac{x^2+1}{x(x^2+2)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+2} = \dfrac{A(x^2+2)+x(Bx+C)}{x(x^2+2)}$	**M1**	
	$A = \dfrac{1}{2}; B = \dfrac{1}{2}; C = 0$	**A1A1A1**	
	$\left(\dfrac{x^2+1}{x(x^2+2)} = \dfrac{1}{2x} + \dfrac{x}{2(x^2+2)}\right)$		
2	$u = \sin^2 x \Rightarrow \mathrm{d}u = 2\sin x \cos x \mathrm{d}x$,	**B1**	
	$[0,\pi/2] \to [0, 1]$	**B1**	
	$I = \int_0^1 \dfrac{\mathrm{d}u}{\sqrt{4-u^2}}$	**M1**	
	$= \left[\sin^{-1}(\dfrac{u}{2})\right]_0^1$	**A1**	FT a multiple of this
	$= \pi/6$ cao	**A1**	
3(a)	Denoting the two functional expressions by f_1, f_2		
	$f_1(0) = 1, f_2(0) = 1$	**M1A1**	
	Therefore f is continuous when $x = 0$.	**A1**	No FT
(b)	$f_1'(x) = 2e^{2x}, f_2'(x) = 2(1+x)$	**M1**	
	$f_1'(0) = 2, f_2'(0) = 2$	**A1**	
	Therefore f' is continuous when $x = 0$.	**A1**	No FT
4(a)	$\lvert z\rvert = 2, \arg(z) = \pi/3$	**B1B1**	
(b)	$\text{Root}1 = \sqrt[3]{2}(\cos\pi/9 + \mathrm{i}\sin\pi/9) = 1.184 + 0.431\mathrm{i}$	**M1A1**	Penalise lack of accuracy once only
	$\text{R}2 = \sqrt[3]{2}(\cos 7\pi/9 + \mathrm{i}\sin 7\pi/9) = -0.965 + 0.810\mathrm{i}$	**M1A1**	
	$\text{R}3 = \sqrt[3]{2}(\cos 13\pi/9 + \mathrm{i}\sin 13\pi/9) = -0.219 - 1.241\mathrm{i}$	**M1A1**	

Ques	Solution	Mark	Notes
5	The equation can be rewritten $2\sin 3\theta\cos 2\theta = \cos 2\theta$ $\cos 2\theta(2\sin 3\theta - 1) = 0$ Either cos2θ = 0, $2\theta = 2n\pi \pm \frac{\pi}{2}$ $\theta = n\pi \pm \frac{\pi}{4}$ Or $\sin 3\theta = 1/2$ $3\theta = n\pi + (-1)^n \frac{\pi}{6}$ or $\theta = \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$	**M1A1** **A1** **M1** **A1** **M1** **A1** **A1**	Accept equivalent answers Accept degrees throughout
6	Consider $\cos 6\theta + \mathrm{i}\sin 6\theta = (\cos\theta + \mathrm{i}\sin\theta)^6$ Expanding and equating imaginary terms, isin6θ = $6\cos^5\theta(\mathrm{i}\sin\theta) + 20\cos^3\theta(\mathrm{i}\sin\theta)^3 + 6\cos\theta(\mathrm{i}\sin\theta)^5$ $\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta$ $+ 6\cos\theta\sin^5\theta$ $\frac{\sin 6\theta}{\sin\theta} = 6\cos^5\theta - 20\cos^3\theta(1-\cos^2\theta)$ $+ 6\cos\theta(1-\cos^2\theta)^2$ = $32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$ Letting $\theta \to \pi$ in the right hand side, Limit = – 32 + 32 – 6 = – 6	**M1** **m1** **A1** **A1** **A1** **A1** **M1** **A1**	 FT their expression in the line above

Ques	Solution	Mark	Notes
7(a)(i)	The equation can be rewritten as $\frac{x^2}{9}+\frac{y^2}{4}=1$	**M1**	
	In the usual notation, a = 3, b = 2.	**A1**	
	$e=\sqrt{\frac{a^2-b^2}{a^2}}=\frac{\sqrt{5}}{3}$	**A1**	FT their a,b
(ii)	The foci are $(\pm ae,0)$, ie $(\pm\sqrt{5},0)$ cao	**A1**	
(b)(i)	Substituting the x,y expressions, $4\times 9\cos^2\theta+9\times 4\sin^2\theta=36(\cos^2\theta+\sin^2\theta)=36$ showing that P lies on the ellipse.	**B1**	
(ii)	EITHER $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta}=-\frac{2\cos\theta}{3\sin\theta}$ OR $8x+18y\frac{\mathrm{d}y}{\mathrm{d}x}=0;\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{8x}{18y}=-\frac{2\cos\theta}{3\sin\theta}$	**M1A1**	
	This equation of the tangent is $y-2\sin\theta=-\frac{2\cos\theta}{3\sin\theta}(x-3\cos\theta)$	**M1**	
	$3y\sin\theta-6\sin^2\theta=-2x\cos\theta+6\cos^2\theta$ $3y\sin\theta+2x\cos\theta=6$ (convincing)	**A1**	
(iii)	Putting y = 0, R is the point $\left(\frac{3}{\cos\theta},0\right)$	**B1**	
	Putting x = 0, S is the point $\left(0,\frac{2}{\sin\theta}\right)$	**B1**	
	So M is the point $\left(\frac{3}{2\cos\theta},\frac{1}{\sin\theta}\right)$	**B1**	
	$x=\frac{3}{2\cos\theta},y=\frac{1}{\sin\theta}$ Eliminating θ,	**M1**	
	$\cos\theta=\frac{3}{2x};\sin\theta=\frac{1}{y}$	**A1**	
	$\frac{9}{4x^2}+\frac{1}{y^2}=\cos^2\theta+\sin^2\theta=1$	**A1**	

Ques	Solution	Mark	Notes
8(a)	(0,2) ; (– 4,0) ; (2,0)	**B1**	
(b)(i)	$x = 4$	**B1**	
(ii)	$f(x) = x + 6 + \dfrac{16}{x-4}$	**M1A1**	M1 any valid method
	Oblique asymptote is $y = x + 6$.	**A1**	
(c)	$f'(x) = 1 - \dfrac{16}{(x-4)^2}$ or $\dfrac{x^2 - 8x}{(x-4)^2}$	**B1**	
	At a stationary point, $f'(x) = 0$	**M1**	
	$(x-4)^2 = 16$ or $x^2 - 8x = 0$	**A1**	
	Stationary points are (0,2) ; (8,18)	**A1**	
(d)		**G1** **G1** **G1**	LH branch RH branch Asymptotes
(e)(i)	$f(-7) = -27/11$; $f(3) = -7$	**M1**	
	$f(S) = [-7,2]$	**A1**	
(ii)	Solve $\dfrac{(x+4)(x-2)}{x-4} = -7$	**M1**	
	$x^2 + 9x - 36 = 0$	**A1**	
	$x = -12, 3$	**A1**	
	$f^{=1}(S) = [-12,3]$	**A1**	

