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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

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MARKING SCHEMES

JANUARY 2006

MATHEMATICS
(New Specification)

WJEC
CBAC

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

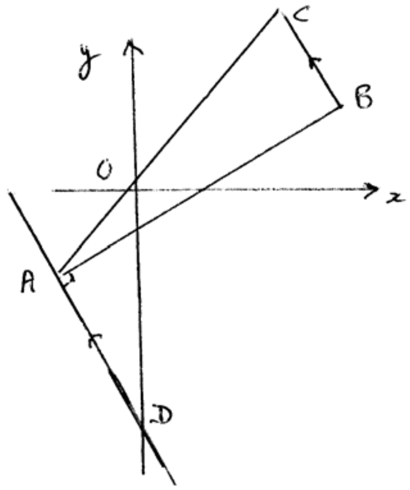
The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS C1

1. *(a)* Gradient of AB = $\frac{1+3}{6+2} = \frac{1}{2}$ — M1 (correct method)

A1 (grad = $\frac{1}{2}$)

(b)



Gradient of $BC = \frac{3-1}{k-6} = \frac{2}{k-6}$ — M1 (use of $m_1m_2 = -1$, seen or implied)

Then $\frac{1}{2} \times \frac{2}{k-6} = -1$ — M1 (method of finding equation in *k*)

$k - 6 = -1$
$k = 5$ — A1 (C.A.O.) (convincing)

(c) Gradient of L = –2 — B1 (F.T. for same gradient as *BC* or $-\frac{1}{\text{grad}AB}$)

Equation of L is $y + 3 = -2(x + 2)$ — B1 (give mark here)

(d) Coords of D : $x = 0, y = -7$ — B1(F.T. unsimplified equation of L)

$CD = \sqrt{(5-0)^2 + (3+7)^2} = \sqrt{125}$ — M1 (correct formula)

A1 ($\sqrt{\text{single no,}}$ F.T. derived coords of D)

10

2. *(a)* $4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3}$ — B1, B1, B1

$= 5\sqrt{3}$ — B1 (F.T. one slip, answer of form $k\sqrt{3}$)

(b) $\dfrac{(2+\sqrt{7})(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \dfrac{6 - 2\sqrt{7} + \sqrt{7} - 7}{9 - 7}$ — M1 (correct rationalising)

A1 numerator with $(\sqrt{7})^2 = 7$, allow 2×3

A1 (denominator with no surds)

$= \dfrac{\sqrt{7} - 1}{2}$ — A1 (F.T. one slip)

8

3. $\dfrac{dy}{dx} = 8x - 7$ — B1 (correct differentiation)

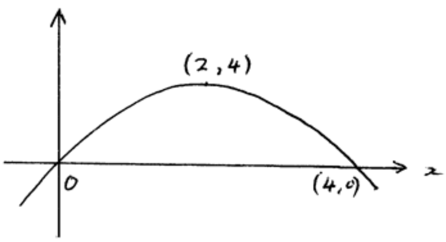
$= 9$ at $(2,\ 4)$ — B1 (numerical result, F.T. one slip)

Gradient of normal = $-\dfrac{1}{9}$ — M1 $\left(\dfrac{-1}{\text{gradient of tgt}}\right)$

Equation is $y - 4 = -\dfrac{1}{9}\ (x - 2)$ — A1 (F.T. candidate's gradient of tangent)

4

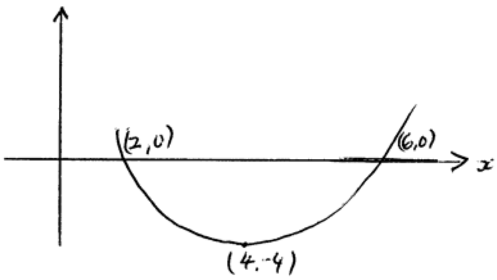
4. *(a)*



M1 (reflection in x – axis)

A1 (correct points)

(b)



M1 (x translation to right)

B1 (st.pt) (on diagram or stated)

A1 (points of intersection)

5

5.		For no real roots,	M1 ($b^2 – 4ac$, correct b, a or c correct)
		$4^2 – 4(k + 2)(k + 5) < 0$	A1 (correct)
		$4 – k^2 – 7k – 10 < 0$	M1 ($b^2 – 4ac < 0$)
		$k^2 + 7k + 6 > 0$	A1 (convincing)
		$(k + 6)(k + 1) > 0$	B1 (fixed pts, –1, –6)
		$k < – 6$ (and/or) $k > – 1$	
or			B2 (F.T. fixed points)
		$k < – 6, k > – 1$	
		$– 1 < k < – 6$ or $k > – 1$ or $k < – 6$	All gain B1
			7
6.	*(a)*	$f(2) = 4$	M1 (remainder theorem or division with remainder equated to 4)
		$8a – 4 – 14 + 6 = 4$	
		$8a = 16$	
		$a = 2$	A1
	(b)	$f(1) = 2 – 1 – 7 + 6 = 0$	M1 (correct use of factor theorem with appropriate factor mentioned)
		$x – 1$ is a factor	A1 (correct factor)
		$2x^3 – x^2 – 7x + 6 = (x – 1)(2x^2 + x – 6)$	m1 ($2x^2 + ax + b$, a or b correct, any method)
			A1 (correct quad. factor)
		$= (x – 1)(2x – 3)(x + 2)$	A1 (3 factors and 3 roots, F.T. one slip)
		Roots are $1, \frac{3}{2}, – 2$	
			7
7.	*(a)*	$(3x + 2)^3 = (3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + 2^3$	
		$= 27x^3 + 54x^2 + 36x + 8$	B3 (–1 for each error, any method)
	(b)	$\frac{n(n-1)}{2}(2)^2 = 2(2n)$	M1 ($^nC_2\, 2^p = 2k\, ^nC_1$, $k=2, \frac{1}{2}$, $p = 1,2$)
			A2 ($\frac{n(n-1)}{2} 2^2 = 2(2n)$)
		$n = 3$	A1 (C.A.O.)
			7
8.	*(a)*	Let $y = 2x^2 – 5x + 3$	
		$y + \Delta y = 2(x + \Delta x)^2 – 5(x + \Delta x) + 3$	B1
		$\Delta y = 4x\Delta x + 2(\Delta x)^2 – 5$	M1 (attempt to find Δy)
			A1
		$\frac{\Delta y}{\Delta x} = 4x + 2\Delta x – 5$	M1 (divide by Δx and let $\Delta x \to$ o)
		Let $\Delta x \to$ o	
		$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = 4x – 5$	A1 (award only if some mention of limit, clear presentation and no abuse of notation)

