GCE MARKING SCHEME

MATHEMATICS
AS/Advanced

JANUARY 2013

INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2013 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1 January 2013

Solutions and Mark Scheme

Final Version

1. (*a*) Gradient of $AB = \dfrac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = {}^4/_2$ (or equivalent) A1

A correct method for finding the equation of AB using the candidate's value for the gradient of AB. M1

Equation of AB : $y - 1 = 2(x - 4)$ (or equivalent) A1

(f.t. the candidate's value for the gradient of AB)

Equation of AB : $2x - y - 7 = 0$

(f.t. one error if both M1's are awarded) A1

(*b*) Gradient of $L = -{}^1/_2$ (or equivalent) B1

An attempt to use the fact that the product of perpendicular lines = –1

(or equivalent) M1

Gradient $AB \times$ Gradient $L = -1 \Rightarrow AB$, L perpendicular

(convincing) A1

(*c*) An attempt to solve equations of AB and L simultaneously M1

$x = 5$, $y = 3$ (convincing) A1

(*d*) A correct method for finding the length of $AB(AC)$ M1

$AB = \sqrt{20}$ A1

$AC = \sqrt{45}$ A1

$k = {}^2/_3$ (c.a.o.) A1

2. (*a*) $\dfrac{6\sqrt{7} - 11\sqrt{2}}{\sqrt{7} - \sqrt{2}} = \dfrac{(6\sqrt{7} - 11\sqrt{2})(\sqrt{7} + \sqrt{2})}{(\sqrt{7} - \sqrt{2})(\sqrt{7} + \sqrt{2})}$ M1

Numerator: $6 \times 7 + 6 \times \sqrt{7} \times \sqrt{2} - 11 \times \sqrt{7} \times \sqrt{2} - 11 \times 2$ A1

Denominator: $7 - 2$ A1

$\dfrac{6\sqrt{7} - 11\sqrt{2}}{\sqrt{7} - \sqrt{2}} = 4 - \sqrt{14}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{7} - \sqrt{2}$

(*b*) $\dfrac{3}{2\sqrt{6}} = p\sqrt{6}$, where p is a fraction equivalent to ${}^1/_4$ B1

$\left(\dfrac{\sqrt{6}}{2}\right)^3 = q\sqrt{6}$, where q is a fraction equivalent to ${}^3/_4$ B1

$\dfrac{3}{2\sqrt{6}} + \left(\dfrac{\sqrt{6}}{2}\right)^3 = \sqrt{6}$ (c.a.o.) B1

3. y-coordinate at $P = -2$ — B1

$\frac{dy}{dx} = 6x - 14$ (an attempt to differentiate, at least one non-zero term correct) — M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ — m1

Use of candidate's numerical value for $\frac{dy}{dx}$ as gradient of tangent at P — m1

Equation of tangent at P: $y - (-2) = 4(x - 3)$ (or equivalent)
(f.t. only candidate's derived value for y-coordinate at P) — A1

4. (*a*) (i) $a = 4$ — B1
$b = -11$ — B1

(ii) least value -33 (f.t. candidate's value for b) — B1
corresponding x-value $= -4$
(f.t. candidate's value for a) — B1

(*b*) $x^2 - x - 9 = 2x - 5$ — M1

An attempt to collect terms, form and use a correct method to solve their quadratic equation — m1

$x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0 \Rightarrow x = 4, x = -1$
(both values, c.a.o.) — A1

When $x = 4$, $y = 3$, when $x = -1$, $y = -7$
(both values, f.t. one slip) — A1

The line [$y = 2x - 5$] intersects the curve [$y = x^2 - x - 9$] at two points $(-1, -7)$ and $(4, 3)$ (f.t. candidate's x and y-values) — E1

5. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct — M1

$b^2 - 4ac = 6^2 - 4 \times 5 \times (-3k)$ — A1

$b^2 - 4ac > 0$ — m1

$k > -{}^{3}/_{5}$ (o.e.)
[f.t. only for $k < {}^{3}/_{5}$ from $b^2 - 4ac = 6^2 - 4 \times 5 \times (3k)$] — A1

(*b*) Finding critical values $x = 2{\cdot}5$, $x = 3$ — B1

$2{\cdot}5 \leq x \leq 3$ **or** $3 \geq x \geq 2{\cdot}5$ **or** $[2{\cdot}5, 3]$ **or** $2{\cdot}5 \leq x$ and $x \leq 3$ **or** a correctly worded statement to the effect that x lies between 2·5 and 3 (both values inclusive) (f.t. candidate's derived critical values) — B2

Note:
$2{\cdot}5 < x < 3$
$2{\cdot}5 \leq x, x \leq 3$
$2{\cdot}5 \leq x \quad x \leq 3$
$2{\cdot}5 \leq x$ or $x \leq 3$
all earn B1

6. (*a*) $y + \delta y = -(x + \delta x)^2 + 4(x + \delta x) - 6$ B1

Subtracting y from above to find δy M1

$\delta y = -2x\delta x - (\delta x)^2 + 4\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -2x + 4$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 5 \times \frac{4}{3} \times x^{1/3} - 9 \times -\frac{1}{2} \times x^{-3/2}$ B1, B1

7. Coefficient of $x = {}^6C_1 \times a^5 \times 4(x)$ B1

Coefficient of $x^2 = {}^6C_2 \times a^4 \times 4^2(x^2)$ B1

$15 \times a^4 \times m = k \times 6 \times a^5 \times 4$ (m = **16** or 4 or 8, k = **2** or $^1/_2$) M1

$a = 5$ (c.a.o.) A1

8. (*a*) Use of $f(-2) = 0$ M1

$-8p + 72 + 8 - 8 = 0 \Rightarrow p = 9$ (convincing) A1

Special case

Candidates who assume $p = 9$ and show $f(-2) = 0$ are awarded B1

(*b*) $f(x) = (x + 2)(9x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x + 2)(9x^2 + 0x - 4)$ A1