Ques	Solution	Mark	Notes
1(a)	Let $y = \sinh^{-1} x$ so that $x = \sinh y = \dfrac{e^y - e^{-y}}{2}$	**M1**	
	$e^{2y} - 2xe^y - 1 = 0$	**A1**	
	$e^y = \dfrac{2x \pm \sqrt{4x^2 + 4}}{2}$	**A1**	
	$y = \ln\left(x + \sqrt{x^2 + 1}\right)$		
	rejecting the negative sign since $e^y > 0$	**A1**	
(b)			
	Substituting for cosh2x,		
	$1 + 2\sinh^2 x = 2\sinh x + 5$	**M1**	
	$\sinh^2 x - \sinh x - 2 = 0$	**A1**	
	Solving for sinh$x$,		
	sinh$x$ = –1, 2	**M1A1**	
	$x = \ln(-1 + \sqrt{2}); \ln(2 + \sqrt{5})$	**A1**	
2(a)	Consider		
	$\dfrac{d}{dx}(3 - x)^{1/3} = \dfrac{-(3 - x)^{-2/3}}{3}$	M1A1	
	$= -0.2295\ldots$ when $x = 1.25$	A1	Allow any x between 1.2 and 1.3 M1A0A1 if negative sign omitted
	The sequence converges because this is less than 1 in modulus.	A1	FT the f' value if M1 awarded
	$x_0 = 1.25$		
	$x_1 = 1.205071132$	M1A1	
	$x_2 = 1.215296967$		
	$x_3 = 1.212984693$		
	$x_4 = 1.213508318$		
	$x_5 = 1.21338978$		
	$x_6 = 1.213416617$	A1	
	α = 1.2134 correct to 4 decimal places.	A1	

Ques	Solution	Mark	Notes
(b)	The Newton-Raphson iteration is		
	$x_{n+1} = x_n - \frac{(x_n{}^3 + x_n - 3)}{3x_n^2 + 1}$ or $\frac{2x_n{}^3 + 3}{3x_n{}^2 + 1}$	M1A1	
	$x_0 = 1.25$		
	$x_1 = 1.214285714$	M1A1	
	$x_2 = 1.213412176$		
	$x_3 = 1.213411663$	A1	
	($x_4 = 1.213411663$)		
	$\alpha = 1.213412$ correct to 6 decimal places	A1	
3(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{sech}x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\cosh x}\right)$		
	$= -\frac{\sinh x}{\cosh^2 x} = -\mathrm{sech}x \tanh x$	B1	Convincing
(b)	$f'(x) = \mathrm{sech}^2 x$	B1	
	$f''(x) = -2\mathrm{sech}^2 x \tanh x$	B1	FT 1 slip
	$f'''(x) = 4\mathrm{sech}^2 x \tanh^2 x - 2\mathrm{sech}^4 x$	B1	
	$f(0) = 0,\ f'(0) = 1,\ f''(0) = 0,\ f'''(0) = -2$	B1	
	The Maclaurin series for tanh$x$ is		
	$x - \frac{x^3}{3} + \ldots$	M1A1	
(c)	$(1 + x)\tanh x \approx x + x^2 - \frac{x^3}{3} - \frac{x^4}{3}$	B1	FT their series
	$\int_0^{0.5} (1 + x)\tanh x\mathrm{d}x \approx \int_0^{0.5} (x + x^2 - \frac{x^3}{3} - \frac{x^4}{3})\mathrm{d}x$	M1	FT 1 slip
	$= \left[\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{12} - \frac{x^5}{15}\right]_0^{0.5}$	A1	
	= 0.159 cao	A1	

Ques	Solution	Mark	Notes
4	$\mathrm{d}x = \dfrac{2\mathrm{d}t}{1+t^2}; [0, \pi/2] \to [0,1]$	B1B1	
	$I = \displaystyle\int_0^1 \dfrac{1}{2-\left(\dfrac{1-t^2}{1+t^2}\right)} \times \dfrac{2\mathrm{d}t}{1+t^2}$	M1A1	
	$= \displaystyle\int_0^1 \dfrac{2}{3t^2+1}\mathrm{d}t$	A1	
	$= \dfrac{2}{3}\displaystyle\int_0^1 \dfrac{1}{t^2+1/3}\mathrm{d}t$	A1	
	$= \dfrac{2\sqrt{3}}{3}\left[\tan^{-1}(t\sqrt{3}\right]_0^1$	A1	
	$= \dfrac{2\sqrt{3}\pi}{9}$ (1.21) cao	A1	
5(a)	$I_n = -\dfrac{1}{2}\displaystyle\int_0^1 x^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{-x^2})\,\mathrm{d}x$	M1	
	$= -\dfrac{1}{2}\left[x^{n-1}\mathrm{e}^{-x^2}\right]_0^1 + \dfrac{n-1}{2}\displaystyle\int_0^1 x^{n-2}\mathrm{e}^{-x^2}\mathrm{d}x$	A1A1	
	$= -\dfrac{\mathrm{e}^{-1}}{2} + \left(\dfrac{n-1}{2}\right)I_{n-2}$		
(b)			
	$I_1 = \displaystyle\int_0^1 x\mathrm{e}^{-x^2}\mathrm{d}x = -\dfrac{1}{2}\left[\mathrm{e}^{-x^2}\right]_0^1$	M1A1	M1A1A1 for evaluating I_1 at any stage
	$= \dfrac{1}{2}\left(1-\mathrm{e}^{-1}\right)$	A1	
	$I_5 = -\dfrac{\mathrm{e}^{-1}}{2} + 2I_3$	M1	
	$= -\dfrac{\mathrm{e}^{-1}}{2} + 2\left(-\dfrac{\mathrm{e}^{-1}}{2} + I_1\right)$	M1	
	$= 1 - 2.5\mathrm{e}^{-1}$	A1	

Ques	Solution	Mark	Notes
6(a)	Consider		
	$y = r\sin\theta$	M1	
	$= (\sin\theta + \cos\theta)\sin\theta$	A1	
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = (\cos\theta - \sin\theta)\sin\theta + \cos\theta(\sin\theta + \cos\theta)$	M1	
	$= \sin 2\theta + \cos 2\theta$	A1	FT 1 slip
	The tangent is parallel to the initial line where		
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$	M1	
	$\tan 2\theta = -1$	A1	
	$\theta = \frac{3\pi}{8}$ (1.18, 67.5°)	A1	
	$r = 1.31$	A1	
(b)	Area = $\frac{1}{2}\int r^2 \mathrm{d}\theta$	M1	
	$= \frac{1}{2}\int_0^{\pi/2} (\sin\theta + \cos\theta)^2 \mathrm{d}\theta$	A1	
	$= \frac{1}{2}\int_0^{\pi/2} (1 + \sin 2\theta) \mathrm{d}\theta$	A1	
	$= \frac{1}{2}\left[\theta - \frac{1}{2}\cos 2\theta\right]_0^{\pi/2}$	A1	
	$= \frac{\pi}{4} + \frac{1}{2}$ (1.29) cao	A1	

Ques	Solution	Mark	Notes
7(a)	$x = a\sinh\theta \rightarrow \mathrm{d}x = a\cosh\theta\mathrm{d}\theta$	B1	
	$I = \int\sqrt{a^2(1+\sinh^2\theta)}\ a\cosh\theta\mathrm{d}\theta$	M1	
	$= a^2\int\cosh^2\theta\,\mathrm{d}\theta$	A1	
	$= \dfrac{a^2}{2}\int(1+\cosh 2\theta)\,\mathrm{d}\theta$	A1	
	$= \dfrac{a^2}{2}\left(\theta+\sinh\theta\cosh\theta\right)$	A1	FT line above
	$= \dfrac{a^2}{2}\left(\sinh^{-1}\left(\dfrac{x}{a}\right)+\dfrac{x\sqrt{x^2+a^2}}{a^2}\right)$ (+ C)		Answer given
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x$	B1	
	$L = \int\sqrt{1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$	M1	
	$= \int_0^1\sqrt{1+4x^2}\ \mathrm{d}x$	A1	
	$= 2\int_0^1\sqrt{(x^2+1/4)}\mathrm{d}x$	A1	
	$= \dfrac{2}{8}\left[\sinh^{-1}2x+4x\sqrt{x^2+1/4}\right]_0^1$	A1	
	$= 1.48$	A1	

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GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3 AS/Advanced