(b) $\frac{dy}{dx} = -\frac{a}{x^2} + 3x^{\frac{1}{2}}$ (o.e.) — B1, B1

$-\frac{a}{4^2} + 3 \times 4^{\frac{1}{2}} = 7$ — M1 ($f'(4)=7$, reasonable diffn)

$a = -16$ — A1 (C.A.O.)

9

9. *(a)* $23 + 6x - x^2 = 32 - (x-3)^2$ — B1 ($-(x-3)^2$)
B1 (32)

Greatest value = 32 — B1 (F.T. candidate's *b* and *a*)
when $x = 3$ — B1

(b) $\frac{1}{30+6x-x^2} = \frac{1}{7+23+6x-x^2}$ — A1

Least value = $\frac{1}{39}$ — A1 (F.T. *b*, *a*)

6

10. *(a)* $\left(\frac{dy}{dx} = 0\right)$ — B1 $\left(\frac{dy}{dx}\right)$

$12x - 6x^2 = 0$ — M1 $\left(\frac{dy}{dx} = 0\right)$

$x = 0, 2$ — A1 (either root, F.T. one slip)

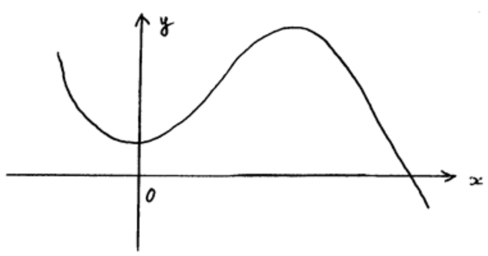
When $x = 0, y = 2$; when $x = 2, y = 10$ — A1 (both C.A.O.)

$\frac{d^2y}{d^2x} = 12 - 12x$ — M1 (any method)

$x = 0, \frac{d^2y}{dx^2} = 12 > 0$ min. — A1 (F.T. *x* values)

$x = 2, \frac{d^2y}{dx^2} = -12 < 0$ max. — A1

(b)



B1 (shape, negative cubic)

B1, B1 (derived stat pts)

(c) One real root as
graph crosses *x*–axis once — M1 (reason)
A1 (F.T. graph)

12

MATHEMATICS C2

1.	*(a)*	$h = 0.2$	M1 (correct formula $h = 0.2$)
		Integral $\approx \frac{0.2}{2}$ [0.5 + 0.3333333 + 2(0.4980080 + 0.48449612	B1 (4 values)
		+ 0.4512635 + 0.3980892)]	B1 (2 values)
		$\approx$ 0.4497	A1 (F.T. one slip)
		<u>S. Case</u> $h = \frac{1}{6}$	M1 (correct formula, $h = \frac{1}{6}$)
		Integral $\approx \frac{1}{12}$ [0.5 + 0.3333333 + 2(0.4988453 + 0.4909091	B1 (all values)
		+ 0.4705882 + 0.4354838 + 0.3877917)]	
		$\approx$ 0.4500	A1 (F.T. one slip)
			4
2.	*(a)*	$4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$	M1 (correct use of $\sin^2\theta + \cos^2\theta = 1$)
		$6\cos^2\theta - \cos\theta - 2 = 0$	M1 (attempt to solve quadratic in $\cos\theta$, correct formula)
		$(3\cos\theta - 2)(2\cos\theta + 1) = 0$	
		$\cos\theta = \frac{2}{3}, -\frac{1}{2}$	A1 (C.A.O.)
		θ = 48.2º, 311.8º, 120º, 240º	B1 (48.2, 311.8º) B1 (120º) B1 (240º)
	(b)	$\tan\theta = -\sqrt{3}$	
		θ = 120º, 300º	B1, B1
	(c)	2θ = 30º, 150º, 390º, 510º	B1 (one value)
		θ = 15º, 75º, 195º, 255º	B1 (2 values) B1 (2 values)
			11

3.

(a)	Sine rule $\frac{10}{\sin 45°} = \frac{12}{\sin A\hat{C}B}$	M1 (correct use of sine rule)
	$\sin\ A\hat{C}B = \frac{12\sin 45°}{10}$	
	$A\hat{C}B = 58.05° \text{ or } 121.95°$	A1 (one value)
	$A\hat{B}C = 180° – (45° + 58.05°) = 76.95°$	A1 (F.T. one slip)
	or $180° – (45° + 121.95°) = 13.05°$	A1
(b)	Area $= \frac{1}{2} \times 12 \times 10 \sin 76.95° \approx 58.4 \text{ cm}^2$ (58.4 – 58.5)	M1 (correct formula)
	or $\frac{1}{2} \times 12 \times 10 \sin 13.05° \approx 13.4 \text{ cm}^2$	A1 (both) (F.T. candidate values)