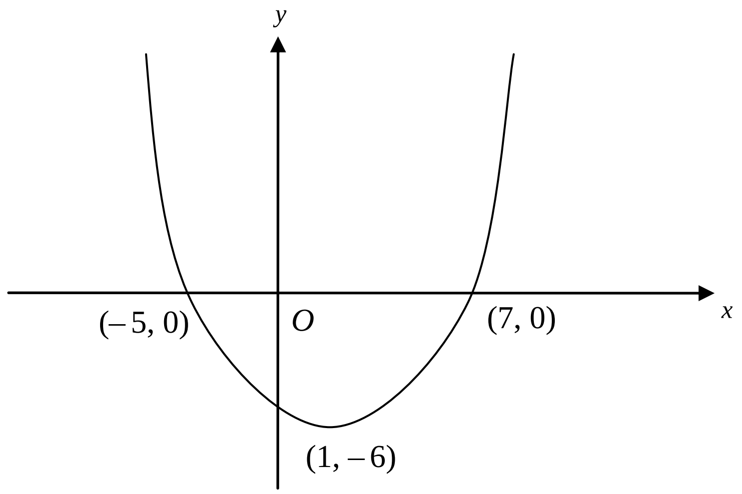
$f(x) = (x + 2)(3x - 2)(3x + 2)$ A1

Roots are $x = -2, 2/3, -2/3$ A1

Special case

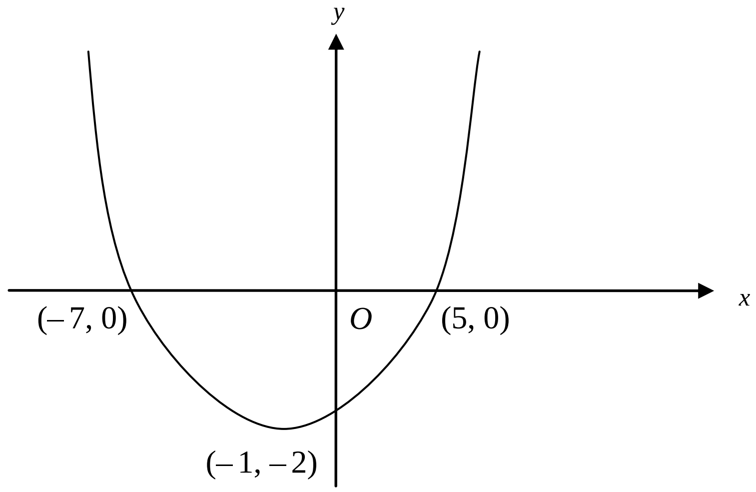
Candidates who find one of the remaining factors, $(3x - 2)$ or $(3x + 2)$, using e.g. factor theorem, are awarded B1

9. (*a*)



Concave up curve and *x*-coordinate of minimum = 1	B1
y-coordinate of minimum = – 6	B1
Both points of intersection with *x*-axis	B1

(*b*)



Concave up curve and *y*-coordinate of minimum = – 2	B1
x-coordinate of minimum = – 1	B1
Both points of intersection with *x*-axis	B1

Note: A candidate who draws a curve with no changes to the original graph is awarded 0 marks (both parts)

10. (*a*) $\frac{dy}{dx} = 3x^2 - 6x - 9$ B1

Putting candidate's derived $\frac{dy}{dx} = 0$ M1

$x = -1, 3$ (both correct) (f.t. candidate's $\frac{dy}{dx}$) A1

Stationary points are $(-1, 19)$ and $(3, -13)$ (both correct) (c.a.o) A1

A correct method for finding nature of stationary points yielding

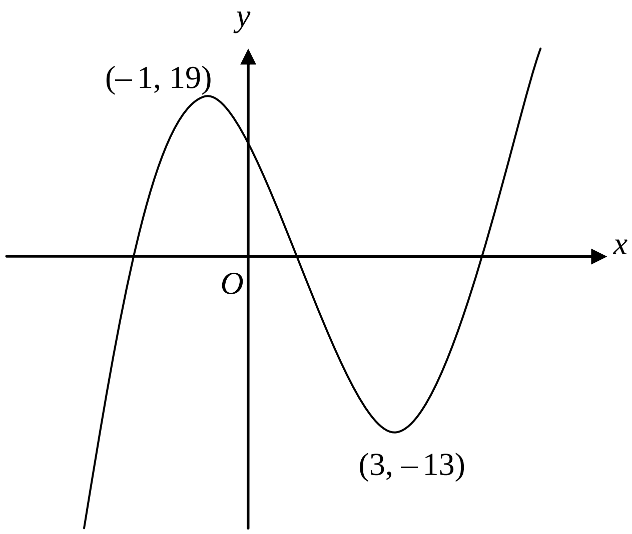
either $(-1, 19)$ is a maximum point

or $(3, -13)$ is a minimum point (f.t. candidate's derived values) M1

Correct conclusion for other point

(f.t. candidate's derived values) A1

(*b*)



Graph in shape of a positive cubic with two turning points M1

Correct marking of both stationary points

(f.t. candidate's derived maximum and minimum points) A1

(*c*) $k < -13$ B1

$19 < k$ B1

(f.t. candidate's *y*-values at stationary points)

Mathematics C2 January 2013

Solutions and Mark Scheme

Final Version

1.

0	3·16227766		
0·5	3·142451272		
1	3		
1·5	2·573907535		
2	1·414213562	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·5$ M1

$I \approx \frac{0·5}{2} \times \{3·16227766 + 1·414213562 + 2(3·142451272 + 3 + 2·573907535)\}$

$I \approx 22·00920884 \times 0·5 \div 2$

$I \approx 5·50230221$

$I \approx 5·5023$ (f.t. one slip) A1

Special case for candidates who put $h = 0·4$

0	3·16227766		
0·4	3·152142129		
0·8	3·080259729		
1·2	2·876108482		
1·6	2·429814808		
2	1·414213562	(all values correct)	B1

Correct formula with $h = 0·4$ M1

$I \approx \frac{0·4}{2} \times \{3·16227766 + 1·414213562 + 2(3·152142129 + 3·080259729 + 2·876108482 + 2·429814808)\}$

$I \approx 27·65314152 \times 0·4 \div 2$

$I \approx 5·530628304$

$I \approx 5·5306$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $7\sin^2\theta - \sin\theta = 3(1 - \sin^2\theta)$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$10\sin^2\theta - \sin\theta - 3 = 0 \Rightarrow (2\sin\theta + 1)(5\sin\theta - 3) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2},\ \sin\theta = \frac{3}{5}$ (c.a.o.) A1

$\theta = 210°, 330°$ B1, B1

$\theta = 36·87°, 143·13°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks
$\sin\theta$ = +, +, f.t. for 1 mark

(*b*) $3x - 20° = 52°, 232°, 412°$ (one value) B1

$x = 24°, 84°, 144°$ B1, B1, B1

Note: Subtract (from final three marks) 1 mark for each additional root in range, ignore roots outside range.

3. (*a*) $x^2 = 10^2 + (x + 4)^2 - 2 \times 10 \times (x + 4) \times \frac{3}{5}$ (correct use of cos rule) M1

$x^2 = 100 + x^2 + 8x + 16 - 12x - 48$ A1

$x = 17$ (f.t. one slip) A1

(*b*) $\sin\alpha = \frac{4}{5}$ B1

Area of triangle $ABC = \frac{1}{2} \times 10 \times 21 \times \frac{4}{5}$

(substituting the correct values in the correct places in the area formula, f.t. candidate's values for x and $\sin\alpha$) M1

Area of triangle $ABC = 84\,\text{cm}^2$ (f.t. candidate's value for x) A1

4. (*a*) (i) nth term $= 1 + 4(n-1) = 1 + 4n - 4 = 4n - 3$ (convincing) B1

(ii) $S_n = 1 + 5 + \ldots + (4n-7) + (4n-3)$

$S_n = (4n-3) + (4n-7) + \ldots + 5 + 1$

Reversing and adding M1

Either:

$2S_n = (4n-2) + (4n-2) + \ldots + (4n-2) + (4n-2)$

Or:

$2S_n = (4n-2) + \ldots$ (n times) A1

$2S_n = n(4n-2)$

$S_n = n(2n-1)$ (convincing) A1

(*b*) $\frac{10}{2} \times [2a + 9d] = 55$ B1

Either: $(a + 3d) + (a + 6d) + (a + 8d) = 27$

Or: $(a + 4d) + (a + 7d) + (a + 9d) = 27$ M1

$3a + 17d = 27$ (seen or implied by later work) A1

An attempt to solve candidate's derived linear equations simultaneously by eliminating one unknown M1