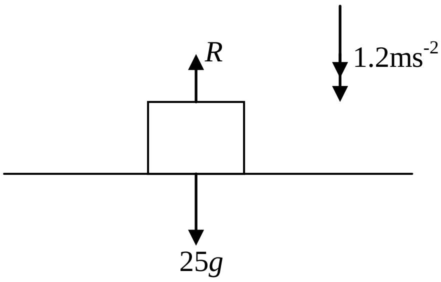
SUMMER 2014

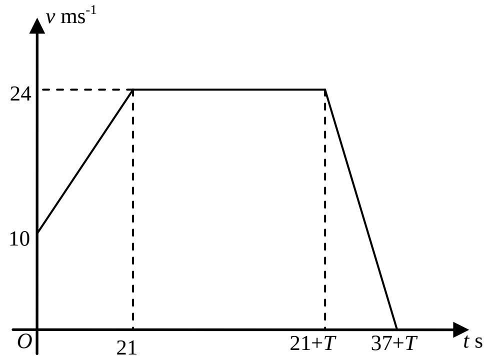
INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2014 examination in GCE MATHEMATICS - M1-M3 & S1-S3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

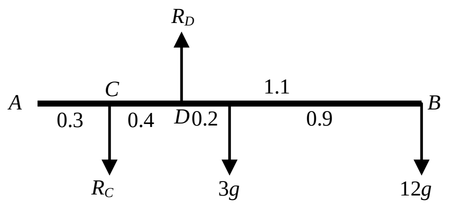
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

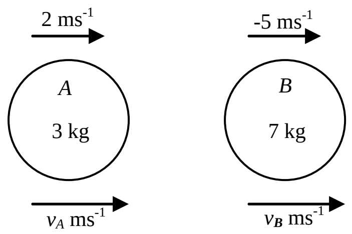
Q	Solution	Mark	Notes
1(a)			
	Apply N2L to crate	M1	*R* and 25*g* opposing. Dim. Correct
	$25g - R = 25 \times 1.2$	A1	correct equation Any form
	$R =$ <u>215 (N)</u>	A1	
1(b)	$R = 25g =$ <u>245 (N)</u>	B1	

Q	Solution	Mark	Notes
2(a)	Use of $v = u + at$ with u=10, v=24, t=21	M1	oe
	$24 = 10 + 21a$	A1	
	$a = \frac{2}{3}$ (ms^{-2})	A1	accept anything derived from $\frac{2}{3}$ rounded correctly
2(b)	$s = \frac{1}{2}(u + v)t$ with v=0, u=24, t=16	M1	oe
	$s = \frac{1}{2} \times 24 \times 16$	A1	
	s = <u>192 (m)</u>	A1	
2(c)			
		B1	(0, 10) to (21, 24)
		B1	(21, 24) to (21+T, 24)
		B1	(21+T, 24) to (37+T, 0)
		B1	all labels, units and shape.
2(d)	Area under graph = 15000	M1	used
	0.5(10+24)21 + 24T + 192 = 15000	A1	ft (b)
		B1	0.5(10+24)21 or 24T Ft graph
	24T = 14451		
	T = <u>602(.125)</u>	A1	Accept 600 from correct working. Cao.

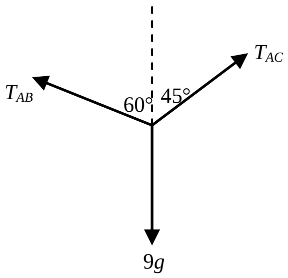
Q	Solution	Mark	Notes
3(a)	Resolve perpendicular to plane		
	$R = mg\cos\alpha$	M1	sin/cos
	$F = \mu mg\cos\alpha$	m1	correct expression
	$F = 0.6 \times 7 \times 9.8 \times \frac{4}{5}$		
	$F = \underline{32.9(28\text{ N})}$	A1	Accept rounding to 32.9.
3(b)	Apply N2L to *A*	M1	dim correct equation Friction opposes motion 4 terms. Accept cos.
	$T + mg\sin\alpha - F = 7a$	A1	ft (a)
	$T + 41.16 - 32.928 = 7a$		
	$T + 8.232 = 7a$		
	Apply N2L to *B*	M1	dim correct equation
	$3g - T = 3a$	A1	
	$3g + 8.232 = 10a$	m1	one variable eliminated Dep on both M's
	$a = \underline{3.7(632\text{ ms}^{-2})}$	A1	cao
	$T = \underline{18.1(104\text{ N})}$	A1	cao

Q	Solution	Mark	Notes
4.			
		B1	any 1 correct moment.
	Take moments about *C*	M1	dim correct equation. oe
	$0.4R_D = 3g\times0.6 + 12g\times1.5$	A1	correct equ any form
	$0.4R_D = 19.8g = 194.04$		
	$R_D = 49.5g = \underline{485.1\text{ (N)}}$	A1	cao
	Resolve vertically	M1	equation attempted. Or 2nd moment equation.
	$R_D = R_C + 15g$	A1	
	$R_C = 34.5g = \underline{338.1\text{ (N)}}$	A1	cao
	Alternative solution		
	Moment equation about *A*/centre/*B*	M1	
	Correct equation	B1	
	Second moment equation	M1	
	Correct equation	A1	
	Correct method for solving simultaneously	m1	Dep on both M's
	$R_C = 34.5g = \underline{338.1\text{ (N)}}$	A1	cao
	$R_D = 49.5g = \underline{485.1\text{ (N)}}$	A1	cao

Q	Solution	Mark	Notes
5(a)	Resolve perpendicular to motion	M1	equation, sin/cos
	20sin60 + *T*sin30 = 28sin60	A1	
	$20\frac{\sqrt{3}}{2} + T \times \frac{1}{2} = 28 \frac{\sqrt{3}}{2}$	A1	convincing
	$T = \underline{8\sqrt{3}}$		
5(b)	N2L in direction of motion	M1	dim correct all forces and No extra force
	20cos60 + *T*cos30 + 28cos60 – 16 = 80*a*	A2	-1 each error
	$20\times\frac{1}{2} + 8\sqrt{3}\times\frac{\sqrt{3}}{2} + 28\times\frac{1}{2} - 16 = 80a$		
	$a = \underline{0.25\ (ms^{-2})}$	A1	cao
5(c)	N2L -16 = 80*a*	M1	no extra force
	a = -0.2	A1	accept +/-
	Use of *v* = *u* + *at*, *v*=4, *u*=12, *a*=(+/-)0.2	m1	
	4 = 12 – 0.2*t*	A1	ft if *a*<0
	t = <u>40 (s)</u>	A1	ft if a<0

Q	Solution	Mark	Notes
6(a)			
	Conservation of momentum	M1	equation required Only one sign error. Ignore common factors
	$2\times3 - 7\times5 = 3v_A + 7v_B$ $3v_A + 7v_B = -29$	A1	
	Restitution	M1	v_B, v_A opposing consistent with diagram, +/-7 with the 0.6.
	$v_B - v_A = -0.6(-5 - 2)$ $v_B - v_A = 4.2$	A1	
	$-7v_A + 7v_B = 29.4$ $3v_A + 7v_B = -29$		
	$10v_A = -58.4$	m1	one variable eliminated. Dep on both M's.
	$v_A =(\underline{-)5.84}$	A1	cao
	$v_B =(\underline{-)1.64}$	A1	cao
6(b)	Impulse = change of momentum	M1	used
	$I = 7v_B - 7(-5)$ $I = -11.48 + 35$		
	I = <u>23.52 (Ns)</u>	A1	ft their v_A or v_B
6(c)	$3.65 = e(5.84)$ $e = \underline{0.625}$	B1	ft v_A if > 3.65.

Q	Solution	Mark	Notes
7.			