6

4. *(a)* nth term $= ar^{n-1}$ — B1

Let $S_n = a + ar + ar^2 + ... + ar^{n-2} + ar^{n-1}$ (1) — B1 (at least 3 terms, one at each end)

$rS_n = ar + ar^2 + ... + ar^{n-2} + ar^{n-1} + ar^n$ (2) — M1 (multiplication by r and subtract)

(1) – (2)

$S_n - rS_n = a - ar^n$

$S_n(1 - r) = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$ — A1 (convincing)

	(b)	(i)	$ar^3 = 2,\ ar^6 = 54$ $r^3 = 27$ $r = 3$	B1 (both) M1 (eliminate *a*) A1
		(ii)	$a = \dfrac{2}{3^3} = \dfrac{2}{27}$	B1 (F.T. one slip)
			$S_{10} = \dfrac{2}{27}\dfrac{(1-3^{10})}{1-3} \approx 2187.0$	M1 (correct formula, F.T. candidate values) A1
		(iii)	$\dfrac{2}{27}3^{n-1} = 125000$	B1 (F.T. candidate values)
			$\therefore\ 3^{n-1} = 1687500$	
			$(n-1)\ln 3 = \ln 1687500$	M1 (attempt to take logs)
			$n = 1 + 13.05 = 14.05$	A1 (C.A.O.)
			Least value is 15	A1 (F.T. candidate's *n*)
				14
5.	*(a)*		$2a + d = 3$ (1) $a + 7d = 47$ (2)	B1 ($a + a + d = 3$) B1
			Solve (1), (2) $d = 7,\ a = -2$	M1 (attempt to solve) A1 (C.A.O.)
	(b)		$S_{20} = \dfrac{20}{2}[2 \times -2 + 19 \times 7]$	M1 (correct formula with candidate values)
			$= 1290$	A1 (F.T. candidate values)
				6
6.	$\dfrac{5x^{\frac{4}{3}}}{\frac{4}{3}} + \dfrac{3x^{-2}}{-2}\ (+C)$			B1, B1
	$\left(\dfrac{15}{4}x^{\frac{4}{3}} - \dfrac{3}{2x^2}\right)$			**2**
7.	*(a)*	<u>B</u>	$y = 0,\quad 4 - x^2 = 0$	M1 (setting $y = 0$)
			$x = \pm 2$	
			$B\,(2, 0)$	A1
		<u>A</u>	$4 - x^2 = 3x$	M1 (equating *ys*)
			$x^2 + 3x - 4 = 0$	M1 (correct attempt to solve quad)
			$x = -4,\ 1$	
			$A(1, 3)$	A1

	(b)	Area $= \displaystyle\int_0^1 3x\,dx + \int_1^2 (4 - x^2)\,dx$	M1 (use of integration to find area)
			M1 (addition of areas)
		$= \left[\dfrac{3x^2}{2}\right]_0^1 + \left[4x - \dfrac{x^3}{3}\right]_1^2$	B3 (integration)
		$= \dfrac{3}{2} + 8 - \dfrac{8}{3} - 4 + \dfrac{1}{3}$	M1 (use of candidate's limits, any order)
		$= \dfrac{19}{6}$	A1 (C.A.O.)
			12
8.	*(a)*	Centre (4, –2)	B1
		Radius $= \sqrt{4^2 + (-2)^2 - 1} = 3$	M1 (correct method of finding radius) A1
	(b)	Centre is (0, 0), radius = a	B1 (both)
		Distance between centres = $\sqrt{4^2 + 2^2} = \sqrt{20}$	B1
		Circles touch if	
		$\sqrt{20} = a + 3$	M1 (F.T. distance)
		$a \approx 1.47$	A1 (F.T. distance)
			7
9.	*(a)*	$\dfrac{1}{2}4^2(\theta + \phi) = 15.2$	M1 (use of correct formula)
		$\theta + \phi = \dfrac{15.2}{8} = 1.9$ (1)	A1 (convincing)
	(b)	$4\theta - 4\phi = 3.2$	M1 (use of correct formula)
		$\theta - \phi = 0.8$ (2)	A1
		Solve (1), (2)	
		$\theta = 1.35, \phi = 0.55$	M1 (attempt to solve) A1 (F.T. one slip)
			6

10.	*(a)*	Let $x = a^p$, $y = a^q$ $\log_a x = p$, $\log_a y = q$ $xy = a^{p+q} = a^{p+q}$	B1 (properties of $\log_a x$ and $x = a^p$) B1 (laws of indices)
		$\log_a (xy) = p + q = \log_a x + \log_a y$	B1 (convincing)
	(b)	$\displaystyle\int_1^3 \log_{10} x dx + \int_1^3 \log_{10} 10 dx$	B1 laws of logs)
		$\displaystyle\approx 0.5628 + \int_1^3 1 dx$	B1
		$= 0.5628 + [x]_1^3$,	B1 (integration)
		$= 0.5628 + 3 - 1$ $= 2.5628$	B1

