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GENERAL CERTIFICATE OF EDUCATION
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MARKING SCHEME

MATHEMATICS
AS/Advanced

JANUARY 2008

INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2008 examination in GCE Mathematics. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1

1. (*a*) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -{}^1/_3$ (or equivalent) A1

(*b*) A correct method for finding the equation of $AB(CD)$ using candidate's gradient for AB M1

Equation of AB : $y - 3 = -{}^1/_3\,[x - (-2)]$ (or equivalent) A1

Equation of AB : $x + 3y - 7 = 0$ (convincing) A1

Use of $m_{AB} \times m_{CD} = -1$ M1

Equation of CD : $y - 8 = 3(x - 3)$ (or equivalent)
(f.t. candidate's gradient of AB) A1

Special case:
Verification of equation of AB by substituting coordinates of **both** A and B into the given equation B1

(*c*) An attempt to solve equations of AB and CD simultaneously M1

$x = 1$, $y = 2$ (convincing) (c.a.o.) A1

Special case
Substituting (1, 2) in equations of **both** AB and CD M1
Convincing argument that coordinates of D are (1, 2) A1

(*d*) A correct method for finding the mid-point of AB M1

$E(4, 1)$ A1

A correct method for finding the length of ED M1

$ED = \sqrt{10}$ (f.t. candidate's coordinates of E) A1

2. (*a*) $\sqrt{20} = 2\sqrt{5}$ B1

$\frac{\sqrt{35}}{\sqrt{7}} = \sqrt{5}$ B1

$\frac{20}{\sqrt{5}} = 4\sqrt{5}$ B1

$\sqrt{20} + \frac{\sqrt{35}}{\sqrt{7}} - \frac{20}{\sqrt{5}} = -\sqrt{5}$ (c.a.o.) B1

(*b*) $\frac{2 + \sqrt{3}}{5 + 2\sqrt{3}} = \frac{(2 + \sqrt{3})(5 - 2\sqrt{3})}{(5 + 2\sqrt{3})(5 - 2\sqrt{3})}$ M1

Numerator: $10 - 4\sqrt{3} + 5\sqrt{3} - 2 \times 3$ A1

Denominator: $25 - 12$ A1

$\frac{2 + \sqrt{3}}{5 + 2\sqrt{3}} = \frac{4 + \sqrt{3}}{13}$ (c.a.o.) A1

Special case
If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5 + 2\sqrt{3}$

3. $\frac{dy}{dx} = 4x - 10$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Gradient of tangent at $P = 2$ (c.a.o.) A1

Equation of tangent at P: $y - 4 = 2(x - 3)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ provided both M1 and m1 awarded) A1

4. (*a*) $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (-1 for each error)

(-1 for any subsequent 'simplification') B2

(*b*) (i) $\left(1 + \frac{x}{2}\right)^5 \approx 1^5 + 5\left(\frac{x}{2}\right)1^4 + \frac{5(5-1)}{2}\left(\frac{x}{2}\right)^2 1^3 + \frac{5(5-1)(5-2)}{2 \times 3}\left(\frac{x}{2}\right)^3 1^2$

Two terms correct B1

Other two terms correct B1

$\left(1 + \frac{x}{2}\right)^5 \approx 1 + \frac{5(x)}{2} + \frac{10(x)^2}{4} + \frac{10(x)^3}{8}$ B1

(ii) An attempt to substitute $x = 0{\cdot}1$ in candidate's expression for $\left(1 + \frac{x}{2}\right)^5$ M1

$1{\cdot}05^5 \approx 1{\cdot}276(25)$ (c.a.o.) A1

5. (*a*) An expression for $b^2 - 4ac$, with $c = \pm k$ and at least one of a or b correct M1

$b^2 - 4ac = 2^2 - 4 \times 3 \times (-k)$ A1

Putting $b^2 - 4ac > 0$ m1

$4 + 12k > 0 \Rightarrow k > -{}^1/_3$ (o.e.)

(f.t. only for $c = k$ in original expression for $b^2 - 4ac$) A1

(*b*) Finding critical points $x = -2$, $x = 7$ B1

$-2 \le x \le 7$ **or** $7 \ge x \ge -2$ **or** [–2, 7] **or** $x \le 7$ and $-2 \le x$ **or** a correctly worded statement to the effect that x lies between –2 and 7 (inclusive)

(f.t. candidate's critical points) B2

Note: $-2 < x < 7$,

$x \le 7, -2 \le x$

$x \le 7 \quad -2 \le x$

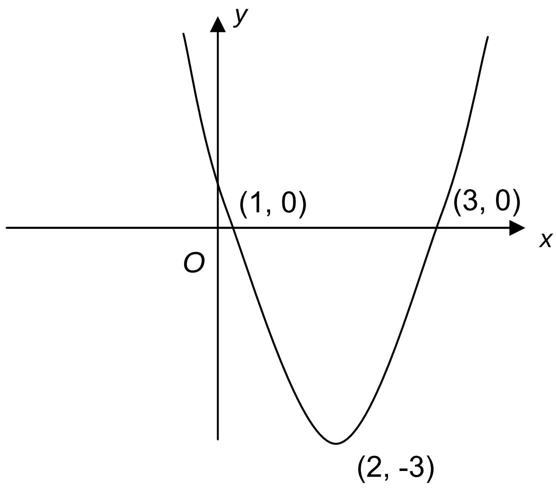
$x \le 7$ or $-2 \le x$ all earn B1

6.	(*a*)	$y + \delta y = 3(x + \delta x)^2 - 4(x + \delta x) + 7$		B1
		Subtracting y from above to find δy		M1
		$\delta y = 6x\delta x + 3(\delta x)^2 - 4\delta x$		A1
		Dividing by δx and letting $\delta x \to 0$		M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 4$	(c.a.o.)	A1
	(*b*)	Required derivative = $5 \times \frac{1}{2} \times x^{-1/2} - 3 \times (-3) \times x^{-4}$		B1, B1

7.	$p = 0{\cdot}9$	B1
	A convincing argument to show that the value 4 is correct	B1
	$x^2 + 1{\cdot}8x - 3{\cdot}19 = 0 \Rightarrow (x + 0{\cdot}9)^2 = 4$	M1
	$x = 1{\cdot}1$	A1
	$x = -2{\cdot}9$	A1