$a = -8$, $d = 3$ (both values) (c.a.o.) A1

5. (*a*) $r = 1{\cdot}5$ B1

A correct method for finding $(p + 4)$ th term M1

$(p + 4)$ th term $= 81$ (c.a.o.) A1

(*b*) Either: $\frac{a(1 - r^3)}{1 - r} = 22{\cdot}8$

Or: $a + ar + ar^2 = 22{\cdot}8$ B1

$\frac{a}{1 - r} = 18{\cdot}75$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$r^3 = -0{\cdot}216$ A1

$r = -0{\cdot}6$ (c.a.o.) A1

$a = 30$ (f.t. candidate's derived value for r) A1

6. (*a*) $5 \times \frac{x^{-3}}{-3} - 7 \times \frac{x^{5/3}}{5/3} + c$ B1, B1

(–1 if no constant term present)

(*b*) (i) $9 - a^2 = 0 \Rightarrow a = 3$ B1

(ii) $\frac{dy}{dx} = \pm 2x$ M1

Gradient of tangent = ± 6 (f.t. candidate's value for a) A1

$b = 18$ (convincing) A1

(iii) **Either:**

Area of triangle = 27 (f.t. candidate's value for a) B1

Use of integration to find the area under the curve M1

$\int (9 - x^2)\, dx = 9x - (1/3)x^3$ (correct integration) B1

Correct method of substitution of candidate's limits m1

$$\left[9x - (1/3)x^3\right]_0^3 = (27 - 9) - 0 = 18$$

Use of 0 and candidate's value for a as limits and trying to find total area by subtracting area under curve from area of triangle m1

Shaded area = 27 – 18 = 9 (c.a.o.) A1

Or:

Equation of tangent is $y = -6x + 18$

Use of integration to find an area M1

$\int (-6x + 18)\, dx = -3x^2 + 18x$ (correct integration)

(f.t. one slip in candidate's equation of tangent) B1

$\int (9 - x^2)\, dx = 9x - (1/3)x^3$ (correct integration) B1

Correct method of substitution of candidate's limits m1

$$\left[-3x^2 + 18x\right]_0^3 = (-27 + 54) - 0 = 27$$

(f.t. one slip in candidate's equation of tangent)

$$\left[9x - (1/3)x^3\right]_0^3 = (27 - 9) - 0 = 18$$

Use of 0 and candidate's value for a as limits and trying to find total area by subtracting area under curve from area under line m1

Shaded area = 27 – 18 = 9 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1

$\log_a x/y = p - q$ (the relationship between log and power)

$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) **Either:**

$(2x + 5)\log_{10} 6 = \log_{10} 7$

(taking logs on both sides and using the power law) M1

$x = \frac{\log_{10} 7 - 5\log_{10} 6}{2\log_{10} 6}$ (o.e.) A1

$x = -1\cdot957$ (f.t. one slip, see below) A1

Or:

$2x + 5 = \log_6 7$ (rewriting as a log equation) M1

$x = \frac{\log_6 7 - 5}{2}$ (o.e.) A1

$x = -1\cdot957$ (f.t. one slip, see below) A1

Note: an answer of $x = 1\cdot957$ from $x = \frac{5\log_{10} 6 - \log_{10} 7}{2\log_{10} 6}$

earns M1 A0 A1

an answer of $x = 3\cdot043$ from $x = \frac{\log_{10} 7 + 5\log_{10} 6}{2\log_{10} 6}$

earns M1 A0 A1

an answer of $x = -3\cdot914$ from $x = \frac{\log_{10} 7 - 5\log_{10} 6}{\log_{10} 6}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

8.	(*a*)	(i)	$A(-3, 5)$	B1
			A correct method for finding radius	M1
			Radius = $\sqrt{20}$	A1
		(ii)	**Either:**	
			A correct method for finding AP^2	M1
			$AP^2 = 18\ (< 20) \Rightarrow P$ is inside C	
			(f.t. candidate's coordinates for A)	A1
			Or:	
			An attempt to substitute $x = -6$, $y = 2$ in the equation of C	M1
			$(-6)^2 + 2^2 + 6 \times (-6) - 10 \times 2 + 14 = -2\ (< 0)$	
			$\Rightarrow P$ is inside C	A1
	(*b*)	(i)	An attempt to substitute $(2x + 1)$ for y in the equation of the circle	M1
			$5x^2 - 10x + 5 = 0$	A1
			Either: Use of $b^2 - 4ac$	m1
			Discriminant = 0 ($\Rightarrow y = 2x + 1$ is a tangent to the circle)	A1
			$x = 1, y = 3$	A1
			Or: An attempt to factorise candidate's quadratic	m1
			Repeated (single) root ($\Rightarrow y = 2x + 1$ is a tangent to the circle)	A1
			$x = 1, y = 3$	A1
		(ii)	**Either:**	
			$RQ = \sqrt{45}$ or $RA = \sqrt{65}$	
			(f.t. candidate's coordinates for A and Q)	B1
			Correct substitution of candidate's values in an expression for sin R, cos R or tan R	M1
			$ARQ = 33·69°$ (f.t. one numerical slip)	A1
			Or:	
			$RQ = \sqrt{45}$ or $RA = \sqrt{65}$	
			(f.t. candidate's coordinates for A and Q)	B1
			Correct substitution of candidate's values in the cos rule to find cos R	M1
			$ARQ = 33·69°$ (f.t. one numerical slip)	A1

9.	(*a*)	$\frac{1}{2} \times 11 \times 11 \times \theta = 43·56$	M1
		$\theta = 0·72$ radians	A1
	(*b*)	$BC = 11\phi$	B1
		$CD = 11(\pi - \phi)$	B1
		$11\phi = 11(\pi - \phi) \pm 13$	M1
		$\phi = 0·98$ radians (c.a.o.)	A1

Mathematics C3 January 2013

Solutions and Mark Scheme

Final Version

1.

1	0·211941557		
1·25	0·182137984		
1·5	0·154280773		
1·75	0·128955672		
2	0·106506978	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·25$ M1

$I \approx \frac{0·25}{3} \times \{0·211941557 + 0·106506978 + 4(0·182137984 + 0·128955672) + 2(0·154280773)\}$

$I \approx 1·871384705 \times 0·25 \div 3$

$I \approx 0·155948725$

$I \approx 0·156$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

2. (*a*) (i) e.g. $\theta = 20°$

$\cos^3\theta = 0·83$ (choice of θ and one correct evaluation) B1

$1 - \sin^3\theta = 0·96$ (both evaluations correct but different) B1

(ii) $\theta = 0°$ or $\theta = 90°$ B1

(*b*) $4(1 + \cot^2\theta) = 9 - 8\cot\theta$. (correct use of $\text{cosec}^2\theta = 1 + \cot^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = candidate's coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant m1

$4\cot^2\theta + 8\cot\theta - 5 = 0 \Rightarrow (2\cot\theta - 1)(2\cot\theta + 5) = 0$

$\Rightarrow \cot\theta = \frac{1}{2}, \cot\theta = -\frac{5}{2}$

$\Rightarrow \tan\theta = 2, \tan\theta = -\frac{2}{5}$ (c.a.o.) A1

$\theta = 63·43°, 243·43°$ B1

$\theta = 158·2°, 338·2°$ B1, B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\tan\theta$ = +, –, f.t. for 3 marks, $\tan\theta$ = –, –, f.t. for 2 marks