7

MATHEMATICS C3

1.	*(a)*	$h = 0.25$	
		Integral $\approx \frac{0.25}{3}$ [1.7320508 + 4.2426407 +	M1 (h= 0.25 use of correct formula)
		4(2.1074644 + 3.3732634)	B1 (3 values)
		+ 2(2.6575365]	B1 (2 further values)
		$\approx$ 2.768	A1 (F.T. one slip)
			4
2.	*(a)*	$\theta = 30º$, for example $\tan 2\theta = \sqrt{3} \approx 1.732$	M1 (substitution of θ in both, one correct value)
		$2 \tan \theta = \frac{2}{3} \approx 1.15$	A1 (both correct and clearly unequal)
		$(\therefore \tan 2\theta \neq 2 \tan \theta)$	
	(b)	$4 (\text{cosec}^2 \theta - 1) = 11 - 4\ \text{cos}ex\ \theta$	M1 (correct use of $\text{cosec}^2 \theta = 1+ \cos^2\theta$)
		$4 \text{cosec}^2\theta + 4 \text{cosec}\ \theta - 15 = 0$	M1 (grouping terms and attempting to solve quad. in cosec θ or sin θ
		$(2 \text{cosec}\ \theta - 3)(2 \text{cosec}\ \theta + 5) = 0$	correct formula or (a cosec^2 θ + b)(cosec θ + d)
		$\text{cosec}\ \theta = \frac{3}{2}, -\frac{5}{2}$	where ac = coefft of cosec^2 θ bd = constant term)
		$\sin \theta = \frac{2}{3}, -\frac{2}{2}$	A1 (CAO)
		θ = 41.8º, 138.2º, 203.6º, 336.4º	B1 (41.5º – 42º) B1 (2035º – 204º) B1 (336º – 336.5º)
			8
3.	*(a)*	$4y^3 \frac{dy}{dx} + x^3 \frac{dy}{dx} + 3x^2 y = 2x + 4$	B1 ($4y^3 \frac{dy}{dx}$) B1 ($x^3 \frac{dy}{dx} + 3x^2 y$) B1 ($2x + 4$)
		$4\frac{dy}{dx} + 8\frac{dy}{dx} + 12 = 8$	
		$\frac{dy}{dx} = -\frac{4}{12}$ (o.e.)	B1 (C.A.O.)

	(b)	(i) $\frac{dy}{dx} = \frac{12t^3}{6t^2}$	(o.e.)	M1 A1
		(ii) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{2}{6t^2}$	(o.e.)	M1 (use of correct formula)
				A1 (F.T. one slip)
				8
4.	*(a)*	$\frac{2}{1+x^2} - \frac{12x}{1+x^2} - 8x = 0$		B1 $\left(\frac{2}{1+x^2}\right)$ M1 $\left(\frac{kx}{1+x^2}\right)$ A1 ($k = 12$)
		$2 - 12x - 8x - 8x^3 = 0$		A1 (correct unsimplified equation, F.T. one slip)
		$4x^3 + 10x - 1 = 0$		A1 (convincing)
	(b)	x : $4x^3 + 10x - 1$ 0 : -1 1 : 13	Change of sign indicates	M1 (attempt to find signs or values)
			presence of root	A1 (correct signs, values and conclusion)
		$x_0 = 0.1$, $x_1 = 0.0996$, $x_2 = 0.0996048$		B1 (x_1)
		$x_3 = 0.0996047 \approx 0.099605$		B1 (x_3, rounded or unrounded)
		Check 0.0996045, 0.0996055		M1 (attempt to find signs or values) A1 (correct)
		x 0.0996045 0.0996055	$4x^3 + 10x - 1$ -0.000002 0.000008	Root lies between 0.0996045 and 0.0996055 and is therefore 0.099605 to 6 decimal places
				A1 (correct conclusion)
				12
5.	*(a)*	$-e^{3x} \sin x + 3e^{3x} \cos x$		M1 ($f(x)e^{3x} + g(x) \cos x$) A1 ($f(x) = -\sin x$, $g(x) = ke^{3x}$) A1 ($k = 3$, correct unambiguous answer)

	(b)	$\dfrac{(3x^2+2)4x-(2x^2+1)6x}{(3x^2+2)^2}$	M1 $\left(\dfrac{(3x^2+2)f(x)-(2x^2+1)g(x)}{(3x^2+2)^2}\right)$
			A1 ($f(x) = 4x$, $g(x) = 6x$)
		$=\dfrac{2x}{(3x^2+2)^2}$	A1 (C.A.O.)
	(c)	$10x\sec^2(5x^2+3)$	M1 ($\sec^2(5x+3)$, allow $g(x) = 1$) A1 (correct unambiguous answer)
	(d)	$\dfrac{1}{2x}\times 2=\dfrac{1}{x}$	M1 ($\dfrac{k}{2x}$, allow $k = 1, 2$) A1 (simplified answer)
	(e)	$\dfrac{3}{\sqrt{1-(3x)^2}}\left(=\dfrac{3}{\sqrt{1-9x^2}}\right)$	M1 ($\dfrac{k}{\sqrt{1-(3x)^2}}$ (o.e.), allow $k = 1$) A1 ($k = 3$)
			12
6.	*(a)*	$3x-8\le 5$	
		$x\le\dfrac{13}{3}$	B1
		$3x-8\ge -5$	M1
		$x\ge 1$	
		$1\le x\le\dfrac{3}{13}$ (or $x\ge 1$ and $x\le\dfrac{3}{13}$)	A1 (must indicate both conditions apply)
	(b)	Graphs	M1 (for $\lvert x\rvert$, V shape through origin) A1 (translation in $+ve$ y direction, cusp at $(\pm 2, 1)$) A1 (cusp at $(-2, 1)$ A1 (correct relative positions)
			7
7.	*(a)*	(i) $\dfrac{4}{7}\ ln\ \lvert 7x+2\rvert-\dfrac{5}{6(3x+1)^2}$	M1 ($k\ ln\ (7x + 2)$)
		($+C$)	A1 $\left(k=\dfrac{4}{7}\right)$
			M1 $\left(\dfrac{k}{(3x+1)^2}\right)$ A1 $\left(k=-\dfrac{5}{6}\text{(o.e)}\right)$