	Solution	Mark	Notes
	Resolve horizontally	M1	equation, no extra force
	$T_{AB}\sin 60 = T_{AC}\sin 45$	A1	
	$\frac{\sqrt{3}}{2}T_{AB} = \frac{1}{\sqrt{2}}T_{AC}$		
	$T_{AB} = \sqrt{\frac{2}{3}}T_{AC}$		
	Resolve vertically	M1	equation, no extra force
	$T_{AB}\cos 60 + T_{AC}\cos 45 = 9g$	A1	
	$T_{AB} + \sqrt{2}\ T_{AC} = 18g$		
	$\sqrt{\frac{2}{3}}T_{AC} + \sqrt{2}\ T_{AC} = 18g$	m1	
	T_{AC} = $\underline{79.(078)\text{ (N)}}$	A1	cao allow 79
	T_{AB} = $\underline{64.(567)\text{ (N)}}$	A1	cao allow 65
	<u>Alternative Method</u>		
	Third angle 75º/105º	B1	
	$\frac{T_{AB}}{\sin 45} = \frac{9g}{\sin 75}$	M1	sine rule attempted
	$T_{AB} = \frac{9g \times \sin 45}{\sin 75}$	A1	si
	T_{AB} = $\underline{64.(567)\text{ (N)}}$	A1	cao allow 65
	$\frac{T_{AC}}{\sin 60} = \frac{9g}{\sin 75}$	M1	sine rule attempted
	$T_{AC} = \frac{9g \times \sin 60}{\sin 75}$	A1	si
	T_{AC} = $\underline{79.(078)\text{ (N)}}$	A1	cao allow 79

Q	Solution				Mark	Notes
8(a)		mass	*AD*	*AB*		
	ABCD	72	6	3	B1	
	XYZ	12	6	2	B1	
	E	24	3	4		
	F	36	9	4	B1	both *E* and *F* correct
	Jewel	120	x	y	B1	masses in correct proportions.

Q	Solution	Mark	Notes
8(a)(i)	Moments about *AD*	M1	masses and moments consistent.
	$120x + 12\times6 = 72\times6 + 24\times3 + 36\times9$	A1	ft table if triangle subt.
	$120x = 756$		
	$x = \frac{63}{10} = \underline{6.3\text{(cm)}}$	A1	cao
8(a)(ii)	Moments about *AB*	M1	masses & moments consistent
	$120y + 12\times2 = 72\times3 + 24\times4 + 36\times4$	A1	ft table if triangle subt.
	$120y = 432$		
	$y = \frac{18}{5} = \underline{3.6\text{ (cm)}}$	A1	cao
8(b)	$PC = 12 - x$		
	$PC = \underline{5.7\text{ (cm)}}$	B1	ft their x if < 12.

Q	Solution	Mark	Notes
1(a)	EE = $\frac{1}{2}\times\frac{\lambda x^2}{l}$, λ=625, *x*=(+/-)0.1, *l*=0.2	M1	
	EE = $\frac{1}{2}\times\frac{625\times 0\cdot 1^2}{0\cdot 2}$		
	EE = <u>15.625 (J)</u>	A1	
1(b)	KE = $\frac{1}{2}\times 0\cdot 8v^2$ (= $0.4v^2$)	B1	
	WD by resistance = 46 × 0.1 (= 4.6)	B1	
	Work-energy Principle	M1	3 terms, no PE.
	$\frac{1}{2}0.8v^2 + 46\times 0.1 = 15.625$	A1	FT their EE
	$0.4v^2 = 15.625 - 4.6$		
	$0.4v^2 = 11.025$		
	$v = \sqrt{\frac{11\cdot 025}{0\cdot 4}}$		
	v = <u>5.25 (ms^{-1})</u>	A1	cao

Q	Solution	Mark	Notes
2(a)	$F - R = ma$	M1	used, F and R opposing.
	$30t^{-2} - 150 = 5a$		
	$6t^{-2} - 30 = a$	A1	
	$\frac{dv}{dt} = 6t^{-2} - 30$		Answer given
(b)	$24 = \frac{6}{t^2} - 30$	M1	Ft (a) if same form
	$\frac{6}{t^2} = 54$		
	$t = \frac{1}{3}$	A1	cao, accept 0.3.
2(c)	Integrate w.r.t. t	M1	Increase in powers
	$v = -6t^{-1} - 30t\ (+ C)$	A1	
	$t = \frac{1}{3}$, $v = 18$	m1	
	$18 = -18 - 10 + C$		
	$C = 46$		
	$v = -6t^{-1} - 30t + 46$		
	When $v = 10$	m1	
	$10 = -\frac{6}{t} - 30t + 46$		
	$5t^2 - 6t + 1 = 0$	m1	recognition of quadratic Some attempt to solve.
	$(5t - 1)(t - 1) = 0$		
	$t = \frac{1}{5}, 1$	A1	cao

Q	Solution	Mark	Notes
3(a)	$T = \frac{P}{v}$, $P = 90\times1000$, $v = 4.8$	M1	si
	$T = \frac{90\times1000}{4\cdot8}$	A1	si
	$T = 18750$		
	N2L	M1	dim correct, all forces *T*, *R* opposing.
	$T - mg\sin\alpha - R = ma$	A1	
	$18750 - 4000\times9.8\times\frac{2}{49} - R = 4000\times1.2$	A1	
	$R = 18750 - 1600 - 4800$		
	$R = \underline{12350\text{ (N)}}$	A1	cao
3(b)	N2L with $a = 0$	M1	all forces.
	$T = \frac{90\times1000}{v}$	B1	si
	$T - 1600 - 12800 = 0$	A1	
	$v = \underline{6.25\text{ ms}^{-1}}$	A1	

Q	Solution	Mark	Notes
4(a)	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$	M1	used
	$\mathbf{r}_A = (3 - t)\mathbf{i} + (5 + 2t)\mathbf{j} + (20 + t)\mathbf{k}$	A1	
	$\mathbf{r}_B = (-2 + 3t)\mathbf{i} + (x - 4t)\mathbf{j} + (15 + 2t)\mathbf{k}$	A1	
4(b)	$\mathbf{r}_B - \mathbf{r}_A =$	M1	
	$(-5 + 4t)\mathbf{i} + (x - 5 - 6t)\mathbf{j} + (-5 + t)\mathbf{k}$	A1	ft (a) similar expressions.
	$AB^2 = x^2 + y^2 + z^2$	M1	
	$AB^2 = (-5 + 4t)^2 + (x - 5 - 6t)^2 + (-5 + t)^2$	A1	cao
4(c)	Differentiate	M1	powers reduced
	$\frac{dAB^2}{dt} = 2(-5 + 4t)(4) + 2(x - 5 - 6t)(-6)$		
	$+ 2(-5 + t)(1)$		
	$-40 + 32t - 12x + 60 + 72t - 10 + 2t = 0$	m1	equating to 0.
	$106t + 10 = 12x$		
	When $t = 5$		
	$x = \underline{45}$	A1	cao

Q	Solution	Mark	Notes
5(a)	$u_H = \dfrac{42}{2\cdot 5}$ = $\underline{16.8\ (\text{ms}^{-1})}$	B1	
	$s = u_V t + 0.5at^2$, $s = 3$, $t = 2.5$, $a=(\pm)9.8$	M1	
	$3 = 2.5u_V - 4.9\times 2.5^2$	A1	
	$u_V = \underline{13.45\ (\text{ms}^{-1})}$	A1	cao, accept 13.4, 13.5.
5(b)	$v_V = u_V + at$, $u_V = 13.45$, $a = (\pm)9.8$, $t=2.5$	M1	
	$v_V = 13.45 - 9.8 \times 2.5$	A1	ft from (a)
	$v_V = -11.05$		
	magnitude of vel = $\sqrt{u_H^{\ 2} + v_V^{\ 2}}$	m1	
	= $\underline{20.11\ (\text{ms}^{-1})}$	A1	cao
	$\theta = \tan^{-1}\left(\dfrac{11\cdot 05}{16\cdot 8}\right)$	m1	
	$\theta = \underline{33.33°}$ (below horizontal)	A1	cao
5(c)	$s = ut + 0.5at^2$, $s = 0$, $u=13.45$, $a=(\pm)9.8$	M1	
	$0 = 13.45t - 4.9t^2$		
	$t = 2.7449$		
	Distance = 2.7449×16.8	m1	
	Distance = 46.11		
	Required distance = 46.11 – 42 = $\underline{4.11\ (\text{m})}$	A1	cao