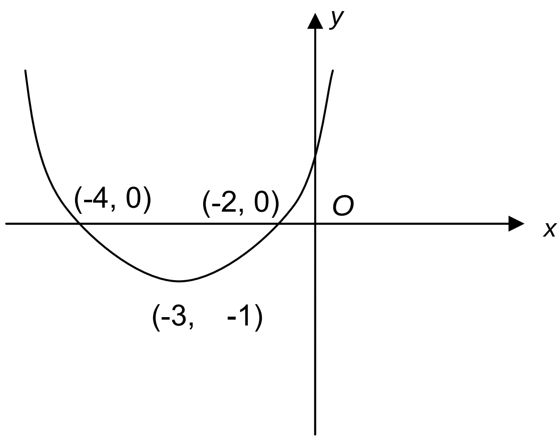
8.	(*a*)	Use of $f(-2) = -24$	M1
		$-48 + 4a + 6 - 2 = -24 \Rightarrow a = 5$	A1
		Special case	
		Candidates who assume $a = 5$ and show $f(-2) = -24$ are awarded	B1
	(*b*)	Attempting to find $f(r) = 0$ for some value of r	M1
		$f(-1) = 0 \Rightarrow x + 1$ is a factor	A1
		$f(x) = (x + 1)(6x^2 + ax + b)$ with one of a, b correct	M1
		$f(x) = (x + 1)(6x^2 - x - 2)$	A1
		$f(x) = (x + 1)(2x + 1)(3x - 2)$ (f.t. only $6x^2 + x - 2$ in above line)	A1
		Special case	
		Candidates who find one of the remaining factors, (2x + 1) or (3x – 2), using e.g. factor theorem, are awarded	B1

9. (*a*)



Concave up curve and minimum point = (2, k) with $k < -1$	B1
Minimum point = (2, –3)	B1
Both points of intersection with x-axis	B1

(*b*)



Concave up curve and y-coordinate of minimum = –1	B1
x-coordinate of minimum = –3	B1
Both points of intersection with x-axis	B1

10. (*a*) $\frac{dy}{dx} = 3x^2 - 12$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$x = 2, -2$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

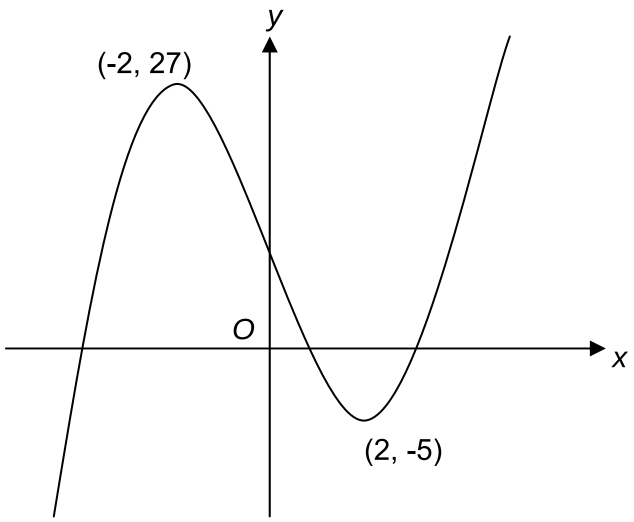
Stationary points are (2, –5) and (–2, 27) (both correct) (c.a.o) A1

A correct method for finding nature of stationary points M1

(–2, 27) is a maximum point (f.t. candidate's derived values) A1

(2, –5) is a minimum point (f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) $k > 27$ B1

$k < -5$ B1

Special case

$k \geq 27$, $k \leq -5$ (both) awarded B1

Mathematics C2

1.

0	0·707106781		
0·25	0·704360725		
0·5	0·68599434		
0·75	0·642575463	(3 values correct)	B1
1	0·577350269	(5 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{2} \times \{0{\cdot}707106781 + 0{\cdot}577350269 + 2(0{\cdot}704360725 + 0{\cdot}68599434 + 0{\cdot}642575463)\}$

$I \approx 0{\cdot}669$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}2$

0	0·707106781		
0·2	0·705696796		
0·4	0·696057558		
0·6	0·671761518		
0·8	0·630943081		
1	0·577350269	(all values correct)	B1

Correct formula with $h = 0{\cdot}2$ M1

$I \approx \frac{0{\cdot}2}{2} \times \{0{\cdot}707106781 + 0{\cdot}577350269 + 2(0{\cdot}705696796 + 0{\cdot}696057558 + 0{\cdot}671761518 + 0{\cdot}630943081)\}$

$I \approx 0{\cdot}669$ (f.t. one slip) A1

2. (*a*) $12(1 - \cos^2\theta) - 5\cos\theta - 9 = 0$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$12\cos^2\theta + 5\cos\theta - 3 = 0 \Rightarrow (3\cos\theta - 1)(4\cos\theta + 3) = 0$

$\Rightarrow \cos\theta = \frac{1}{3}, -\frac{3}{4}$ A1

$\theta = 70{\cdot}53°, 289{\cdot}47°, 138{\cdot}59°, 221{\cdot}41°$ (70·53°, 289·47°) B1

(138·59°) B1

(221·41°) B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks
$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $3x + 15° = 30°, 150°, 390°, 510°$ (one value) B1

$x = 5°, 45°, 125°, 165°,$ (one value) B1

(three values) B1

(four values) B1

Note: Subtract 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-2)d] + [a + (n-1)d]$
(at least 3 terms, one at each end) B1
$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + [a + d] + a$
$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$
(reverse and add) M1
$2S_n = n[2a + (n-1)d]$
$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $S_n = \frac{n}{2}[2 + (n-1)2]$ B1
$S_n = n^2$ (c.a.o.) B1

(*c*) $a + 19d = 98$ B1
$\frac{20}{2} \times [2a + 19d] = 1010$ B1
An attempt to solve the above equations simultaneously by eliminating one unknown M1
$a = 3$, $d = 5$ (both values) (c.a.o.) A1

4. (*a*) $ar^7 = 5$ and $ar^4 = 135$ M1
$r^3 = \frac{5}{135}$ m1
$r = \frac{1}{3}$ A1
$a \times \frac{1}{3^4} = 135$ M1
$a = 10935$ (c.a.o.) A1

(*b*) $S_\infty = \frac{10935}{1 - {}^1/_3}$ (use of formula for sum to infinity) M1
$S_\infty = 16402 \cdot 5$ (f.t. candidate's derived value for *a*) A1