$\tan\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(2y^3) = 6y^2 \frac{dy}{dx}$ B1

$\frac{d}{dx}(5x^4y) = 5x^4\frac{dy}{dx} + 20x^3y$ B1

$\frac{d}{dx}(x^3) = 3x^2$, $\frac{d}{dx}(7) = 0$ B1

$\frac{dy}{dx} = \frac{20x^3y + 3x^2}{6y^2 - 5x^4}$ (o.e.) (c.a.o.) B1

(*b*) (i) candidate's *x*-derivative = $3t^2$ B1

candidate's *y*-derivative = $4t^3 + 35t^4$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{4t^3 + 35t^4}{3t^2}$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{4 + 70t}{3}$ (o.e.) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

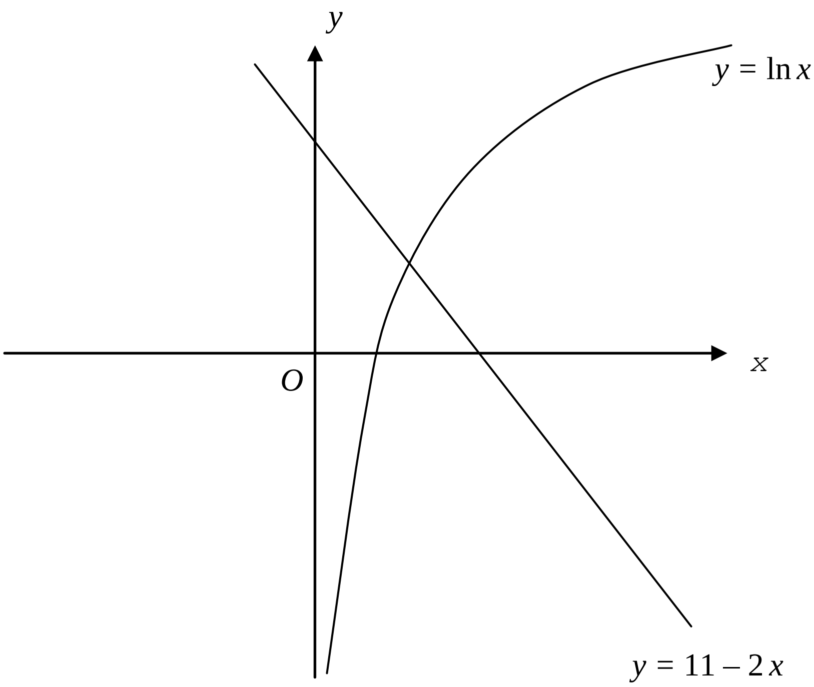
(f.t. candidate's expression for $\frac{dy}{dx}$) M1

$\frac{d^2y}{dx^2} = \frac{4 + 70t}{9t^2}$ (o.e.) A1

(iii) An attempt to solve $t^3 - 5 = 3$ and substitution of answer in candidate's expression for $\frac{d^2y}{dx^2}$ M1

$\frac{d^2y}{dx^2} = 4$ (c.a.o.) A1

4. (*a*)



Correct shape for $y = \ln x$, including the fact that the y-axis is an asymptote at $-\infty$ B1

A straight line with positive intercept and negative gradient intersecting once with $y = \ln x$ in the first quadrant. B1

Equation has one root (c.a.o.) B1

(*b*) $x_0 = 4{\cdot}7$

$x_1 = 4{\cdot}726218746$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 4{\cdot}723437268$

$x_3 = 4{\cdot}723731615$

$x_4 = 4{\cdot}723700458 = 4{\cdot}72370$ (x_4 correct to 5 decimal places) B1

Let $h(x) = \ln x + 2x - 11$

An attempt to check values or signs of $h(x)$ at $x = 4{\cdot}723695$, $x = 4{\cdot}723705$ M1

$h(4{\cdot}723695) = -1{\cdot}87 \times 10^{-5} < 0$, $h(4{\cdot}723705) = 3{\cdot}45 \times 10^{-6} > 0$ A1

Change of sign $\Rightarrow \alpha = 4{\cdot}72370$ correct to five decimal places A1

5. (*a*) (i) $\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = \frac{1}{2} \times (5x^2 - 3x)^{-1/2} \times (10x - 3)$ A1

(ii) $\frac{dy}{dx} = \frac{\pm 7}{\sqrt{(1 - (7x)^2)}}$ **or** $\frac{1}{\sqrt{(1 - (7x)^2)}}$ **or** $\frac{7}{\sqrt{(1 - 7x^2)}}$ M1

$\frac{dy}{dx} = \frac{7}{\sqrt{(1 - 49x^2)}}$ A1

(iii) $\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$ M1

$\frac{dy}{dx} = e^{3x} \times f(x) + \ln x \times g(x)$
(either $f(x) = 1/x$ or $g(x) = 3e^{3x}$) A1

$\frac{dy}{dx} = \frac{e^{3x}}{x} + 3e^{3x} \ln x$ (all correct) A1

(*b*) $\frac{d}{dx}(\cot x) = \frac{\sin x \times m \sin x - \cos x \times k \cos x}{\sin^2 x}$ ($m = 1$, **–1**, $k =$ **1**, –1) M1

$\frac{d}{dx}(\cot x) = \frac{\sin x \times (-\sin x) - \cos x \times (\cos x)}{\sin^2 x}$ A1

$\frac{d}{dx}(\cot x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$\frac{d}{dx}(\cot x) = \frac{-1}{\sin^2 x} = -\text{cosec}^2 x$ (convincing) A1

6. (*a*) (i) $\int \cos\left(\frac{4x+5}{3}\right) dx = k \times \sin\left(\frac{4x+5}{3}\right) + c$ ($k = 1, {}^4/_3, \mathbf{{}^3/_4}, -{}^3/_4, {}^1/_4$) M1

$\int \cos\left(\frac{4x+5}{3}\right) dx = \frac{3}{4} \times \sin\left(\frac{4x+5}{3}\right) + c$ A1

(ii) $\int e^{2x+9} dx = k \times e^{2x+9} + c$ ($k = 1, 2, \mathbf{{}^1/_2}$) M1

$\int e^{2x+9} dx = {}^1/_2 \times e^{2x+9} + c$ A1

(iii) $\int \frac{3}{(7-2x)^6} dx = \frac{3}{-5k} \times (7-2x)^{-5} + c$ ($k = 1, 2, \mathbf{-2}, -{}^1/_2$) M1

$\int \frac{3}{(7-2x)^6} dx = \frac{3}{-5 \times -2} \times (7-2x)^{-5} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \frac{1}{3x-4} dx = k \times \ln|3x-4|$ ($k = 1, 3, \mathbf{{}^1/_3}$) M1

$\int \frac{1}{3x-4} dx = \left[{}^1/_3 \times \ln|3x-4|\right]$ A1

A correct method for substitution of limits 2, 44, in an expression of the form $k \times \ln|3x-4|$ ($k = 1, 3, {}^1/_3$) m1