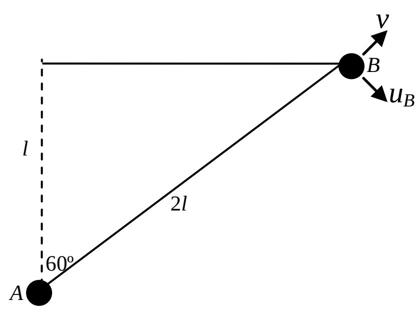
Q	Solution	Mark	Notes
6(a)	$\mathbf{a} = \frac{dv}{dt}$	M1	differentiation attempted.
			Vectors required.
	$\mathbf{a} = 8\cos 2t\ \mathbf{i} - 75\sin 5t\ \mathbf{j}$	A1	
	At $t = \frac{3\pi}{2}$, ($\mathbf{a} = -8\mathbf{i} + 75\mathbf{j}$)	m1	substitution of t.
	Magnitude of force = $3\times\sqrt{8^2 + 75^2}$	M1	or $\mathbf{F} = 3(-8\mathbf{i} + 75\mathbf{j})$
	= <u>226.28 (N)</u>	A1	cao
6(b)	$\mathbf{r} = \int 4\sin 2t\ \mathbf{i} + 15\cos 5t\ \mathbf{j}\ dt$	M1	integration attempted
	$\mathbf{r} = -2\cos 2t\ \mathbf{i} + 3\sin 5t\ \mathbf{j}\ (+\ \mathbf{c})$	A1	
	At $t = 0$,		
	$-2\mathbf{i} + 3\mathbf{j} = -2\mathbf{i} + \mathbf{c}$	m1	
	$\mathbf{c} = 3\mathbf{j}$	A1	
	$\mathbf{r} = -2\cos 2t\ \mathbf{i} + 3\sin 5t\ \mathbf{j} + 3\mathbf{j}$		
6(c)	Particle crosses the y-axis when		
	$-2\cos 2t = 0$	M1	
	$2t = \frac{\pi}{2}$		
	$t = \frac{\pi}{4}$	A1	cao
	Distance from origin = $3\sin(5\times\frac{\pi}{4}) + 3$	m1	substitute t into $\mathbf{r}$
	= <u>0.88 (m)</u>	A1	cao

Q	Solution	Mark	Notes
7(a)	Conservation of energy	M1	
	$0.5m(4u)^2 = mg(2l) + 0.5mu^2$	A1	
	$16u^2 = 4gl + u^2$		
	$u^2 = \frac{4}{15}gl$	A1	convincing
7(b)(i)	Conservation of energy	M1	
	$0.5m(4u)^2 = 0.5mv^2 + mgl(1 - \cos\theta)$	A1	
	$v^2 = 16u^2 – 2gl + 2gl\cos\theta$		
	$v^2 = \frac{34}{15}gl + 2gl\cos\theta$	A1	
	N2L towards centre of circle	M1	
	$T - mg\cos\theta = \frac{mv^2}{l}$	A1	
	$T = \frac{34}{15}mg + 3mg\cos\theta$	m1	If M1s gained, substitute for v^2.
	$T = \frac{mg}{15}(34 + 45\cos\theta)$	A1	any correct form
7(b)(ii)	when $T = 0$, $\cos\theta = -\frac{34}{45}$	M1	putting $T = 0$ in acos ± b
	$\theta = 139.1°$	A1	Ft cos = a, a<0.

Q	Solution	Mark	Notes
1(a)	N2L $500 - 100v = 1200\frac{dv}{dt}$	M1	
	$\frac{dv}{dt} = \frac{500-100v}{1200} = \frac{5-v}{12}$	A1	convincing
1(b)	$\int 12\frac{dv}{5-v} = \int dt$	M1	sep. var. (5-v) together.
	$-12\ln(5 - v) = t + (C)$	A1	correct integration
	When $t = 0$, $v = 0$, $C = -12\ln 5$	m1	allow +/-, oe
	$t = 12\ln\left(\frac{5}{5-v}\right)$		
	$\frac{5}{5-v} = e^{\frac{t}{12}}$	m1	inversion ft similar exp.
	$v = 5(1 - e^{-t/12})$	A1	cao
	limiting speed = 5 (ms^{-1})	B1	Ft similar expression
1(c)	When $v = 4$, $t = 12\ln\left(\frac{5}{5-4}\right)$	M1	
	$t = 12\ln 5$ (= 19.31s)	A1	cao

Q	Solution	Mark	Notes
2(a)	Period = $\dfrac{2\pi}{\omega}$ = 2	M1	
	$k = \omega = \pi$	A1	
2(b)	$x = 0.52\cos\pi t$	B1	for amp=0.52
	When $t = \dfrac{1}{3}$, $x = 0.52\cos\dfrac{\pi}{3}$	M1	allow asin/acos, c's a
	$x = 0.26$	A1	cao
2(c)	$0.4 = 0.52\cos\pi t$	M1	allow sin/cos
	$\cos\pi t = \dfrac{0.4}{0.52}$		
	$t = 0.22$	A1	cao
	$t = 1.78$	A1	FT t,ie 2-first t.
2(d)	$v^2 = \omega^2(0.52^2 - x^2)$	M1	used. oe
	$v^2 = \pi^2(0.52^2 – 0.2^2)$	m1	sub $x = 0.2$
	$v = \pi(0.48)$ (= 1.508 ms^{-1})	A1	cao
2(e)	max $v = a\omega$	M1	used
	$=0.52\pi$ (= 1.634 ms^{-1})	A1	cao

Q	Solution	Mark	Notes
3			

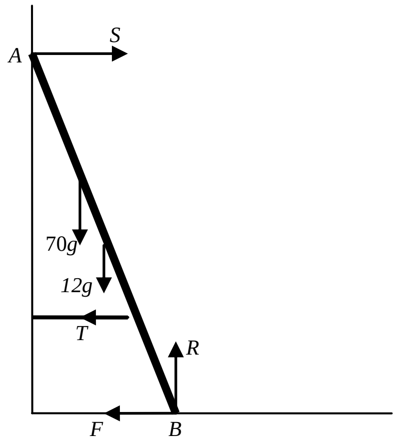


Impulse = change in momentum	M1	used
$J = 2u\cos30 - 2v$	A1	
$J = 3v$	B1	
Eliminating J	m1	one variable eliminated
$3v = 2u\cos30 - 2v$		
$5v = 2u\cos30$		
$v = 0.4u\cos30$		
$v = 2.77$ (ms^{-1})(speed of A)	A1	cao
$J = 1.2\,u\cos30 = 8.31$ (Ns)	A1	ft 3 x c's v.
$u_B = u\sin30 = 4$ (ms^{-1})	B1	
Speed of $B = \sqrt{(2.77^2 + 4^2)}$	m1	
Speed of $B = 4.87$ (ms^{-1})	A1	cao