	(ii)	$\frac{1}{2}\sin 2x\ (+C)$	M1 ($k \sin 2x$, $k = \frac{1}{2}, -\frac{1}{2}$, 1,2) A1 ($k = \frac{1}{2}$)
	(b)	$\left[2e^{\frac{x}{2}}\right]_0^4$	M1 ($ke^{\frac{x}{2}}, k = 2, \frac{1}{2}, 1$) A1 ($k = 2$)
		$= 2e^2 - 2e^0$	M1 ($ke^2 - ke^0$, allowable *k*s)
		~ 12.8	A1 (C.A.O. at least 3 sig. figs)
			10
8.	*(a)*	Let $y = 3x^2 + 4$	M1 ($x^2 = f(y)$)
		$x^2 = \frac{y-4}{3}$	
		$x = \pm\sqrt{\frac{y-4}{3}}$	A1
		Choose + ∵ domain of f is $x \geq 0$	
		$x = \sqrt{\frac{y-4}{3}}$	A1 (F.t. one slip)
		$f^{-1}(x) = \sqrt{\frac{x-4}{3}}$	A1 (F.T. one slip)
		Domain is $x \geq 4$, range is $f^{-1}(x) \geq 0$ (o.e.) Domain is $x \geq 4$, range is $f^{-1}(x) > 0$	B1, B1
	(b)	Graphs	M1 (full or half parabola passing through (0,b)) A1 (r.h. branch and minimum at (0,4)) A1 F.T. b, full or half parabola))
			9
9.	*(a)*	Domain is $(2, \infty)$	B1
	(b)	$(fg(x) = 5)$	
		$e^{\ln(x^2-4)} = 5$	= 5

M1 (correct order)

		$x^2 - 4 = 5$ or $ln\ (x^2 - 4) = ln\ 5$	A1 (either)
		$x^2 = 9$	A1
		$x = 3$ (– 3 not in domain)	A1 (with reason)
			5

MATHEMATICS FP1

1. Mod = $\sqrt{3+1} = 2$ B1

Arg = $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ (30°) B1

$\sqrt{3} + \mathrm{i} = 2(\cos\left(\frac{\pi}{6}\right) + \mathrm{i}\sin\left(\frac{\pi}{6}\right))$ B1

Arg of $(\sqrt{3} + \mathrm{i})^n = \frac{n\pi}{6}$ M1A1

Power is real when

$\frac{n\pi}{6}$ is an integer multiple of π A1

Smallest $n = 6$. A1

2. Det $= 1.2 + 2\lambda + \lambda(3\lambda - 4)$ M1m1A1

$= 3\lambda^2 - 2\lambda + 2$ A1

EITHER $= 3\left((\lambda - 1/3)^2 + 5/9\right)$ M1

> 0 for all λ A1

OR $'b^2 - 4ac' = -20$ M1

So determinant not equal to zero for any real λ. A1

3. $f(x + h) - f(x) = \frac{1}{1-(x+h)^2} - \frac{1}{1-x^2}$ M1A1

$= \frac{1 - x^2 - [1-(x+h)^2]}{[1-(x+h)^2](1-x^2)}$ m1

$= \frac{h(2x+h)}{[1-(x+h)^2](1-x^2)}$ A1

$f'(x) = \lim_{h \to 0} \frac{(2x+h)}{[1-(x+h)^2](1-x^2)}$ M1

$= \frac{2x}{(1-x^2)^2}$ A1

4. $\frac{11+7\mathrm{i}}{1+\mathrm{i}} = \frac{(11+7\mathrm{i})(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}$ M1A1

$= \frac{18-4\mathrm{i}}{2} = 9 - 2\mathrm{i}$ A1

$2(x + \mathrm{i}y) + (x - \mathrm{i}y) = 9 - 2\mathrm{i}$ M1

Equating real and imaginary parts m1

$x = 3$ and $y = -2$ A1

5. *(a)*

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ B1

$$\mathbf{T}_2 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ B1

$$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ M1

$$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$ A2

Note: Multiplying the wrong way gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ Award M1A1

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

giving $-y-2 = x$ and $x+1 = y$ A1

Fixed point is (–3/2,–1/2). M1A1

FT from wrong answer to (a) – the incorrect matrix above leads to (–1/2,3/2).

6. *(a)* Assume the proposition is true for $n = k$, that is

$$\sum_{r=1}^{k}(2r+1) = (k+1)^2$$ B1

Consider

$$\sum_{r=1}^{k+1}(2r+1) = (k+1)^2 + 2k + 3$$ M1A1

$$= (k+2)^2$$ A1

So, if the proposition is true for $n = k$, it is also true for $n = k+1$. A1

(b) Since 3 ≠ 4, P is false for $n = 1$ is therefore false. B2

7. *(a)* Using reduction to echelon form,

$$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & \lambda-2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu-4 \end{bmatrix}$$ M1A1A1

$$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & \lambda-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu-10 \end{bmatrix}$$ A1

The solution will not be unique if $\lambda = 3$. A1

(b) The equations are consistent if $\mu = 10$. B1

Put $z = \alpha$ M1

$y = \alpha - 6$ A1

$x = 16 - 4\alpha$ M1A1

8. *(a)* Let the roots be $\alpha - 1, \alpha, \alpha + 1$. M1

Then $\alpha(\alpha - 1) + \alpha(\alpha + 1) + (\alpha - 1)(\alpha + 1) = 47$ m1

$\alpha^2 = 16$ A1

$\alpha = -4$ A1

The roots are –3, –4 and –5. A1

(b) $p = 12$, $q = 60$B1B1

9. *(a)* $\ln f(x) = \frac{1}{x}\ln(x)$ M1

$$\frac{f'(x)}{f(x)} = -\frac{1}{x^2}\ln(x) + \frac{1}{x^2}$$ m1A1

$$= \frac{1}{x^2}(1 - \ln(x))$$ A1

At the stationary point,

$\ln(x) = 1$ M1

so $x = \mathrm{e}$ (2.72) and $y = \mathrm{e}^{\frac{1}{\mathrm{e}}}$ (1.44) A1A1

(b) We now need to determine its nature.