$\int_2^{44} \frac{1}{3x-4} dx = \ln 4$ (c.a.o.) A1

7. (*a*) Trying to solve either $3x - 4 > 5$ or $3x - 4 < -5$ M1

$3x - 4 > 5 \Rightarrow x > 3$

$3x - 4 < -5 \Rightarrow x < -{}^{1}/_{3}$ (both inequalities) A1

Required range: $x < -{}^{1}/_{3}$ or $x > 3$ (f.t. one slip) A1

Alternative mark scheme

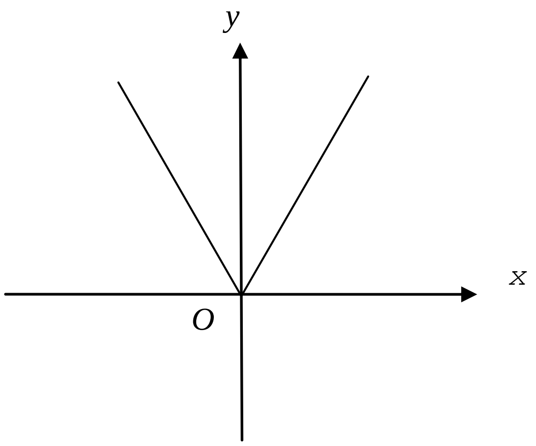
$(3x - 4)^2 > 25$

(squaring both sides, forming and trying to solve quadratic) M1

Critical values $x = -{}^{1}/_{3}$ and $x = 3$ A1

Required range: $x < -{}^{1}/_{3}$ or $x > 3$ (f.t. one slip in critical values) A1

(*b*) (i)



G1

(ii) $a = -2$ B1

$b = -4$ B1

8. (*a*) $y + 2 = \ln(4x + 5)$ B1

An attempt to express candidate's equation as an exponential equation M1

$x = \dfrac{(e^{y+2} - 5)}{4}$ (f.t. one slip) A1

$f^{-1}(x) = \dfrac{(e^{x+2} - 5)}{4}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [-2, \infty)$ B1

9.	(*a*)	(i)	$D(fg) = (0, \infty)$		B1
		(ii)	$R(fg) = [a, b)$ with		
			$a = -25$		B1
			$b = \infty$		B1
		(iii)	$fg(x) = (2x - 3)^2 - 25$		B1
		(iv)	Putting candidate's expression for $fg(x)$ equal to 0 and using a correct method to try and solve the resulting quadratic in x		M1
			$x = 4, x = -1,$	(c.a.o.)	A1
			$x = 4$	(c.a.o.)	A1
	(*b*)	(i)	$hh(x) = \dfrac{2 \times \dfrac{2x+7}{5x-2} + 7}{5 \times \dfrac{2x+7}{5x-2} - 2}$		M1
			$hh(x) = \dfrac{4x + 14 + 35x - 14}{10x + 35 - 10x + 4}$		
			$hh(x) = x$	(convincing)	A1
		(ii)	$h^{-1}(x) = h(x)$		B1

Mathematics FP1 January 2013

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1	$f(x+h)-f(x)=\frac{1}{2+(x+h)^2}-\frac{1}{2+x^2}$	M1A1	
	$=\frac{2+x^2-2-(x+h)^2}{(2+(x+h)^2)(2+x^2)}$	A1	
	$=\frac{-2xh-h^2}{(2+(x+h)^2)(2+x^2)}$	A1	
	$f'(x)=\lim_{h\to 0}\left(\frac{-2xh-h^2}{h(2+(x+h)^2)(2+x^2)}\right)$	M1	
	$=\frac{-2x}{(2+x^2)^2}$	A1	
2(a)	By row reduction,	M1	
	$\begin{bmatrix}1&2&3\\0&5&5\\0&5&5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\6\\k-4\end{bmatrix}$	A1 A1	
	It follows now that $k-4=6$ $k=10$	A1	
(b)	Put $z=\alpha$.	M1	
	Then $y=\frac{6}{5}-\alpha$	A1	FT their k if used
	and $x=\frac{8}{5}-\alpha$	A1	
3(a)	$\frac{4+6\text{i}}{1+\text{i}}=\frac{(4+6\text{i})(1-\text{i})}{(1+\text{i})(1-\text{i})}$	M1	
	$=\frac{10+2\text{i}}{2}\quad(5+\text{i})$	A1A1	
	$\text{i}(x+\text{i}y)+2(x-\text{i}y)=5+\text{i}$	M1	Award M1 for substituting for $z,\bar{z}$
	$2x-y=5$ $x-2y=1$	A1	
	$x=3,\ y=1$ $(z=3+\text{i})$	A1	
(b)	Mod(z) = $\sqrt{10}$ (3.16)	B1	
	Arg(z) = $\tan^{-1}(1/3)=0.322$ (18.4°)	B1	FT their z

Ques	Solution	Mark	Notes
4(a)	Det = $\lambda(15-7\lambda)+4\lambda-5-5=-7\lambda^2+19\lambda-10$ **A** is singular when det(**A**) = 0 $\lambda=\frac{-19\pm\sqrt{81}}{-14}=2,\frac{5}{7}$	**M1A1** **M1** **M1A1**	
(b)(i)	Cofactor matrix = $\begin{bmatrix}8 & -1 & -5\\ 2 & 1 & -3\\ -2 & 0 & 2\end{bmatrix}$	**M1** **A1**	Award M1 if at least 5 cofactors are correct.
	Adjugate matrix = $\begin{bmatrix}8 & 2 & -2\\ -1 & 1 & 0\\ -5 & -3 & 2\end{bmatrix}$	**A1**	No FT on cofactor matrix.
(ii)	Determinant = 2	**B1**	
	Inverse matrix = $\frac{1}{2}\begin{bmatrix}8 & 2 & -2\\ -1 & 1 & 0\\ -5 & -3 & 2\end{bmatrix}$	**A1**	FT the adjugate or determinant
5(a)	$\alpha+\beta+\gamma=-4,\beta\gamma+\gamma\alpha+\alpha\beta=3,\alpha\beta\gamma=-2$	**B1**	
	$\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}+\frac{1}{\alpha\beta}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ $=2$	**M1A1**	
(b)	$\frac{1}{\gamma\alpha}\times\frac{1}{\alpha\beta}+\frac{1}{\alpha\beta}\times\frac{1}{\beta\gamma}+\frac{1}{\beta\gamma}\times\frac{1}{\gamma\alpha}=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha^2\beta^2\gamma^2}$	**M1A1**	
	$=\frac{3}{4}$	**A1**	
	$\frac{1}{\beta\gamma}\times\frac{1}{\gamma\alpha}\times\frac{1}{\alpha\beta}=\frac{1}{\alpha^2\beta^2\gamma^2}=\frac{1}{4}$	**M1A1**	
	The required cubic equation is $x^3-2x^2+\frac{3}{4}x-\frac{1}{4}=0$ $\left(4x^3-8x^2+3x-1=0\right)$	**B1**	FT their previous values