5. (*a*) Substituting the correct values in the correct places in the cos rule M1
$(2 \times 6 \times 9) \times \cos B\hat{A}C = 6^2 + 9^2 - 13^2$ A1
$\cos B\hat{A}C = -\frac{52}{108} = -\frac{13}{27}$ (c.a.o) A1

(*b*) $\sin B\hat{A}C = \frac{\sqrt{560}}{27}$ (o.e.) B1
Area of triangle *ABC* = $\frac{1}{2} \times 6 \times 9 \times \sin B\hat{A}C$
(correct use of area formula) M1

Area of triangle *ABC* = $\sqrt{560} = 4\sqrt{35}$ (convincing) A1

6. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between p and $\log_a x$) B1

$\dfrac{x}{y} = \dfrac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1

$\therefore \log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) (i) $(2x - 1)\log 3 = \log 11$ (taking logs on both sides) M1

An attempt to isolate x (no more than 1 algebraic error) m1

$x = 1{\cdot}591$ (c.a.o.) A1

(ii) ${}^3/_2 \log_a 16 = \log_a 16^{3/2}$, $2\log_a 12 = \log_a 12^2$

(one use of power law) B1

${}^3/_2 \log_a 16 + \log_a 6 - 2\log_a 12 = \log_a \dfrac{16^{3/2} \times 6}{12^2}$ (addition law) B1

(subtraction law) B1

${}^3/_2 \log_a 16 + \log_a 6 - 2\log_a 12 = \log_a {}^8/_3$ (o.e.) (c.a.o.) B1

7. (*a*) $4 \times \dfrac{x^{5/3}}{5/3} - 7 \times \dfrac{x^{1/2}}{1/2}$ $(+\,c)$ B1,B1

(*b*) (i) $x^2 - 6x + 11 = -x + 7$ M1

An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b$ = candidate's constant

m1

$(x - 1)(x - 4) = 0 \Rightarrow x = 1, x = 4$ (both values, c.a.o.) A1

$y = 6$, $y = 3$ (both values, f.t. candidate's x-values) A1

(ii)

Either:

Total area $= \displaystyle\int_1^4 (-x + 7)\,dx - \int_1^4 (x^2 - 6x + 11)\,dx$

(use of integration) M1

(subtraction of integrals with candidate's x_A, x_B as limits in at least one integral) m1

$= \left[(5/2)x^2 - (1/3)x^3 - 4x\right]_1^4$ (o.e.)

(correct integration) B3

$= [40 - 4^3/3 - 16] - [5/2 - 1/3 - 4]$ (o.e.)

(use of candidate's x_A, x_B as limits in all integrals) m1

$= 9/2$ (c.a.o.) A1

Or:

Area of trapezium = 27/2
(f.t. candidate's coordinates for *A* and *B*) B1

Area under curve $= \int_1^4 (x^2 - 6x + 11)\,dx$
(use of integration) M1

$= \left[(1/3)x^3 - 3x^2 + 11x\right]_1^4$

(correct integration) B2

$= [(64/3 - 48 + 44) - (1/3 - 3 + 11)]$
(use of candidate's x_A, x_B as limits) m1

$= 9$

Finding total area by subtracting values m1

Total area = 27/2 – 9 = 9/2 (c.a.o.) A1

8. (*a*) $A(2, -3)$ B1

A correct method for finding the radius M1

Radius = 5 A1

(*b*) Gradient $AP = \dfrac{\text{inc in } y}{\text{inc in } x}$ M1

Gradient $AP = \dfrac{1 + 3}{5 - 2} = \dfrac{4}{3}$ (f.t. candidate's coordinates for *A*) A1

Use of $m_{\text{tan}} \times m_{\text{rad}} = -1$ M1

Equation of tangent is:

$y - 1 = -\dfrac{3}{4}(x - 5)$ (f.t. candidate's gradient for *AP*) A1

(c) An attempt to substitute $(x + 3)$ for y in the equation of the circle and form quadratic in x M1

$x^2 + (x + 3)^2 - 4x + 6(x + 3) - 12 = 0 \Rightarrow 2x^2 + 8x + 15 = 0$ A1

An attempt to calculate value of discriminant M1

Discriminant = $64 - 120 < 0 \Rightarrow$ no points of intersection A1

9. $r\theta = 6$ B1

$\dfrac{r^2\theta}{2} = 22{\cdot}5$ B1

An attempt to eliminate θ M1

$\dfrac{r}{2} = \dfrac{22{\cdot}5}{6} \Rightarrow r = 7{\cdot}5$ (c.a.o.) A1

$\theta = \dfrac{6}{7{\cdot}5} \Rightarrow \theta = 0{\cdot}8$ (f.t. candidate's value for *r*) A1

Mathematics C3

1.		$h = 0{\cdot}2$	M1 (Correct formula $h = 0.2$)
		Integral $\approx \frac{0\cdot 2}{3}$ [1 + 1·8964809 + 4 (1·0408108	
		+ 1·4333294)	B1 (3 correct values)
		+ 2 (1·1735109)]	B1 (2 correct values)
		$\approx 1{\cdot}0093$	A1 (F.T. one slip)
			(4)
2.	*(a)*	$\theta = \frac{\pi^c}{2}$ or degrees	B1 (choice of θ and use correct evaluation)
		$\sin 3\theta = \sin \frac{3\pi}{2} = -1$	B1 (2 correct evaluations)
		$4 \sin \frac{\pi}{2} - 3 \sin^3 \frac{\pi}{2} = +1$	
		$(\therefore \sin 3\theta \neq 4 \sin\theta - 3 \sin^3 \theta)$	
	(b)	see $\theta = 1 - 2 (\sec^2 \theta - 1)$	M1 ($\tan^2 \theta = \sec^2 \theta - 1$)
		$2 \sec^2 \theta + \sec\theta - 3 = 0$	M1((a sec θ +b)(c sec θ + d)
		$(2 \sec\theta + 3) (\sec\theta - 1) = 0$	with ac = coefft of $\sec^2 \theta$
		$\sec\theta = -\frac{3}{2}$, $\sec\theta = 1$	bd = constant term or use of correct formula)
		$\cos\theta = -\frac{3}{2}$, $\cos\theta = 1$	A1
		$\theta = 131{\cdot}8^\circ, 228{\cdot}2^\circ, 0^\circ, 360^\circ$	B1 (131·8°) B1(228·2°) B1 (0°, 360°)
			(8)

3. *(a)* $\frac{dy}{dx} = \frac{2e^{2t}}{4t^3}$

M1 ($\dot{y} = ke^{2t}$, k = 1 or 2 or $2e^{2t}+5$)

A1 ($2e^{2t}$)

M1 $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$, A1 (all correct C.A.O.)