We see from above that

For $x < \mathrm{e}$, $f'(x) > 0$ and for $x > \mathrm{e}$, $f'(x) < 0$ M1

Showing it to be a maximum. A1

10. $$w = \frac{z+3}{z+1}$$

$wz + w = z + 3$ M1

$z(w-1) = 3 - w$ A1

$z = \dfrac{3-w}{w-1}$ A1

Since $|z| = 1$, it follows that

$|3-w| = |w-1|$ M1

$\sqrt{(3-u)^2 + v^2} = \sqrt{(u-1)^2 + v^2}$ A1

$9 - 6u + u^2 + v^2 = u^2 - 2u + 1 + v^2$ A1

leading to $u = 2$. A1

Straight line parallel to the v-axis (passing through (2,0)). B1

ALTERNATIVE METHOD

$u + \mathrm{i}v = \dfrac{x+3+\mathrm{i}y}{x+1+\mathrm{i}y} \cdot \dfrac{x+1-\mathrm{i}y}{x+1-\mathrm{i}y}$ M1

$= \dfrac{(x+1)(x+3) + y^2 + \mathrm{i}y(x+1-x-3)}{(x+1)^2 + y^2}$ A1

Equating real and imaginary parts, M1

$u = \dfrac{x^2 + y^2 + 4x + 3}{x^2 + y^2 + 2x + 1}$ A1

$v = \dfrac{-2y}{x^2 + y^2 + 2x + 1}$ A1

Putting $x^2 + y^2 = 1$, M1

$u = 2$ A1

which is a straight line parallel to the v-axis (passing through (2,0)).

B1

MATHEMATICS M1

1(a) (i) Use of $v^2 = u^2 + 2as$ with $u = 0$, $a = (\pm)9.8$, $s = (\pm)160$ — M1

$v^2 = 0 + 2 \times 9.8 \times 160$ — A1

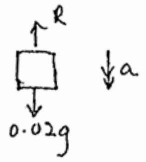
$v = \underline{56 \text{ ms}^{-1}}$ — A1

(ii) Use of $s = ut + \frac{1}{2}at^2$ with $s = (\pm)160$, $u = 0$, $a = (\pm)9.8$ — M1

$(-)160 = \frac{1}{2} \times (-)9.8t^2$ — ft v — A1√

$t = \underline{\frac{40}{7} \text{ s}} = 5\frac{5}{7} = 5.71$ — cao A1

(b)



use of 0.2 for mass mark as MR-1

(i) Use of N2L — dim. correct — M1

$0.02g - 0.096 = 0.02a$ — A1

$a = \underline{5 \text{ ms}^{-2}}$ — A1

(ii) Use of $s = ut + \frac{1}{2}at^2$ with $u = 0$, $a = (\pm)5c$, $t = 4$ — M1

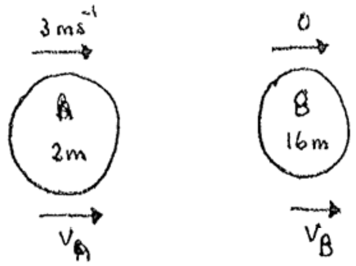
$s = \frac{1}{2} \times 5 \times 4^2$ — ft a — A1√

$= 40$

$\therefore$ Height above ground $= 160 - 40$

$= \underline{120 \text{ m}}$ — A1√

2(a)



Conservation of momentum — M1

$$2\times3(m) + 16\times0(m) = 2V_A(m) + 16V_B(m)$$ — A1

$$V_A + 8V_B = 3$$

Restitution — allow u instead of v — M1

$$V_B - V_A = -\tfrac{1}{2}(0-3)$$ — A1

$$-V_A + V_B = \tfrac{3}{2}$$

Adding — dep on both Ms — M1

$$9V_B = \tfrac{9}{2}$$

$$V_B = \tfrac{1}{2}\ \text{ms}^{-1}$$ — ft 1 slip — A1

$$V_A = V_B - \tfrac{3}{2}$$

$$= -1\ \text{ms}^{-1}$$ — ft 1 slip — A1

Allow V_A going to left. Consistent equations

(b) Impulse $I = 2m(-1-3)$ — M1

$= -8m$ Ns $=$ kg m s⁻¹ — allow no − — A1 B1

In the direction opposite to the original motion of A — A1

3. (a) $T = \dfrac{600}{20}$

$= \underline{30}$ B1

(b) Using $s = ut + \frac{1}{2}at^2$ with $u = 15$, $s = 600$, $t = 30$ M1

$600 = 15 \times 30 + \frac{1}{2}a \times 30^2$ A1

$a = \underline{\frac{1}{3}\ ms^{-2}}$ accept 0.3, 0.33 etc A1

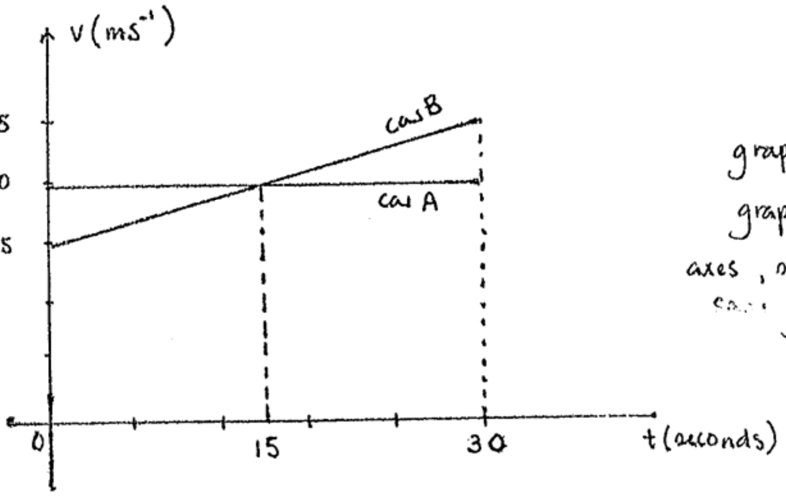
(c) Using $s = \frac{1}{2}(u+v)t$ with $s = 600$, $u = 15$, $t = 30$ M1