Ques	Solution	Mark	Notes
6	Putting n = 1, the expression gives 1 which is correct so the result is true for n = 1 Assume that the formula is true for $n = k$. $(\sum_{r=1}^{k} r^3 = \frac{k^2(k+1)^2}{4}$). Consider (for $n = k + 1$) $\sum_{r=1}^{k+1} r^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$ $= \frac{(k+1)^2}{4}(k^2 + 4k + 4)$ $= \frac{(k+1)^2(k+2)^2}{4}$ Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**B1** **M1** **M1A1** **A1** **A1** **A1**	
7(a)	Taking logs, $\ln f(x) = (\ln x)^2$ Differentiating, $\frac{f'(x)}{f(x)} = \frac{2\ln x}{x}$ $f'(x) = 2x^{\ln x}\frac{\ln x}{x}$	**B1** **B1B1** **B1**	 B1 for LHS, B1 for RHS
(b)	At a stationary point, $f'(x) = 0$ $\ln x = 0$ $x = 1, y = 1$ EITHER Consider $f(0.9) = 1.011…$, $f(1.1) = 1.009..$ It is a minimum. OR Consider $f'(0.9) = –0.236..$, $f'(1.1) = 0.174..$ It is a minimum.	**M1** **A1** **A1** **M1** **A1** **M1** **A1**	 Accept correct analysis leading to $f''(1) = 2$

Ques	Solution	Mark	Notes
8(a)	Rotation matrix = $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$	B1	
	Reflection matrix = $\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$	B1	
	$\mathbf{T} = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$	B1	
	$= \frac{1}{\sqrt{2}}\begin{bmatrix}-1 & -1\\ -1 & 1\end{bmatrix}$		
(b)(i)	Consider		
	$\frac{1}{\sqrt{2}}\begin{bmatrix}-1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\lambda\\ m\lambda\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}-(m+1)\lambda\\ (m-1)\lambda\end{bmatrix}$	M1	
	The image line satisfies		
	$x = -(m+1)\lambda/\sqrt{2}; y = (m-1)\lambda/\sqrt{2}$	A1	The $\sqrt{2}$ need not be present
	Eliminating λ,		
	$y = \left(\frac{1-m}{1+m}\right)x$	A1	
(ii)	We are given that		
	$\frac{1-m}{1+m} = m \Rightarrow m^2 + 2m - 1 = 0$	M1	
	Solving,	m1	
	$m = -1 \pm \sqrt{2}$	A1	
9(a)	$u + \mathrm{i}v = (x + \mathrm{i}y)^2 + x + \mathrm{i}y$	M1	
	$= x^2 + 2\mathrm{i}xy + \mathrm{i}^2y^2 + x + \mathrm{i}y$	A1	
	whence $v = 2xy + y = (2x+1)y$		
	and $u = x^2 - y^2 + x$	A1	
(b)	Substituting $y = x + 1$,	M1	
	$u = x^2 - (x+1)^2 + x = -x - 1$	A1	FT their expression for u from (a)
	$v = (2x + 1)(x + 1) = 2x^2 + 3x + 1$	A1	Accept substitution of x in terms of y and subsequent elimination of y
	Attempting to eliminate x,	M1	
	$x = -u - 1$	A1	
	$v = 2(-u-1)^2 + 3(-u-1) + 1$	A1	No further FT for incorrect u
	$= 2u^2 + u$	A1	

7. (*a*) Trying to solve either $3x - 4 > 5$ or $3x - 4 < -5$ M1

$3x - 4 > 5 \Rightarrow x > 3$

$3x - 4 < -5 \Rightarrow x < -{}^1/_3$ (both inequalities) A1

Required range: $x < -{}^1/_3$ or $x > 3$ (f.t. one slip) A1

Alternative mark scheme

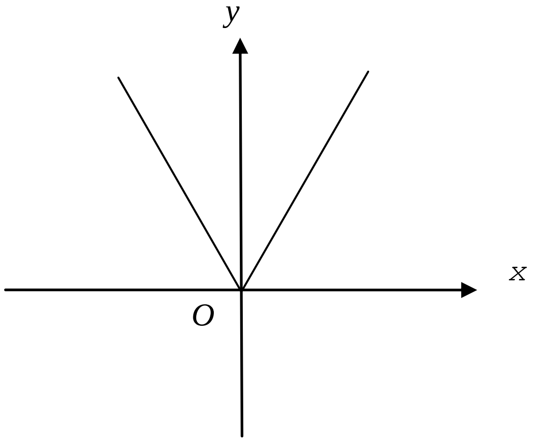
$(3x - 4)^2 > 25$

(squaring both sides, forming and trying to solve quadratic) M1

Critical values $x = -{}^1/_3$ and $x = 3$ A1

Required range: $x < -{}^1/_3$ or $x > 3$ (f.t. one slip in critical values) A1

(*b*) (i)



G1

(ii) $a = -2$ B1

$b = -4$ B1

8. (*a*) $y + 2 = \ln(4x + 5)$ B1

An attempt to express candidate's equation as an exponential equation M1

$x = \frac{(e^{y+2} - 5)}{4}$ (f.t. one slip) A1

$f^{-1}(x) = \frac{(e^{x+2} - 5)}{4}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [-2, \infty)$ B1

9. (*a*) (i) $D(fg) = (0, \infty)$ B1

(ii) $R(fg) = [a, b)$ with

$a = -25$ B1

$b = \infty$ B1

(iii) $fg(x) = (2x - 3)^2 - 25$ B1

(iv) Putting candidate's expression for $fg(x)$ equal to 0 and using a correct method to try and solve the resulting quadratic in x M1

$x = 4, x = -1,$ (c.a.o.) A1

$x = 4$ (c.a.o.) A1

(*b*) (i)

$$hh(x) = \frac{2 \times \frac{2x+7}{5x-2} + 7}{5 \times \frac{2x+7}{5x-2} - 2}$$ M1

$$hh(x) = \frac{4x + 14 + 35x - 14}{10x + 35 - 10x + 4}$$

$hh(x) = x$ (convincing) A1

(ii) $h^{-1}(x) = h(x)$ B1

Mathematics M1 January 2013

Solutions and Mark Scheme

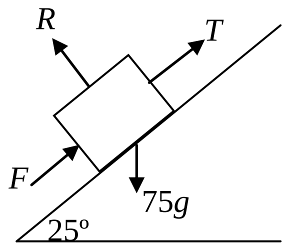
Final Version

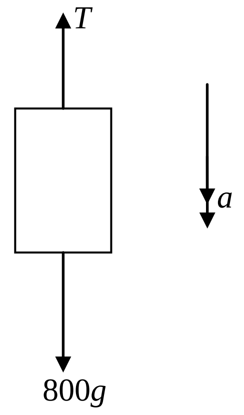
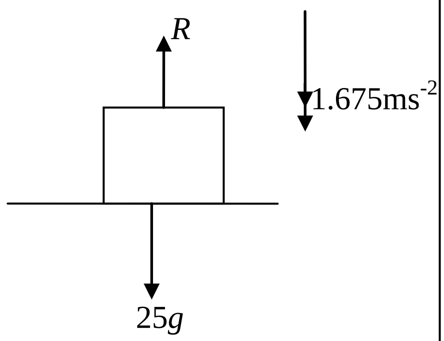
Q	Solution	Mark	Notes
1(a).	Using $v = u + at$ with $u=12$, $v=32$, $t=4$	M1	o.e.
	$32 = 12 + 4a$	A1	
	$a = \underline{5 \text{ ms}^{-2}}$	A1	cao
1(b)	Using $s = ut + 0.5at^2$, $u=12, t=4, a=5$	M1	
	$s = 12 \times 4 + 0.5 \times 5 \times 4^2$	A1	
	$s = \underline{88 \text{ m}}$	A1	cao
	OR		
	Using $v^2 = u^2 + 2as$, $u=12$, $v=32$, $a=5$	M1	
	$32^2 = 12^2 + 2 \times 5s$	A1	
	$s = \underline{88 \text{ m}}$	A1	cao
	OR		
	Using $s = 0.5(u + v)t$, $u=12$, $v=32$, $t=4$	M1	
	$s = 0.5(12 + 32) \times 4$	A1	
	$s = \underline{88\text{m}}$	A1	cao
1(c)	Using $v^2 = u^2 + 2as$, $u=12$, $a=5$, $s=44$	M1	ft answer in (b) for s
	$v^2 = 12^2 + 2 \times 5 \times 44$	A1	ft (b)
	$v = \underline{24.2 \text{ ms}^{-1}}$	A1	ft (b)