Q	Solution	Mark	Notes
4(a)	Auxiliary equation		
	$2m^2 + 6m + 5 = 0$	B1	
	$m = -1.5 \pm 0.5i$	B1	
	C.F. is $x = e^{-1.5t}(A\sin 0.5t + B\cos 0.5t)$	B1	ft complex roots
	For PI, try $x = a$		
	$5a = 1$		
	$a = 0.2$	B1	ft CF + a
	GS is $x = e^{-1.5t}(A\sin 0.5t + B\cos 0.5t) + 0.2$	B1	
4(b)	$e^{-1.5t} \to 0$ as $t \to \infty$	M1	si
	x tends to 0.2 as t tends to infinity		
	Limiting value = 0.2	A1	ft similar expression
4(c)(i)	$x = 0.5$ and $\frac{dx}{dt} = 0$ when $t = 0$	M1	used
	$B + 0.2 = 0.5$		
	$B = 0.3$	A1	cao
	$\frac{dx}{dt} = -1.5e^{-1.5t}(A\sin 0.5t + B\cos 0.5t)$		
	$+ e^{-1.5t}(0.5A\cos 0.5t - 0.5B\sin 0.5t)$	B1	ft similar expressions
	$0 = -1.5B + 0.5A$		
	$A = 3B = 0.9$	A1	cao
	$x = e^{-1.5t}(0.9\sin 0.5t + 0.3\cos 0.5t) + 0.2$		
4(c)(ii)	When $t = \frac{\pi}{3}$		
	$x = e^{-\pi/2}(0.9\sin\frac{\pi}{6} + 0.3\cos\frac{\pi}{6}) + 0.2$		
	$x = 0.348$	A1	cao

Q	Solution	Mark	Notes
5(a)	Using F = ma		
	$1200(v+3)^{-1} = 800a$	M1	
	$2v\frac{dv}{dx} = \frac{3}{v+3}$	A1	convincing
5(b)	$\int 3dx = \int 2v(v+3)dv$	M1	separate variables
	$3x = \frac{2v^3}{3} + 3v^2 + (C)$	A1	correct integration
	$x = 0$, $v = 0$, hence C = 0	B1	
	When $v = 3$, $3x = 18 + 27$	m1	
	$x = 15$	A1	convincing
5(c)	$\frac{dv}{dt} = \frac{3}{2(v+3)}$		
	$\int 2(v+3)dv = \int 3dt$	M1	
	$v^2 + 6v = 3t + (C)$	A1	
	$t = 0$, $v = 0$, hence C = 0	B1	
	When $v = 3$		
	$3t = 9 + 18 = 27$		
	$t = 9$	A1	cao
5(d)(i)	$v^2 + 6v - 3t = 0$	M1	recognition of quadratic And attempt to solve
	$v = 0.5(-6 \pm \sqrt{(6^2 - 4 \times -3t)})$	A1	si
	$v = -3 + \sqrt{(9 + 3t)}$	A1	
(ii)	$\frac{dx}{dt} = -3 + (9+3t)^{\frac{1}{2}}$	M1	
	$x = -3t + \frac{2}{9}(9+3t)^{\frac{3}{2}} + (C)$	A1	correct integration
	$x = 0$, $t = 0$, (hence C = -6)	m1	
	$x = -3t + \frac{2}{9}(9+3t)^{\frac{3}{2}} + (-6)$		
	When $t = 7$		
	$x = -21 - 6 + 2 \times 30^{1.5}/9 = 9.5148$	A1	cao
	x is approximately 9.5		

Q	Solution	Mark	Notes
5(d)(ii)	$v = -3 + \sqrt{(9 + 3t)}$		
	When $t=7$, $v = -3+\sqrt{(9+21)}$	M1	
	$v = -3+\sqrt{30}$	A1	si
	$v = 2.4723$		
	$x = \frac{2}{9}(-2.4723)^3 + (2.4723)^2$	m1	
	$x =$ <u>9.51 (m)</u>	A1	cao

Q	Solution	Mark	Notes
6(a)		B2 B0	B1 if one error. more than one error.
6(b)	Resolve vertically $R = 12g + 70g = 82g$	M1 A1	all forces
6(c)	Moments about B	M1	dim correct equation All terms
	$3T\sin 75 + 12g\times 4\cos 75 + 70gx\times\cos 75$ $= 8S\sin 75$	A4	-1 each incorrect term Accept T=100.
	Resolve horizontally $T + F = S$		
	$F = 0.1R = 8.2g$	B1	ft R
	$S = T + 8.2g$	B1	ft F
	$8(8.2g+T)\sin 75 – 3T\sin 75 – 48g\cos 75$ $= 70gx\cos 75$ $5T\sin 75 =$ $48g\cos 75 – 65.6g\sin 75 + 70gx\cos 75$ $T = 100$		
	$x = 5.53$ m	A1	cao

Q	Solution	Mark	Notes
	<u>OR</u>		
	Moments about A	M1	dim correct equation All terms
	$5T\sin 75 + 12g \times 4\cos 75 + 70g(8-x) \times \cos 75$ $+8F\sin 75 = 8R\cos 75$	A5	-1 each incorrect term Accept T=100.
	$F = 0.1R = 80.36$ N	B1	Ft R
	$T = 100$ $x = 5.53$ m	A1	cao
6(d)	Ladder modelled as a rigid rod.	B1	

Ques	Solution	Mark	Notes
1(a)	EITHER P(A∩B) = P(A) + P(B) – P(A∪B) = 0.2	M1 A1	Award M1 for using formula
	This is not equal to P(A) × P(B) therefore not independent.	A1	
	OR Assume A,B are independent so that P(A∩B) = P(A) + P(B) – P(A)P(B) = 0.58	M1 A1	Award M1 for using formula
	Since P(A∪B) ≠ 0.58, A,B are not independent.	A1	
(b)	$P(A \mid B') = \frac{P(A \cap B')}{P(B')}$	M1	Award M1 for using formula
	$= \frac{0.3 - 0.2}{0.6}$	A1	FT their P(A∩B) if independence not assumed
	$= \frac{1}{6}$	A1	Accept Venn diagram
2	$np = 0.9, \quad npq = 0.81$ Dividing, $q = 0.9$, $p = 0.1$ $n = 9$	B1B1 M1A1 A1	
3(a)	P(1 of each) = $\frac{3}{9}\times\frac{3}{8}\times\frac{3}{7}\times 6$ or $\binom{3}{1}\times\binom{3}{1}\times\binom{3}{1}\div\binom{9}{3}$	M1A1	M1A0 if 6 omitted
	$= \frac{9}{28}$	A1	
(b)	P(2 particular colour and 1 different) = $\frac{3}{9}\times\frac{2}{8}\times\frac{6}{7}\times 3$ or $\binom{3}{2}\times\binom{6}{1}\div\binom{9}{3}$	M1A1	M1A0 if 3 omitted
	$= \frac{3}{14}$	A1	Allow 3/28
	P(2 of any colour and 1 different) = $\frac{9}{14}$	B1	FT previous line
4(a)	Let X denote the number of goals scored in the first 15 minutes so that X is Po(1.5) si	B1	
	$P(X = 2) = \frac{e^{-1.5}\times 1.5^2}{2!}$ = 0.251	M1 A1	Award M0 if no working seen
(b)	$P(X > 2) = 1 - e^{-1.5}\left(1 + 1.5 + \frac{1.5^2}{2!}\right)$	M1A1	
	= 0.191	A1	