$600 = \frac{1}{2}(15 + v) \times 30$ A1√

$v = \underline{25\ ms^{-1}}$ cao A1

PA if 0.3 or 0.33 used

(d)



graph A B1

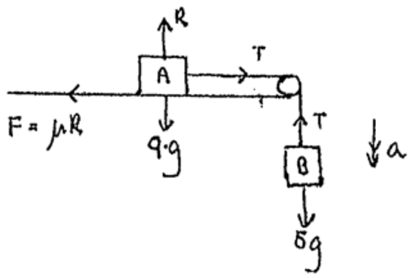
graph B B1√

axes, scales etc B1

required time when A and B have the same speed is $\underline{15\ s}$

ft c's v B1√

4 (a)



N2L to B	$T = 5g = 49$		B1
N2L to A	$T = F = 49$		B1
Resolve ↑ for A	$R = 9g = 88.2$		B1
System in equilibrium	$F \le \mu R$	for $F = \mu R$ used, $\le$ and $a = 0$	M1
	$\mu \geq \frac{5g}{9g} = \frac{5}{9}$	convincing	A1

(b)

$R = 9g = 88.2$

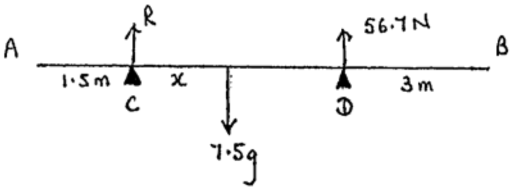
$F = \mu R$

$= 0.5 \times 9 \times 9.8$

$= 4.5g = 44.1$ B1

N2L to B	$5g - T = 5a$		M1 A1
N2L to A	$T - F = 9a$		M1 A1
	$T - 4.5g = 9a$		
Adding	$5g - 4.5g = 14a$	dep on both Ms.	M1
	$a = \frac{0.5 \times 9.8}{14}$		
	$= \underline{0.35 \text{ ms}^{-2}}$		A1
	$T = 9 \times 0.35 + 4.5 \times 9.8$		
	$= \underline{47.25 \text{ N}}$	cao	A1
		dep. only on one M	

5.



(a) Moment about C — dim. correct — M1

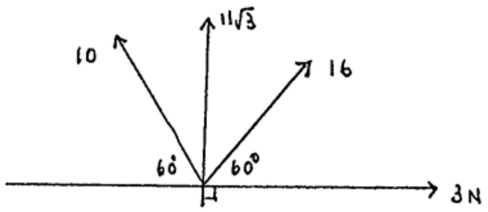
$7.5g\, x = 56.7 \times (5 - 1.5)$ — A1 B1

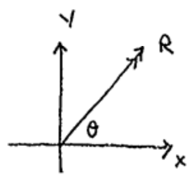
$x = \underline{2.7 \text{ m}}$ — cao A1

(b) Resolve ↑ $R + 56.7 = 7.5g$ — M1

$R = \underline{16.8 \text{ N}}$ — A1

6.





Resolve → $X = 3 + 16\cos 60^\circ - 10\cos 60^\circ$ — resolution — M1 A1 all [illegible] (not [illegible])

$= \underline{6 \text{ N}}$ — A1

Resolve ↑ $Y = 11\sqrt{3} + 10\sin 60^\circ + 16\sin 60^\circ$ — resolution — M1 A1 all [illegible] ([illegible])

$= \underline{24\sqrt{3} \text{ N}} = 41.5692$ — A1

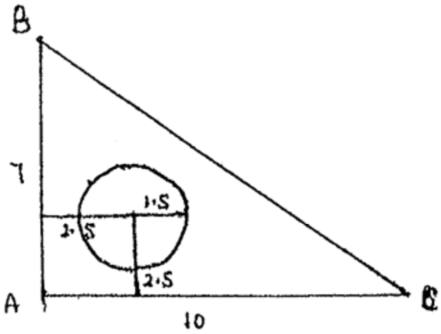
$R = \sqrt{6^2 + (24\sqrt{3})^2}$ — M1

$= \underline{42 \text{ N}}$ — ft X, Y penalise PA. — A1√

$\tan\theta = \dfrac{24\sqrt{3}}{6}$ — M1

$\theta = \underline{81.8^\circ}$ — ft X, Y. — A1√

7.



			from AB	from AC	
(a)	Δ ABC	35	$\frac{10}{3}$	$\frac{7}{3}$	B1 B1 B1 Areas
	Circle	$\pi\left(\frac{3}{2}\right)^2 = 7.069$	$\frac{5}{2}$	$\frac{5}{2}$	B1
	Lamina	$35 - \frac{9}{4}\pi = 27.93$	x	y	

(i) $35 \times \frac{10}{3} = \frac{9}{4}\pi \times \frac{5}{2} + \left(35 - \frac{9}{4}\pi\right)x$ M1 A1√

$x = \underline{3.54}$ cao A1

(ii) $35 \times \frac{7}{3} = \frac{9}{4}\pi \times \frac{5}{2} + \left(35 - \frac{9}{4}\pi\right)y$ M1 A1√