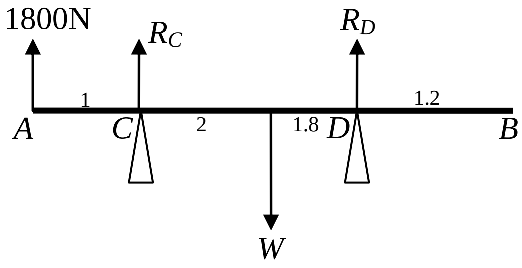
Q	Solution	Mark	Notes
2(a)(i)	e = 0	B1	
2(a)(ii)	Conservation of momentum equation	M1	
	3 x 4 + 7 x 0 = $3v_A + 7v_B$	A1	zero term not required
	12 = 10v		
	v = <u>1.2 ms^{-1}</u>	A1	$v = v_A = v_B$
2(b)(i)	v' = 0.25 x 5	M1	
	v' = <u>1.25</u>	A1	
2(b)(ii)	I = 6(5 + 1.25)	M1	allow –I
	I = <u>37.5</u>	A1	Ft answer in (b(i))
	Units for I is Ns	B1	allow dimensions kgms^{-1}

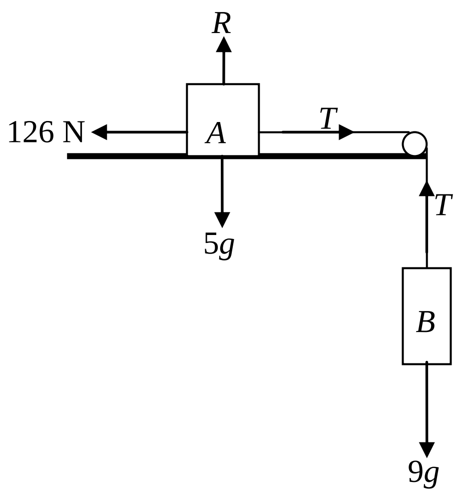
Q	Solution	Mark	Notes
3(a)	$s = ut + 0.5at^2$, s=(±)1.2, a=(±)9.8, u=15 $-1.2 = 15t + 0.5 \times (-9.8)t^2$ $4.9t^2 - 15t - 1.2 = 0$ Use of correct formula to solve quad eq t = 3.139 t = 3.1 s (to one d. p.)	M1 A1 m1 A1	complete method
3(b)	For the model used, the time taken for the particle to reach the ground is independent of the weight of the particle. I would expect the time to be the same as that in (a).	E1	no reason given gets E0

Q	Solution	Mark	Notes
4.	Resolve in direction of 12 N Psin45 + Qsin30 = 12	M1 A1	equation required
	Resolve in direction of 8N Pcos45 = Qcos30 + 8	M1 A1	equation required
	Attempt to eliminate one variable Q(sin30 + cos30) = 4 $Q = \frac{8}{1+\sqrt{3}} = 2.928$	m1	sensible method
	Q = <u>2.9 N</u>	A1	
	$\frac{1}{\sqrt{2}}P = 12 - 0.5 \times Q$		
	P = <u>14.9 N</u>	A1 PA-1	 if coefficients approximated

Q	Solution	Mark	Notes
5.	 		
5(a)	Resolve perp. to plane R = 75g cos α F = μR F = 0.3 x 75 x 9.8 cos25° F = 199.84 N	 M1 M1 A1	 used
	N2L parallel to plane	M1	dim correct, all forces eq Allow –F, 75a on RHS
	T + F – 75g sin 25° = 0 T = 75 x 9.8 x sin 25° - 199.84 T = 110.78 N	A1 A1	 cao
5(b)	N2L parallel to plane	M1	dim correct eq Comp wt and F opposing
	75g sin 25° - F = 75a 75a = 75 x 9.8 x sin25° - 199.84 a = 1.48 ms^{-2}	A1 A1	 Ft T in (a), allow consistent –ve ans

Q	Solution	Mark	Notes
6(a).			
	Apply N2L to lift	M1	dim correct, ±(T-800g)
	800g – T = 800a	A1	
	800a = 800 x 9.8 – 6500		
	a = $1.675\ ms^{-2}$	A1	allow 1.68
6(b)			
	Apply N2l to parcel	M1	dim correct ±(25g-R)
	25g – R = 25a	A1	ft (a)
	R = 25 x 9.8 – 25 x 1.675		
	R = 203.125 N	A1	ft (a)

Q	Solution	Mark	Notes
7.			
7(a)	When beam about to tilt about D, R_C=0 Moments about D 1800 x (6 – 1.2) + (R_C x 3.8) = W x 1.8 W = <u>4800 N</u>	B1 M1 B1 A1 A1	 equation required (or 2 equations) correct moment correct equation (or 2 correct equations) cao
7(b)	Moments about C R_D x 3.8 = 4800 x 2 R_D = <u>2526.32 N</u> Resolve vertically R_C + R_D = 4800 R_C = <u>2273.68 N</u>	M1 A1 A1 M1 A1	dim correct equation ft W ft W ft W

Q	Solution	Mark	Notes
8.			
	Apply N2L to particle A/B	M1	dim correct
	126 – T = 5a	A1	correct eq allow ±a
	Apply N2L to B/A	M1	dim correct
	T – 9g = 9a	A1	consistent with 1^{st} eq
	Eliminating T	m1	reasonable method
	a = <u>2.7 ms^{-2}</u>	A1	cao allow – if correct
	T = <u>112.5 N</u>	A1	cao

Q	Solution	Mark	Notes
9(a)	shape Area fr AD fr AB		
	ABCD 30 2.5 3	B1	one correct row/column
	XYZ 1.5 3.5 2	B1	c of m all correct
	Lamina 28.5 x y	B1	correct areas
	Moments about AD	M1	equation required
	28.5x + 1.5 x 3.5 = 30 x 2.5	A1	Ft table
	$x = \frac{93}{38} = \underline{2.447}$	A1	cao
	Moments about AB	M1	equation required
	28.5y + 1.5 x 2 = 30 x 3	A1	Ft table
	$y = \frac{58}{19} = \underline{3.053}$	A1	cao
9(b)	$\theta = \tan^{-1}\left(\frac{116}{93}\right) = \tan^{-1}\left(\frac{3.053}{2.447}\right)$	M1	correct triangle
		A1	ft (a) correct values
	$\theta = \underline{51.3°}$	A1	ft (a) PA-1 if 1 dp used
9(c)	$DP = \frac{93}{38} = \underline{2.447}$	B1	Ft x