Ques	Solution	Mark	Notes
5(a) (i)	Let X = number of female dogs so X is B(20,0.55) $P(X=12)=\binom{20}{12}\times 0.55^{12}\times 0.45^{8}$ $= 0.162$	**B1** **M1** **A1**	si Accept 0.4143 – 0.2520 or 0.7480 – 0.5857
(ii)	Let Y = number of male dogs so Y is B(20,0.45) $P(8 \le X \le 16) = P(4 \le Y \le 12)$ $= 0.9420 - 0.0049$ or $0.9951 - 0.0580$ $= 0.9371$	**M1** **A1** **A1A1** **A1**	Award M0 if no working seen
(b)	Let U = number of yellow dogs so U is B(60,0.05) ≈ Po(3) $P(U < 5) = 0.8153$	**M1** **m1A1**	
6(a)	P(head) = $\frac{3}{4}\times\frac{1}{2}+\frac{1}{4}\times 1$ $= \frac{5}{8}$	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram)
(b)(i)	P(DH|head) = $\frac{1/4}{5/8}$ $= \frac{2}{5}$ cao	**B1B1** **B1**	B1 num, B1 denom FT denominator from (a)
(ii)	EITHER P(head) = $\frac{3}{5}\times\frac{1}{2}+\frac{2}{5}\times 1$ $= \frac{7}{10}$ OR P(Head) $= \dfrac{3/4\times 1/2\times 1/2+1/4\times 1}{5/8}$ $= \frac{7}{10}$	**M1A1** **A1** **B1B1** **B1**	M1 Use of Law of Total Prob (Accept tree diagram) B1 num, B1 denom FT denominator from (a)

Ques	Solution	Mark	Notes
7(a)	[0,0.4]	**B1**	Allow(0,0.4)
(b)	$E(X) = 0.1 + 0.6 + 3\theta + 0.8 + 5(0.4 - \theta)$	**M1**	
	$= 3.5 - 2\theta$	**A1**	
	The range is [2.7,3.5]	**A1**	FT the range from (a)
(c)	$E(X^2) = 0.1 + 1.2 + 9\theta + 3.2 + 25(0.4 - \theta)$	**M1A1**	Must be in terms of θ
	Var(X) = $0.1 + 1.2 + 9\theta + 3.2 + 25(0.4 - \theta) - (3.5 - 2\theta)^2$	**M1**	
	$= 2.25 - 2\theta - 4\theta^2$	**A1**	
	Var(X) = 1.5 gives	**M1**	
	$4\theta^2 + 2\theta - 0.75 = 0$	**A1**	
	$16\theta^2 + 8\theta - 3 = 0$		
	$(4\theta + 3)(4\theta - 1) = 0$	**M1**	Allow use of formula
	$\theta = 0.25$	**A1**	
8(a)	EITHER the sample space contains 64 pairs of which 8 are equal OR whatever number one of them obtains, 1 number out of 8 obtained by the other one gives equality.	**M1**	
	P(equal numbers) = $\frac{1}{8}$	**A1**	
(b)	The possible pairs are (4,8);(5,7);(6,6);(7,5);(8,4)	**B1**	
	EITHER the sample space contains 64 pairs of which 5 give a sum of 12 OR each pair has probability 1/64.	**M1**	
	P(sum = 12) = $\frac{5}{64}$	**A1**	
(c)	EITHER reduce the sample space to (4,8);(5,7);(6,6);(7,5);(8,4) OR $P(\text{equal numbers}) = \frac{P(6,6)}{P(\text{sum} = 12)} = \frac{1/64}{5/64}$	**M1**	
	Therefore P(equal numbers) = $\frac{1}{5}$	**A1**	

Ques	Solution	Mark	Notes
9(a)(i)	$P(0.4 \le X \le 0.6) = F(0.6) - F(0.4)$ $= 0.261$	**M1** **A1**	
(ii)	The median m satisfies $2m^3 - m^6 = 0.5$ $2m^6 - 4m^3 + 1 = 0$ $m^3 = \dfrac{4 \pm \sqrt{8}}{4}$ (0.293) $m = 0.664$	**B1** **M1A1** **A1**	Award M1 for a valid attempt to solve the equation Do not award A1 if both roots given
(b)(i)	Attempting to differentiate $F(x)$ $f(x) = 6x^2 - 6x^5$	**M1** **A1**	
(ii)	$E(X^3) = \int_0^1 x^3(6x^2 - 6x^5)dx$ $= \left[\dfrac{6x^6}{6} - \dfrac{6x^9}{9}\right]_0^1$ $= 1/3$	**M1A1** **A1** **A1**	M1 for the integral of $x^3 f(x)$ A1 for completely correct although limits may be left until 2nd line. FT their $f(x)$ if M1 awarded in (i)

S2

Ques	Solution	Mark	Notes
1	$\bar{x} = \frac{405.6}{8}$ (= 50.7)	**B1**	
	SE of $\bar{X} = \frac{4}{\sqrt{8}}$ (= 1.4142…)	**M1A1**	
	90% conf limits are 50.7 ± 1.645 × 1.4142… giving [48.4, 53.0] cao	**M1A1** **A1**	M1 correct form, A1 correct *z*. Award M0 if no working seen
2(a)	Upper quartile = mean + 0.6745 × SD = 86.0	**M1** **A1**	
(b)	Let *X*=weight of an orange, *Y*=weight of a lemon		
	$E(\Sigma X) = 1984$	**B1**	
	$\text{Var}(\Sigma X) = 512$	**B1**	
	$z = \frac{2000-1984}{\sqrt{512}} = 0.71$	**M1A1**	Award M0 if no working seen
	Prob = 0.7611 cao	**A1**	
(c)	Let $U = X - 3Y$	**M1**	
	E(U) = – 7	**A1**	
	Var(U) = 64 + 9 × 2.25 = 84.25	**M1A1**	
	We require P(U >0)		
	$z = \frac{0+7}{\sqrt{84.25}} = 0.76$	**m1A1**	Award m0 if no working seen
	Prob = 0.2236	**A1**	
3(a)	$H_0 : \mu_M = \mu_F ; H_1 : \mu_M \neq \mu_F$	**B1**	
(b)	Let *X*= male weight, *Y*=female weight ($\sum x = 39.2; \sum y = 46.6$)		
	$\bar{x} = 4.9; \bar{y} = 4.66$	**B1B1**	
	SE of diff of means= $\sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{10}}$ (0.237…)	**M1A1**	
	Test statistic = $\frac{4.9 - 4.66}{0.237...}$	**m1**	Award m0 if no working seen
	= 1.01	**A1**	
	Prob from tables = 0.1562	**A1**	
	p-value = 0.3124	**B1**	FT line above
	Insufficient evidence to conclude that there is a difference in mean weight between males and females.	**B1**	FT their *p*-value