$y = \underline{2.29}$ cao. A1

(b) Required angle $\theta = \tan^{-1}\left(\frac{2.29}{3.54}\right)$ M1

$= \underline{32.9^\circ}$ ft x, y. A1√

M1 – ASSESSMENT OBJECTIVES

Q.No.	AO1	AO2	AO3	AO4	AO5	Total
1	2	2	6	2		12
2	3	2	4	2		11
3	2	3	4	2		11
4	3	3	4	1	2	13
5	1	1	2	2		6
6	2	2	2	1	3	10
7	3	3	3	2	1	12
TOTAL	16	16	25	12	6	75

MATHEMATICS S1

1.	*(a)*	P(Score at least 3 on one die) = $\frac{4}{6}$	B1
		P(Score at least 3 on both dice) = $\left(\frac{4}{6}\right)^2 = \frac{4}{9}$	M1A1
		[Award B1 for the 6 × 6 array of possible scores, M1 for counting the number of these satisfying the required condition and A1 for the correct answer]	
	(b)	Possibilities are 6,3;5,2;4,1 and reverse, ie 6 possibilities	M1A1
		Prob = $\frac{6}{36}$ $\left(\frac{1}{6}\right)$ (cao)	A1
2.	*(a)*	P($A \cup B$) = P(A) + P(B) gives P(B) = 0.2	M1A1
	(b)	0.5P(B) = 0.5 + P(B) – 0.7	M1A1
		0.5P(B) = 0.2 gives P(B) = 0.4	M1A1
	(c)	0.5 × 0.3 = 0.5 + P(B) – 0.7	M1A1
		P(B) = 0.35	A1
3.	*(a)*	2.	B1
	(b)	(i) Prob = $e^{-4}.\frac{4^3}{3!} = 0.195$	1A1
		[Accept 0.7619 – 0.5665 or 0.4335 – 0.2381 from tables, M1 for at least 1 correct value]	
		(ii) Prob = 0.9084 – 0.1107 or 0.8893 – 0.0916 = 0.7977 (cao)	B1B1B1
	(c)	E(C) = 5 + 4 × 4 = 21	M1A1
		SD = √(16 × Var(X)) = 8 [Award M1 for 64]	M1A1
4.	*(a)*	Prob = $\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}$ or $\frac{\binom{5}{3}}{\binom{8}{3}} = \frac{5}{28}$	M1A1

	(b)	P(3 blue) = $\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}$ or $\frac{\binom{3}{3}}{\binom{8}{3}}$ $\quad (=\frac{1}{56})$	B1
		P(2 blue) = $3\times\frac{3}{8}\times\frac{2}{7}\times\frac{5}{6}$ or $\frac{\binom{3}{2}\times\binom{5}{1}}{\binom{8}{3}}$ $\quad (=\frac{15}{56})$	M1A1
		Reqd prob = $\frac{16}{56}$ $\quad (\frac{2}{7})$	B1
5.		$np = 20$; $np(1-p) = 16$	M1A1A1
		Valid attempt to solve equations (FT)	M1
		$p = 0.2$	B1
		$n = 100$	B1
6.	*(a)*	P(4) = $\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{4}$	M1A1
		$= \frac{5}{24}$	A1
	(b)	P(cube$\mid$4) = $\frac{1/12}{5/24} = \frac{2}{5}$ (cao)	B1B1B1
7.	*(a)*	Number of defective glasses X is B(24,0.05) (si)	B1
		P(X = 2) = $\binom{24}{2}\times 0.05^2\times 0.95^{22} = 0.223$	M1A1
	(b)	P(3 ≤ X ≤ 5) = 0.9622 – 0.5405 or 0.4595 – 0.0378 = 0.422 (cao)	B1B1B1
	(c)	Number of defective glasses is now B(120,0.05) ≈ Poi(6)	B1
		P(Y < 8) = 0.744 (or 1 – 0.256)	M1A1
8.	*(a)*	[0,0.7] [Accept (0.0.7)]	B1B1
		[Award B1 for {0,0.1,…0.7} or similar]	
	(b)	(i) E(X) = 0.1 + 0.4 + 3θ + 4(0.7 – θ) = 3.3 – θ	M1A1
		3.3 – θ = 3 gives θ = 0.3	M1A1
		(ii) E(X^3) = 0.1 × 1 + 0.2 × 8 + 0. 3 × 27 + 0.4 × 64	M1A1
		= 35.4	A1

9.	*(a)*	(i)	$\int_1^4 kx^2\,\mathrm{d}x = \frac{k}{3}\left[x^3\right]_1^4 = 21k$	M1A1
			Int = 1 gives $k = \frac{1}{21}$	A1
		(ii)	$\mathrm{E}(X) = \frac{1}{21}\int_1^4 x.x^2\,\mathrm{d}x$ (Limits required, here or later)	M1
			$= \frac{1}{21}.\frac{1}{4}\left[x^4\right]_1^4$	B1
			$= \frac{85}{28}$ (3.04)	A1
	(b)	(i)	$\mathrm{F}(x) = \frac{1}{21}\int_1^x y^2\,\mathrm{d}y$	M1
			$= \frac{1}{63}\left[y^3\right]_1^x$	A1
			$= \frac{1}{63}\left(x^3 - 1\right)$	A1
		(ii)	Prob $= \mathrm{F}(3) - \mathrm{F}(2)$	M1
			$= \frac{19}{63}$ (0.302)	A1
		(iii)	The median m satisfies	
			$\frac{1}{63}\left(m^3 - 1\right) = 0.5$	M1
			$m^3 = 32.5$	A1
			$m = 3.19$ (cao)	A1

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