(b) $4x^3 + \cos y \frac{dy}{dx} + 2xy^3 + 3x^2y^2\frac{dy}{dx} = 0$

B1 (cos $y\frac{dy}{dx}$)

B1 ($2xy^3 + 3x^2\frac{dy}{dx}$)

$$\frac{dy}{dx} = -\frac{4x^3 + 2xy^3}{\cos y + 3x^2y^2}$$

B1 $\left(\text{F.T } \frac{d}{dy}(\sin y) = -\cos y\frac{dy}{dx} \right)$

(7)

4.

x	$2\ln(x+70)-x$
8	0.71
9	−0·26

M1 (attempt to find values)

Change of sign indicates α is between 8 and 9

A1 (correct values or signs and conclusion)

x_0 = 8·8, x_1 = 8·7338 ….. , x_2 = 8·7321 ..

B1 (x_1)

$x_3 \approx$ 8·7321

B1 (x_3)

Check 8·73205, 8·73215

x	$f(x)$
8.73205	0.00005
8.73215	−0.0005

M1 (attempt to find relevant values or signs)

A1 (correct values)

Change of sign indicates α is 8·7321 correct to four decimal places

A1 (FT for incorrect values)

(7)

5. *(a)* $\dfrac{x^2 \frac{1}{x} - (\ln x)(2x)}{x^4}$ = $\dfrac{1 - 2\ln x}{x^3}$

M1 $\dfrac{(x^2)(f(x) - (\ln x)(g(x))}{x^4}$

$\text{A1}(f(x) = \frac{1}{x}, g(x) = 2x)$

A1 (simplified answer)

(b) $\dfrac{-5}{\sqrt{1 - 25x^2}}$ (o.e)

M1 $\left(\dfrac{-k}{\sqrt{1 - (5x)^2}} \right)$ k = 1, ± 5)

$\left(\text{B1 for } \dfrac{-5}{\sqrt{1 - 5x^2}} \right)$

A1 $k = 5$

(c) $\frac{1}{2}\left(1 + 6x^4\right)^{-\frac{1}{2}} 24x^3$

M1 ($\frac{1}{2}(1 + 6x^4)^{-\frac{1}{2}} f(x)$

$f(x) = 24x^n, n = 1,2,3$

$f(x) = kx^3$)

$= 12x^3(1 + 6x^4)^{-\frac{1}{2}}$

A1 ($f(x) = 24x^3$, unambiguous simplified statement)

(d) $2x^3 \sec^2 2x + 3x^2 \tan 2x$

M1 ($x^3 f(x) + \tan 2x g(x)$)

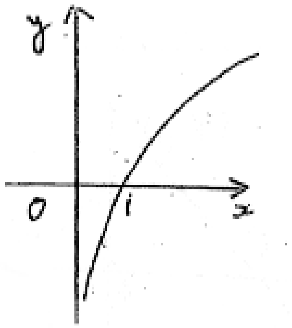
A1 ($f(x) = k\sec^2 2x, k = 1,2$)

A1 ($k = 2$, unambiguous answer)

(10)

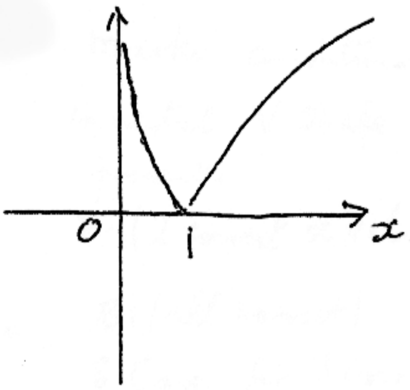
6.

(a) (i)



(i) M1 (shape)
A1((1,0), all correct)

(ii)



M1 (graph above x −axis and former shape for x >1)
A1 (x <1 correct)
F.T −ve parts of graph from (i))

(b) $3x-2 < 4$ B1

$x<2$

$3x-2>-4$ M1 ($3x-2>-4$)

$x>-\dfrac{2}{3}$ A1

$x>-\dfrac{2}{3}$ *and* $x<2$ A1 (must indicate both conditions, C.A.O)

(8)

7 *(a)*

(i) $\dfrac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}.2}$ = $\dfrac{(2x+3)^{\frac{3}{2}}}{3}$ $(+C)$

M1 ($\dfrac{k(2x+3)^{\frac{3}{2}}}{\frac{3}{2}}$,k = 1, 2, ½)

A1 (k = ½)

(ii) $\frac{3}{7}\ln|7x+2|$ *(+C)*

M1 $k\ln(7x+2)$

A1 $\left(k=\frac{3}{7} \right)$

(iii) $\frac{5}{2}e^{2x-7}(+C)$

M1 (ke^{2x-7})

A1 $\left(k=\frac{5}{2} \right)$

(b) $\left[-\frac{1}{4}\cos(4x+\frac{\pi}{6}) \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

M1($k\cos\left(4x+\frac{\pi}{6} \right)$

$k=\frac{1}{4},\ -\frac{1}{4},-1,-4$ **)**

A1 $\left(k=-\frac{1}{4} \right)$

= $\frac{1}{4}\left[-\cos\left(\frac{4\pi}{3}+\frac{\pi}{6} \right)+\cos\left(\frac{4\pi}{6}+\frac{\pi}{6} \right) \right]$

$=\frac{1}{4}\left[-\cos\frac{3\pi}{2}+\cos\frac{5\pi}{6} \right]$

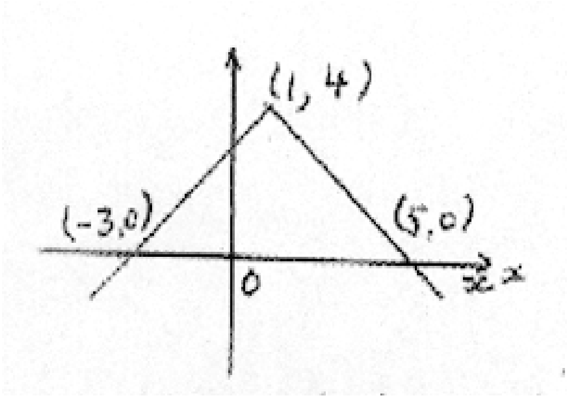
A1 $\left(k(\cos\frac{3\pi}{2}-\cos\frac{5\pi}{6} \right)$ allowable k

$=\frac{1}{4}\left(0+\left(-\frac{\sqrt{3}}{2} \right) \right)=\frac{-\sqrt{3}}{2}\approx -0.217$

A1 (C.A.O) (either form)

(10)

8.