Ques	Solution	Mark	Notes
4(a)(i)	$H_0: p = 0.6; H_1: p < 0.6$	B1	
(ii)	Let X = Number of games won Under H_0, X is B(20,0.6) si Let Y = Number of games lost Under H_0, Y is B(20,0.4) p-value = P($X \leq 7$|(X is B(20,0.6))) = P($Y \geq 13$|Y is B(20,0.4) = 0.021 Strong evidence to reject Gwilym's claim (or to accept Huw's claim).	 B1 B1 M1 A1 A1 B1	 Award M0 if no working seen FT on p-value
(b)	X is now B(80,0.6) (under H_0) ≈ N(48,19.2) p-value = P($X \leq 37$|X is N(48,19.2)) $z = \frac{37.5 - 48}{\sqrt{19.2}}$ = – 2.40 p-value = 0.0082 Very strong evidence to reject Gwilym's claim (or to accept Huw's claim).	B1B1 M1 A1 A1 A1 B1	 Award M0 if no working seen Award M1A0A1 for incorrect or no continuity correction No cc ; z = – 2.51, p = 0.00604 36.5 ; z = – 2.62, p = 0.0044 FT on p-value only if less than 0.01
5(a)	$E(X) = E(Y) = 1.2$ E(U) = E(X)E(Y) = 1.44 cao	B1 B1	
(b)	Var(X) = Var(Y) = 0.96 $E(X^2)(= E(Y^2)) = \text{Var}(X) + [E(X)]^2 = 2.4$ Var(U) = $E(X^2Y^2) - [E(XY)]^2$ = $E(X^2)E(Y^2) - [E(X)E(Y)]^2$ = 3.69 cao	B1 M1A1 M1 A1 A1	 FT their values from (a)
6(a)(i)	Under H_0, X is Po(15) si $P(X \leq 10) = 0.1185; P(X \geq 20) = 0.1248$ Significance level = 0.2433	B1 B1 B1	 Award B1 for either correct
(ii)	X is now Poi(10) P(accept H_0) = $P(11 \leq X \leq 19)$ = $0.9965 - 0.5830$ or $0.4170 - 0.0035$ = 0.4135 cao	B1 M1 A1 A1	 Award M0 if no working seen
(b)	Under H_0, X is now Po(75) ≈ N(75,75) $z = \frac{91.5 - 75}{\sqrt{75}} = 1.91$ Prob from tables = 0.0281 p-value = 0.056 Insufficient evidence to reject H_0	B1 M1A1 A1 A1 B1	 Award M1A0 for incorrect or no continuity correction but FT further work. FT from line above FT from line above No cc gives z = 1.96, p = .05 92.5 gives z = 2.02, p = 0.0434

Ques	Solution	Mark	Notes
7(a)	$P(L \le 4) = P(A \le 4^2)$ $= \frac{16-15}{20-15}$ $= 0.2$	**M1** **A1** **A1**	
(b)	$E(L) = E(A^{1/2})$ $= \int_{15}^{20} a^{1/2} \times \frac{1}{5}\,\mathrm{d}a$ $= \frac{2}{15}\left[a^{3/2}\right]_{15}^{20}$ $= 4.18$	**M1A1** **A1** **A1**	Limits can be left until next line Do not accept $\sqrt{17.5} = 4.18$
(c)	$\mathrm{Var}(L) = E(L^2) - [E(L)]^2$ $= 17.5 - 4.18^2$ $= 0.03$	**M1** **A1** **A1**	FT their E(L)

S3

Ques	Solution	Mark	Notes
1	$\bar{x} = 52.0$ si	B1	
	Variance estimate = $\frac{162480}{59} - \frac{3120^2}{60 \times 59} = 4.068$	M1A1	
	(Accept division by 60 which gives 4.0)		
	90% confidence limits are		
	$52 \pm 1.645\sqrt{4.068/60}$	M1A1	
	giving [51.6,52.4]	A1	
2(a)	$H_0 : \mu = 4.5; H_1 : \mu \neq 4.5$	B1	
(b)	$\sum x = 43.6; \sum x^2 = 190.3428$	B1B1	
	UE of $\mu = 4.36$	B1	No working need be seen
	UE of $\sigma^2 = \frac{190.3428}{9} - \frac{43.6^2}{90}$	M1	
	$= 0.0274(22\ldots)$	A1	Answer only no marks
(c)	test-stat = $\frac{4.36 - 4.5}{\sqrt{0.0274222../10}}$	M1A1	FT their values from (b)
	= – 2.67 (Accept +2.67)	A1	Answer only no marks
	DF = 9 si	B1	
	Crit value = 3.25	B1	
	This result suggests that we should accept H_0, ie that the mean weight is 4.5 kg	B1	FT their *t*-statistic
	because 2.67 < 3.25	B1	
3(a)	$\hat{p} = \frac{654}{1500} = 0.436$ si	B1	
	ESE = $\sqrt{\frac{0.436 \times 0.564}{1500}} = 0.0128..$ si	M1A1	
	95% confidence limits are	M1	M1 correct form
	$0.436 \pm 1.96 \times 0.0128..$	A1	A1 correct *z*
	giving [0.41,0.46]	A1	
(b)	$\hat{p} = \frac{0.4348 + 0.4852}{2} = 0.46$	B1	
	Number of people = 0.46 × 1200 = 552	B1	
	$0.4852 - 0.4348 = 2z\sqrt{\frac{0.46 \times 0.54}{1200}}$	M1A1	
	$z = 1.75$	A1	
	Prob from tables = 0.0401 or 0.9599	A1	
	Confidence level = 92%	B1	FT line above

Ques	Solution	Mark	Notes
4(a)	$H_0 : \mu_a = \mu_b ; H_1 : \mu_a \neq \mu_b$	**B1**	
(b)	SE = $\sqrt{\frac{0.115}{80} + \frac{0.096}{70}}$ (= 0.053)	**M1A1**	
	Test stat = $\frac{3.65 - 3.52}{0.053}$	**M1A1**	
	= 2.45 (Accept 2.46)	**A1**	
	Tabular value = 0.00714 (0.00695)	**A1**	
	p-value = 0.01428 (0.0139)	**A1**	
	Strong evidence to conclude that there is a difference in mean weight.	**B1**	FT their p-value Accept the conclusion that the Variety B mean is greater than the Variety A mean
(c)	Estimates of the variances of the sample means are used and not exact values.	**B1**	
	The sample means are assumed to be normally distributed (using the Central Limit Theorem).	**B1**	
5(a)	$\sum x = 42, \sum x^2 = 364, \sum y = 340.6, \sum xy = 2906.4$	**B2**	Minus 1 each error
	$S_{xy} = 2906.4 - 42 \times 340.6/6 = 522.2$	**B1**	
	$S_{xx} = 364 - 42^2/6 = 70$	**B1**	
	$b = \frac{522.2}{70} = 7.46$	**M1** **A1**	Answers only no marks
	$a = \frac{340.6 - 7.46 \times 42}{6} = 4.55$	**M1** **A1**	
(b)(i)	Unbiased estimate = $a + 5b$ = 41.85	**B1**	FT their values of and a,b if answer between 33.9 and 49.9
(ii)	SE of $a + 5b$ = $0.5\sqrt{\frac{1}{6} + \frac{(5-7)^2}{70}}$ (0.2365…)	**M1A1**	And FT their value of S_{xx}
	95% confidence limits for $\alpha + 5\beta$ are $41.85 \pm 1.96 \times 0.2365...$	**m1A1**	
	giving [41.4,42.3]	**A1**	
(iii)	Test stat = $\frac{7.6 - 7.46}{\sqrt{0.5^2/70}} = 2.34$	**M1A1**	FT their values of b and S_{xx} if possible.
	Critical value = 1.96 or p-value = 0.01928	**A1**	FT their test statistic
	We conclude that β = 7.6 is not consistent with the tabular values.	**B1**	FT the line above

Ques	Solution	Mark	Notes
6(a)(i)	$E(Y) = kE(\bar{X}) = kE(X) = \frac{k\theta}{2}$	M1A1	
	For an unbiased estimator, $k = 2$.	A1	
(ii)	$\text{Var}(Y) = 4\text{Var}(\bar{X})$	M1	FT their k
	$= \frac{4}{n}\text{Var}(X)$	A1	
	$= \frac{4}{n} \times \frac{\theta^2}{12}$	A1	
	$= \frac{\theta^2}{3n}$	A1	
	$\text{SE} = \frac{\theta}{\sqrt{3n}}$	A1	
(b)(i)	Using $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$	M1	
	$E(Y^2) = \frac{\theta^2}{3n} + \theta^2$	A1	
	$\neq \theta^2$ therefore not unbiased	B1	FT the line above
(ii)	$E(Y^2) = \theta^2\left(\frac{3n+1}{3n}\right)$	M1	
	$E\left(\frac{3nY^2}{3n+1}\right) = \theta^2$	A1	
	Therefore $\frac{3nY^2}{3n+1}$ is an unbiased estimator for θ^2	A1	

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