Mathematics S1 January 2013

Solutions and Mark Scheme

Final Version

Ques	Solution	Mark	Notes
1(a)	Use of $P(A\cup B) + P(A\cap B) = P(A) + P(B)$	**M1**	
	Use of $P(A\cap B) = P(A)P(B)$	**m1**	
	$0.4 + 0.2P(B) = 0.2 + P(B)$	**A1**	
	$P(B) = 0.25$	**A1**	
(b)	EITHER		
	We require $P(A\cap B') + P(A'\cap B)$	**M1**	FT their P(B)
	$= 0.2 \times (1 - 0.25) + 0.25 \times (1 - 0.2)$	**A1**	
	$= 0.35$	**A1**	
	OR		
	We require $P(A\cup B) - P(A\cap B)$	**M1**	FT their P(B)
	$= 0.4 - 0.2\times0.25$	**A1**	
	$= 0.35$	**A1**	
2(a)	$E(X) = 3.2$, $Var(X) = 2.56$	**B1B1**	
	$E(Y) = 2 \times 3.2 + 5 = 11.4$ cao	**M1A1**	
	$Var(Y) = 4 \times 2.56 = 10.24$ cao	**M1A1**	
(b)	$Y = 11 \Rightarrow X = 3$	**B1**	
	$P(X = 3) = \binom{16}{3}\times0.2^3\times0.8^{13} = 0.246$	**M1A1**	FT their derived value of X M0 if no working
3(a)	$P(\text{2 red}) = \frac{6}{11}\times\frac{5}{10}\times\frac{5}{9}\times3$ or $\binom{6}{2}\binom{5}{1}\div\binom{11}{3}$	**M1A1**	
	$= \frac{5}{11}$ (0.455)	**A1**	
(b)	$P(\text{2 green}) = \frac{4}{11}\times\frac{3}{10}\times\frac{7}{9}\times3$ or $\binom{4}{2}\binom{7}{1}\div\binom{11}{3}$	**M1A1**	
	$= \frac{14}{55}$ (0.255)	**A1**	
	$P(\text{2 the same}) = \frac{5}{11}+\frac{14}{55}$	**M1**	
	$= \frac{39}{55}$ (0.709)	**A1**	FT on their probs

Ques	Solution	Mark	Notes
4(a)(i)	Poisson mean = 6	**B1**	
	P(4 arrivals) = $e^{-6}\times\frac{6^4}{4!}=0.134$ cao	**M1A1**	Accept 0.2851 – 0.1512 or 0.8488 – 0.7149 M0 if no working
(ii)	EITHER P(between 2 and 8) = 0.8472 – 0.0174 or 0.9826 – 0.1528	**B1B1**	
	= 0.8298 cao	**B1**	M0 if no working
	OR P(between 2 and 8) = $\sum_{x=2}^{8} e^{-6}\times\frac{6^x}{x!}$	**M1**	
	= 0.0446 + 0.0892 + 0.1339 + 0.1606 + 0.1606 + 0.1377 + 0.1033	**A1**	M0 if no working
	= 0.83 cao	**A1**	
(b)	E(X) = 12	**B1**	
	$E(X^2) = \text{E}(X) + [E(X)]^2 = 156$	**M1A1**	M1 requires Var(X) = E(X) FT their mean
5(a)(i)	Let X denote the number of seeds producing red flowers so that X is B(20,0.7) si	**B1**	
	P(X = 15) = $\binom{20}{15}\times 0.7^{15}\times 0.3^{5}$	**M1**	M0 if no working Accept 0.4164 – 0.2375 or 0.7625 – 0.5836
	= 0.179	**A1**	
(ii)	The number of seeds not producing red flowers, X', is B(20,0.3)	**M1**	
	We require P(X > 12) = P(X' < 8)	**m1**	
	= 0.7723	**A1**	
(b)	Number of seeds producing white flowers Y is B(150,0.09) ≈ Poi(13.5) si	**B1**	
	P(Y = 10) = $e^{-13.5}\times\frac{13.5^{10}}{10!}$	**M1**	Do not accept use of interpolation in tables
	= 0.076	**A1**	M0 if no working

Ques	Solution	Mark	Notes
6(a)	$k(2 + 3 + 4 + 5) = 1$ $14k = 1$ $k = 1/14$	**M1** **A1**	 Must be convincing
(b)	$\text{E}(X) = \frac{2}{14}\times 1+\frac{3}{14}\times 2+\frac{4}{14}\times 3+\frac{5}{14}\times 4$ $= \frac{20}{7}$ (2.86) $E(X^2) = \frac{2}{14}\times 1+\frac{3}{14}\times 4+\frac{4}{14}\times 9+\frac{5}{14}\times 16$ (65/7) $\text{Var}(X) = 65/7-(20/7)^2$ $= 1.12$ (55/49)	**M1** **A1** **B1** **M1** **A1**	 Accept 40k Accept in terms of k Numerical value required
(c)	The possibilities are $(x_1, x_2) = (1,2),(2,3),(3,4)$ si Prob = $\frac{2}{14}\times\frac{3}{14}+\frac{3}{14}\times\frac{4}{14}+\frac{4}{14}\times\frac{5}{14}$ $= 0.194$ (19/98)	 **B1** **M1A1** **A1**	 Numerical value required
7(a)	$\text{P}(+) = 0.02\times 0.96+0.98\times 0.01$ $= 0.029$	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram)
(b)(i)	$\text{P}(\text{Disease}\|+) = \frac{0.02\times 0.96}{0.029}$ $= 0.662$ (96/145) cao	**B1B1** **B1**	FT denominator from (a) B1 num, B1 denom
(ii)	EITHER P(+) = 0.662 × 0.96 + 0.338 × 0.01 = 0.639 OR $\text{P}(+) = \frac{0.02\times 0.96^2+0.98\times 0.01^2}{0.029}$ $= 0.639$	 **M1A1** **A1** **M1A1** **A1**	 M1 Use of Law of Total Prob (Accept tree diagram) FT from (b)(i) M1 valid attempt to use conditional probability

Ques	Solution	Mark	Notes
8(a)(i)	P(0.25 ≤ X ≤ 0.75) = F(0.75) – F(0.25)	**M1**	
	= 0.6875 (11/16)	**A1**	
(ii)	The median satisfies		
	$2m^2 - m^4 = 0.5$	**B1**	
	$2m^4 - 4m^2 + 1 = 0$		
(iii)	(Root) $= \dfrac{4 \pm \sqrt{16-8}}{4}$ (= 0.29289..)	**M1A1**	Condone the omission of the redundant root
	$m = \sqrt{0.29289..} = 0.541$	**M1A1**	
(b)(i)	$f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(2x^2 - x^4)$	**M1**	
	$= 4x - 4x^3$	**A1**	
(ii)	$E\left(\sqrt{X}\right) = \int_0^1 \sqrt{x}(4x - 4x^3)\mathrm{d}x$	**M1A1**	M1 for the integral of $\sqrt{x}\, f(x)$ A1 for completely correct although limits may be left until 2nd line.
	$= \left[4x^{5/2} \times \dfrac{2}{5} - 4x^{9/2} \times \dfrac{2}{9}\right]_0^1$	**A1**	FT their $f(x)$ from (b)(i) if M1 awarded there.
	$= \dfrac{32}{45}$ (0.711)	**A1**	

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