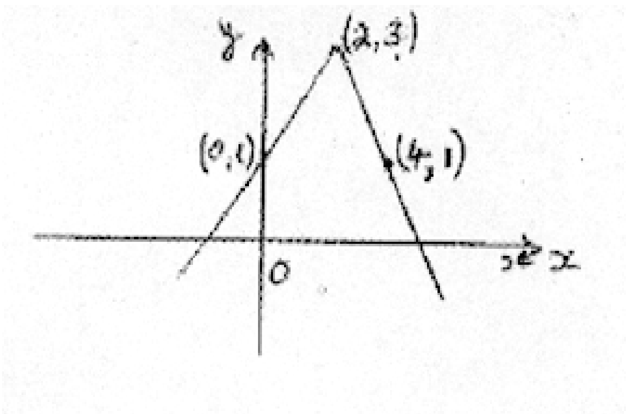
Marks conditional on inverted V shape being present,

B1 (2 correct *x* values),

B1 (*y* = 4 for highest pt.)

B1 (all correct)

S.Case for 3 correct points and incorrect or no graph, B1



B1 (left line of graph intersects *y*- axis at (0,1))

B1 (Second point)

B1 (all correct)

S. Case for 3 correct pts and incorrect or no graph, B1

(6)

9. *(a)*	$fg(x) = \ln e^{4x} = 4x$	M1 (correct order)
		A1
(b)	$gf(x) = e^{4\ln x}$	M1 (correct order)
	$= e^{\ln x^4}$	A1 (power laws)
	$= x^4$	A1

(5)

10. *(a)*	Range is $(0, \infty)$	B1
(b)	Let y = $\frac{1}{\sqrt{x-2}}$	
	$y^2 = \frac{1}{x-2}$	M1 $\left(y^2 = \frac{1}{x-2} \right)$(and attempt to solve)
	$x - 2 = \frac{1}{y^2}$	A1
	$x = 2 + \frac{1}{y^2}$	A1 (C.A.O,)
	$f^{-1}(x) = 2 + \frac{1}{x^2}$	B1 (F.T candidate's $x = f(y)$)
Domain	$(0, \infty)$, range $(2, \infty)$	B1 (both values correct or F.T. from (a))
(c)	$2 + \frac{1}{x^2} = -\frac{3}{x}$	M1 (equating and attempting to set up quadratic equation)
	$2x^2 + 1 = -3x$	
	$2x^2 + 3x + 1 = 0$	A1 (F.T one $\pm$ slip in $f^{-1}(x)$)
	$(2x+1)(x+1) = 0$	
	$x = -\frac{1}{2}, -1$	A1
Not in domain of f^{-1}		A1(F.T candidate's values)
$\therefore$ No solutions		

(10)

Mathematics - FP1

1.

$$x + 3y + 2z = 14$$
$$5y + 3z = 21$$
$$7y + 7z = 35$$

M1A1A1

$$x + 3y + 2z = 14$$
$$5y + 3z = 21$$
$$14z = 28$$

A1

z = 2, *y* = 3, *x* = 1 — A1A1A1

2. *(a)* $\alpha + \beta + \gamma = 1, \beta\gamma + \gamma\alpha + \alpha\beta = 3$ — B1

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\beta\gamma + \gamma\alpha + \alpha\beta)$ — M1A1

$= 1 - 6 = -5$ — A1

A cubic equation has either 1 or 3 real roots. This one has 1 real root since the sum of squares of 3 real numbers cannot be negative. — B2

(b) A second root is 1 + 2i. — B1

The third root is – 1 since the sum of the roots is 1. — M1A1

3. *(a)* detA = 2 - λ +2(1 – 4) + 2λ - 1 = λ - 5 — M1A1

The matrix is singular when λ = 5. — A1

(b)(i)

Cofactor matrix = $\begin{bmatrix} -2 & -3 & 7 \\ 0 & 1 & -2 \\ 1 & 1 & -3 \end{bmatrix}$ — M1A1

Adjugate matrix = $\begin{bmatrix} -2 & 0 & 1 \\ -3 & 1 & 1 \\ 7 & -2 & -3 \end{bmatrix}$ — A1

(ii) det**A** = -1 — B1

Inverse matrix = $\begin{bmatrix} 2 & 0 & -1 \\ 3 & -1 & -1 \\ -7 & 2 & 3 \end{bmatrix}$ — M1A1

4. *(a)* Let

$$\frac{2}{(4x^2-1)} = \frac{A}{2x-1} + \frac{B}{2x+1} = \frac{A(2x+1)+B(2x-1)}{4x^2-1}$$ M1A1

x = ½ gives A = 1 , x = -½ gives B = - 1. A1A1

(b) $S_n = \frac{1}{1} - \frac{1}{3}$ M1

$+\frac{1}{3} - \frac{1}{5}$

…………

$+\frac{1}{2n-3} - \frac{1}{2n-1}$

$+\frac{1}{2n-1} - \frac{1}{2n+1}$ A1

$= 1 - \frac{1}{2n+1}$ ($= \frac{2n}{2n+1}$) A1

5. *(a)* Reflection matrix $= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Translation matrix = $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ B1

T = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ M1

= $\begin{bmatrix} 0 & 1 & b \\ 1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}$ AG

(b)(i) Fixed points satisfy

$$\begin{bmatrix} 0 & 1 & b \\ 1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ M1

giving

$y + b = x$

$x + a = y$ A1

Or $y - a = x$

(i) If $a + b = 0$, these equations are consistent A1
and fixed points lie on the line (or satisfy)
$y = x + a$ (or $y = x - b$). A1

(ii) $\mathbf{T}^2 = \begin{bmatrix} 0 & 1 & b \\ 1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & b \\ 1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ M1A1

Under **T** followed by **T**, every point is transformed to itself. A1
[Award Bi for correct answer without justification]

6. *(a)* **EITHER**

$(3 + 2i)^2 = 5 + 12i$ M1A1

$\left|(3+2i)^2\right| = \sqrt{5^2 + 12^2} = 13$ A1

$\arg(3 + 2i)^2 = \tan^{-1}(12/5) = 1.18$ (67.4°) A1

OR $|3 + 2i| = \sqrt{13}$, $\arg(3 + 2i) = \tan^{-1}(2/3)$ B1B1

$|(3 + 2i)^2| = (\sqrt{13})^2 = 13$ B1

$\arg(3 + 2i)^2 = 2\tan^{-1}(2/3) = 1.18$ (67.4°) B1

(b) **EITHER**

$\frac{1}{2+i} + \frac{1}{1-2i} = \frac{1-2i+2+i}{(2+i)(1-2i)}$ M1A1

$= \frac{3-i}{4-3i}$ A1

$u = \frac{(4-3i)(3+i)}{(3-i)(3+i)}$ M1

$= \frac{15-5i}{10}$ A1

$= 1.5 - 0.5i$ A1

OR

$\frac{1}{2+i} = \frac{2-i}{5}; \frac{1}{1-2i} = \frac{1+2i}{5}$ M1A1A1

$u = \frac{1}{\text{sum}} = \frac{5}{3+i} = \frac{15-5i}{10}$ M1A1

$= 1.5 - 0.5i$ A1

.The statement is true for n = 1 since the formula gives the correct answer 2. B1

Let the statement be true for $n = k$, ie

$$\sum_{r=1}^{k} r \times 2^r = 2^{k+1}(k-1) + 2$$ M1

Consider $$\sum_{r=1}^{k+1} r \times 2^r = 2^{k+1}(k-1) + 2 + (k+1)2^{k+1}$$ M1A1

$$= 2^{k+1}(k-1+k+1) + 2$$ A1

$$= 2^{k+2}k + 2$$ A1

Thus, true for $n = k \Rightarrow$ True for $n = k + 1$. A1

Since the statement is true for n = 1 and true for k implies true for k + 1, the statement is proved to be true by mathematical induction. A1

8 Putting $z = x + iy$, M1

$$\sqrt{(x-1)^2 + y^2} = \sqrt{2}\sqrt{x^2 + (y-1)^2}$$ A1

$$x^2 - 2x + 1 + y^2 = 2(x^2 + y^2 - 2y + 1)$$ A1

$$x^2 + y^2 + 2x - 4y + 1 = 0$$ A1

This is the equation of a circle. A1

[Award above A1if transformed to $(x+1)^2 + (y-2)^2 = 4$]

Centre = (-1,2), radius = 2 A1A1

9. *(a)* ln$y$ = $-\sqrt{x}\ln x$ M1

$$\frac{1}{y}\frac{dy}{dx} = -\frac{\sqrt{x}}{x} - \frac{\ln x}{2\sqrt{x}}$$ A1A1

$$f'(x) = -x^{-\sqrt{x}}.\frac{(2+\ln x)}{2\sqrt{x}}$$ A1

(b) At a stationary point,

ln$x$ = -2 M1

giving x = e^{-2} A1

Since $f'(x)$ is positive for $x < e^{-2}$ and negative for $x > e^{-2}$,

the point is a maximum. M1A1

Mathematics M1

1. *(a)* Using $v^2 = u^2 + 2as$ with $v = 20$, $u = 12$, $s = 1000$ — M1

$20^2 = 12^2 + 2 \times 1000a$ — A1

$a = \dfrac{20^2 - 12^2}{2 \times 1000} = 0.128 \text{ ms}^{-1}$ convincing — A1

(b) Using $s = 0.5(v + u)t$ with $s = 1000$, $u = 12$, $v = 20$ — M1

$1000 = 0.5(12 + 20)t$ — A1

$t = 62.5 \, s$ — A1

(c) Using $v = u + at$ with $u = 12$, $a = 0.128$, $t = 25$ — M1

$v = 12 + 0.128 \times 25$ — A1

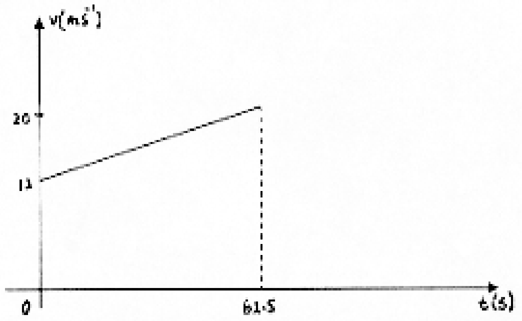
$= 15.2 \text{ ms}^{-1}$ — A1

(d) Using $s = ut + 0.5at^2$ with $u = 12$, $t = 30$, $a = 0.128$ — M1

$s = 12 \times 30 + 0.5 \times 0.128 \times 30^2$ — A1

$= 417.6 \text{ m}$ — A1

(e) — M1 A1



2. Using $v^2 = u^2 + 2as$ with $u = 0$, $a = (-)9.8$, $s = 3.6$ — M1

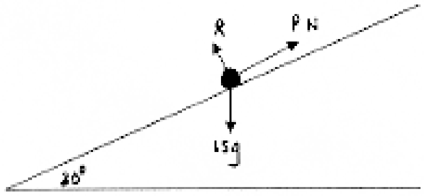
$v^2 = 2 \times 9.8 \times 3.6$ — A1

$v = 8.4 \text{ ms}^{-1}$ — A1

Therefore speed after rebound $= 0.3 \times 8.4$ — M1

$= 2.52 \text{ ms}^{-1}$ ft v — A1

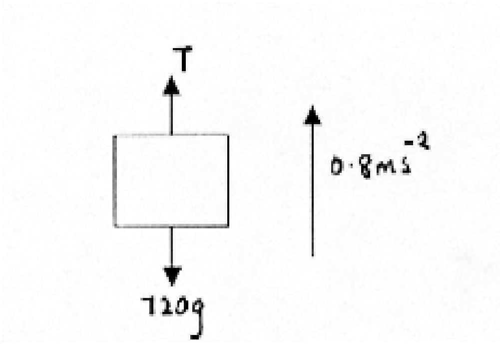
3.



Resolve perpendicular to plane

R = 15 g cos30°			B1
F = 0.2 × 15 g cos30°	ft R		B1
= 29.4 cos30°			
N2L parallel to plane		dim. correct	M1
15 g sin30° - F – P = 15a			A1
15a = 147 sin30° - 29.4 cos30° - 12			A1
a = 2.4 ms^{-2}		cao	A1

4. *(a)*

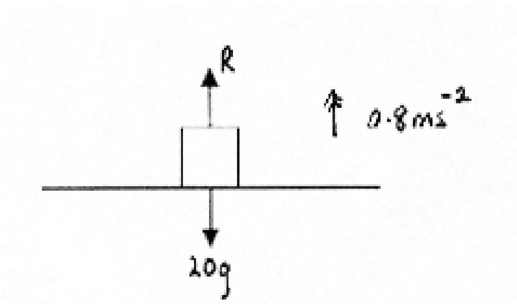


N2L $T - 720g = 720a$ M1 A1

$T = 720 \times 9.8 - 720 \times 0.8$

$= 7632$ N A1

(b)

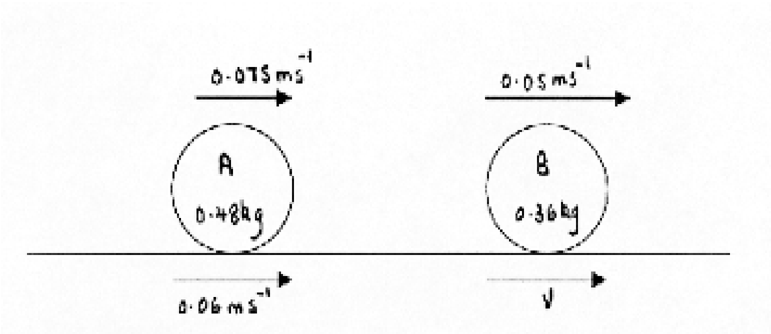


$R - 20g = 20 \times 0.8$ M1A1

$R = 20(9.8 + 0.8)$

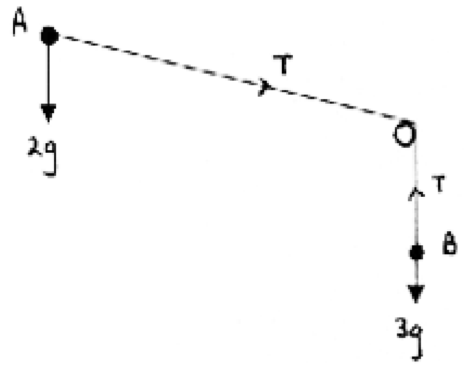
$= 212$ N A1

5.



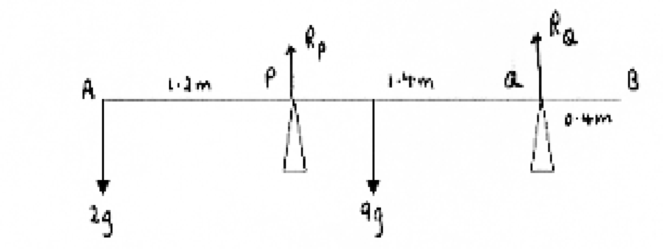
(a)	Conservation of momentum		M1
	$0.48 \times 0.075 + 0.36 \times 0.05 = 0.48 \times 0.06 + 0.36v$		A1
	$0.054 = 0.0288 + 0.36v$		
	$v = 0.07 \text{ ms}^{-1}$		A1
(b)	Restitution		M1
	$0.07 - 0.06 = -e(0.05 - 0.075)$	ft v	A1
	$0.01 = 0.025e$		
	$e = 0.4$		A1
(c)	Impulse on B = change in momentum for B		
	$I = 0.36(0.07 - 0.05)$		M1
	$= 0.0072$ (Ns)	ft v	A1

6.



N2l applied to B		dim. correct	M1
	$3g - T = 3a$		A1
N2L applied to A		dim. correct	M1
	$T + 2g\sin 30° = 2a$		A1
Solving	$3g + g = 5a$		m1
	$a = 0.8g$		
	$= 7.84 \text{ ms}^{-2}$	cao	A1
	$T = 3(g - a)$		
	$= 3(9.8 - 7.84)$		
	$= 5.88$ N	cao	A1

7.



(a) Moments about *P* — M1

$2g \times 1.2 = 9g \times 0.3 - R_Q \times 1.4$ — B1 A1

$1.4\, R_Q = 2.7g - 2.4g$

$= 0.3g$

$R_Q = \dfrac{0.3 \times 9.8}{1.4}$

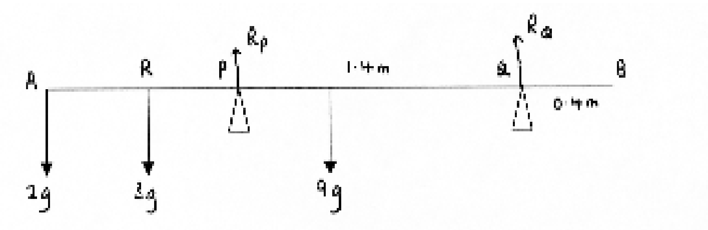
$= 2.1$ N — cao — A1

$R_P + R_Q = 11g$ — M1 A1

$R_P = 11g - 2.1$

$=$ 105 7 N — cao — A1

(b)



$R_Q = 0$ — B1

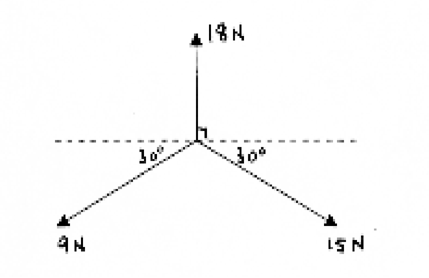
Moments about *P*

$3g(1.2 - x) = 9g \times 0.3 - 2g \times 1.2$ — B1

$1.2 - x = 0.9 - 0.8$

$x = 1.1$m — cao — B1

8.



Resolve in 18 N direction	M1
$Y = 18 - 15 \sin 30^{\circ\circ\circ} - 9 \sin 30$	A1
$= 18 - 7.5 - 4.5$	
$= 6$ N	
Resolve in direction perpendicular to 18 N	M1
$X = 15 \sin 60 - 9 \sin 60°$	A1
$= 6 \times \frac{\sqrt{3}}{2}$	
$= 3\sqrt{3}$ N	
Therefore resultant $= \sqrt{6^2 + (3\sqrt{3})^2}$	m1
$= \sqrt{63}$ ft X, Ys.i.A1	A1
Angle between 18 N force and resultant $= \theta = \tan^{-1}\left(\frac{3\sqrt{3}}{6}\right)$ oe	m1
$= 40.9°$ ft X,Y	A1

9.*(a)*

	Mass	from *DE*	from *DB*		
rectangle	8	2	1	si	B1
triangle	9	5	2	si	B1
lamina	17	*x*	*y*	si	B1

Moments about *AB* M1

$17\ x = 8 \times 2 + 9 \times 5$ A1

$x = \frac{61}{17} = 3.588$ cao A1

Moments about *BC* M1

$17\ y = 8 \times 1 + 9 \times 2$ A1

$y = \frac{26}{61} = 1.529$ cao A1

(c) $\theta = \tan^{-1}\left(\frac{26}{61}\right)$ M1

$= 23.08°$ ft *x*, *y* A1

Mathematics S1

1.	*(a)*	$P(A\cap B)$ = 0.3 + 0.1 – 0.35 = 0.05	M1A1
	(b)	$P(A)P(B)=0.03\neq P(A\cap B)$	M1A1
		A and *B* are not independent.	A1
	(c)	$P(A\cap B')=0.3-0.05$	B1
		$P(B')=1-0.1$	B1
		$P(A\mid B')=\dfrac{0.3-0.05}{1-0.1}$	M1
		$=\dfrac{5}{18}$	A1
2.	*(a)(i)*	P(3G) $=\dfrac{6}{10}\times\dfrac{5}{9}\times\dfrac{4}{8}$ or $\binom{6}{3}\div\binom{10}{3}=\dfrac{1}{6}$	M1A1
	(ii)	P(3B) $=\dfrac{4}{10}\times\dfrac{3}{9}\times\dfrac{2}{8}$ or $\binom{4}{3}\div\binom{10}{3}$	B1
		P(2B,1G) = $\dfrac{4}{10}\times\dfrac{3}{9}\times\dfrac{6}{8}\times3$ or $\binom{4}{2}\times\binom{6}{1}\div\binom{10}{3}$	B1
		Reqd prob = sum = $\dfrac{1}{3}$	M1A1
	(b)	P(Ann chosen) = $\dfrac{1}{10}+\dfrac{9}{10}\times\dfrac{1}{9}+\dfrac{9}{10}\times\dfrac{8}{9}\times\dfrac{1}{8}$ or $\binom{9}{2}\div\binom{10}{3}$ = $\dfrac{3}{10}$	M1A1
3.	*(a)(i)*	Prob = $e^{-0.95}=0.387$	M1A1
	(ii)	Prob = $e^{-0.95}\left(\dfrac{0.95^3}{6}+\dfrac{0.95^4}{24}\right)$	M1m1
		[M1 for individual probabilities, m1 for adding]	
		= 0.0684	A1
	(b)(i)	Reqd prob = $(e^{-0.95})^4$ = 0.0224	M1A1
	(ii)	Reqd prob = $e^{-0.95}\times e^{-0.95}\times(1-e^{-0.95})$	M1A1
		= 0.0917	A1
4.		E(*X*) = 3, Var(*X*) = 2.1	B1B1
	(a)	E(*Y*) = 3 × 3 + 4 =13	M1A1
	(b)	Var(*Y*) = 9 × 2.1 = 18.9	M1A1
	(c)	*Y* = 16 $\Rightarrow$ *X* = 4	M1
		Prob = $\binom{10}{4}\times0.3^4\times0.7^6$	m1
		= 0.200	A1
		[Special case : Award B1 for E(Y) = 3, Var(Y) = 2.1]	

5. *(a)* P(Def) = $0.4\times0.02+0.35\times0.025+0.25\times0.005$ M1A1
= 0.018 (9/500) A1

(b) P(A | Def) = $\dfrac{0.4\times0.02}{0.018}$ B1B1
= 0.444 (4/9) cao, but see note B1

[If 0.5% is interpreted as 0.05, award M1A0A1 for 0.02925. Then award B1B1B1 for an answer of 0.274 in (b)]

6. *(a)* $0 \le \theta \le 1/3$ with either or both ≤ replaced by <. B1B1

(b)(i) E(X) = $\theta+4\theta+3(1-3\theta)$ = 2.2 M1A1
$4\theta = 0.8$ A1
$\theta = 0.2$ AG

[Award M1A1A0 for a verification]

(ii) $E(X^2)=0.2+0.4\times4+0.4\times9=5.4$ M1A1
Var(X) = 5.4 – 4.84 (0.56) A1
SD = 0.748 A1

(iii) $E\left(\dfrac{1}{X}\right)=0.2\times1+0.4\times\dfrac{1}{2}+0.4\times\dfrac{1}{3}$ M1A1
= 0.533 (8/15) A1

7. *(a)* The number of female chicks, X, is B(20, 0.3). (si) B1

(i) P(X =8)= $\binom{20}{8}\times0.3^8\times0.7^{12}$ or 0.8867 – 0.7723 or 0.2277 – 0.1133 M1
= 0.1144 A1

(ii) P(X > 5) = 0.5836 M1A1

[Award M0 for a Poisson approximation in (a) but the B1 may be awarded]

(b) Number of eggs failing to hatch, Y, is B(1000, 0.01) ≈ Poi(10) B1
P(Y < 9) = 0.3328 M1A1

8.	*(a)*	$E(X) = \int_1^2 x(4-2x)\mathrm{d}x$	M1A1
		[Limits can be inserted later]	
		$= \left[2x^2 - \frac{2x^3}{3}\right]_1^2$	A1
		$= \frac{4}{3}$	A1
	(b)	$F(x) = \int_1^x (4-2y)\mathrm{d}y$	M1
		[Limits not required for M1]	
		$= \left[4y - y^2\right]_1^x$	A1
		$= \left(4x - x^2 - 4 + 1\right)$	A1
		$= 4x - x^2 - 3$	AG
	(c)	$F(1.2) = 0.36$	B1
		$P(X > 1.2) = 1 - F(1.2) = 0.64$	M1A1
	(d)	The median satisfies	
		$4m - m^2 - 3 = 0.5$	M1
		or $2m^2 - 8m + 7 = 0$	
		$m = \frac{8 \pm \sqrt{64-56}}{4}$	m1
		= 1.29	A1

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