GCE AS/A LEVEL

WJEC GCE AS/A Level in

FURTHER MATHEMATICS

APPROVED BY QUALIFICATIONS WALES

SAMPLE ASSESSMENT MATERIALS

Teaching from 2017

For teaching from 2017
For AS award from 2018
For A level award from 2019

GCE AS AND A LEVEL FURTHER MATHEMATICS

SAMPLE ASSESSMENT MATERIALS

Contents

GCE

FURTHER MATHEMATICS

UNIT 1: FURTHER PURE MATHEMATICS A

SAMPLE ASSESSMENT MATERIALS

(1 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use mathematical induction to prove that $4^n + 2$ is divisible by 6 for all positive integers n. [7]

2. Solve the equation $2z + \mathrm{i}\bar{z} = \dfrac{-1+7\mathrm{i}}{2+\mathrm{i}}$.

   (a) Give your answer in Cartesian form [7]

   (b) Give your answer in modulus-argument form. [4]

3. Find an expression, in terms of n, for the sum of the first n terms of the series

$$1.2.4 + 2.3.5 + 3.4.6 + \ldots + n(n+1)(n+3) + \ldots$$

   Express your answer as a product of linear factors. [6]

4. The roots of the equation

$$x^3 - 4x^2 + 14x - 20 = 0$$

   are denoted by α, β, γ.

   (a) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = -12.$$

   Explain why this result shows that exactly one of the roots of the above cubic equation is real. [3]

   (b) Given that one of the roots is $1+3\mathrm{i}$, find the other two roots.
   Explain your method for each root. [4]

5. The complex number z is represented by the point $P(x, y)$ in an Argand diagram and

$$|z-3| = 2|z+\mathrm{i}|.$$

   Show that the locus of P is a circle and determine its radius and the coordinates of its centre.

   [9]

6. The transformation T in the plane consists of a reflection in the line $y = x$, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y - 2)$,followed by an anticlockwise rotation through 90° about the origin.

   (a) Find the 3×3 matrix representing T. [6]

   (b) Show that T has no fixed points. [3]

7. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u,v)$ in Argand diagrams and

$$w = z(1+z)$$

(a) Show that

$$v = y(1+2x)$$

and find an expression for u in terms of x and y. [4]

(b) The point P moves along the line $y = x+1$. Find the Cartesian equation of the locus of Q, giving your answer in the form $v = au^2 + bu$, where a and b are constants whose values are to be determined. [5]

8. The line L passes through the points A(1, 2, 3) and B(2, 3, 1).

(a) (i) Find the vector **AB**.

(ii) Write down the vector equation of the line L. [3]

(b) The plane Π has equation $x + 3y - 2z = 5$.

(i) Find the coordinates of the point of intersection of L and Π.

(ii) Find the acute angle between L and Π. [9]

AS Further Mathematics Unit 1: Further Pure Mathematics A
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ($\surd$ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

AS Further Mathematics Unit 1: Further Pure Mathematics A

Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1.	When $n = 1$, $4^n + 2 = 6$ which is divisible by 6 so the proposition is true for $n = 1$.	B1	AO2	
	Assume the proposition to be true for $n = k$ so that $4^k + 2$ is divisible by 6 and equals $6N$	M1	AO2	
	Consider (for $n = k + 1$)	M1	AO2	
	$4^{k+1} + 2 = 4 \times 4^k + 2$	A1	AO2	
	$= 4 \times (6N - 2) + 2$	A1	AO2	
	$= 24N - 6$	A1	AO2	
	This is divisible by 6 so true for $k \Rightarrow$ true for $k + 1$. Since true for $n = 1$, the result is proved by induction.	A1	AO2	
		[7]		
2.(a)	$\frac{-1+7\mathrm{i}}{2+\mathrm{i}} = \frac{(-1+7\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})}$	M1	AO3	
	$= \frac{5+15\mathrm{i}}{5}$	A1	AO3	
	$= 1 + 3\mathrm{i}$	A1	AO3	
	$2(x + \mathrm{i}y) + \mathrm{i}(x - \mathrm{i}y) = 1 + 3\mathrm{i}$	M1	AO3	FT from line above
	$2x + y = 1$	A1	AO3	
	$x + 2y = 3$	A1	AO3	
	$(x = -\frac{1}{3}; y = \frac{5}{3})$			
	$z = \frac{-1+5\mathrm{i}}{3}$	A1	AO1	
(b)	$\lvert z\rvert = \frac{\sqrt{26}}{3} = 1.70 \ (1.699673171)$	B1	AO1	
	$\tan^{-1}(-5) = -1.3734...$	B1	AO1	
	$\arg(z) = -1.3734 + \pi = 1.77 \ (1.768191887)$	B1	AO1	
	$z = 1.70(\cos 1.77 + \mathrm{i}\sin 1.77)$	B1	AO1	
		[11]		

Qu. No.	Solution	Mark	AO	Notes
3.	$S_n = \sum_{r=1}^{n} r(r+1)(r+3)$	M1	AO1	
	$= \sum_{r=1}^{n} (r^3 + 4r^2 + 3r)$	A1	AO1	
	$= \frac{n^2(n+1)^2}{4} + 4\frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2}$	A1	AO1	
	$= \frac{n(n+1)}{12}\left(3n(n+1) + 8(2n+1) + 18\right)$	A1	AO1	
	$= \frac{n(n+1)}{12}\left(3n^2 + 19n + 26\right)$	A1	AO1	
	$= \frac{n(n+1)(n+2)(3n+13)}{12}$	A1 **[6]**	AO1	
4.(a)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\beta\gamma + \gamma\alpha + \alpha\beta)$	M1	AO2	
	$= 4^2 - 2\times 14$ $= -12$	A1	AO2	
	A cubic equation either has 3 real roots or 1 real root. Since the sum of squares is negative, the 3 roots cannot all be real so there is just 1 real root	B1	AO2	
(b)	A second root is $1 - 3i$,	B1	AO1	
	since complex roots occur in conjugate pairs.	E1	AO2	
	The third root is 2	B1	AO1	
	since the sum of the 2 complex roots is 2 and the sum of the 3 roots is 4,	E1 **[7]**	AO2	

Qu. No.	Solution	Mark	AO	Notes
5.	Putting $z = x + iy$,	M1	AO3	
	$\|x - 3 + iy\| = 2\|x + i(y + 1)\|$	A1	AO3	
	$(x-3)^2 + y^2 = 4\left(x^2 + (y+1)^2\right)$	m1	AO3	
	$x^2 - 6x + 9 + y^2 = 4x^2 + 4y^2 + 8y + 4$			
	$3x^2 + 3y^2 + 6x + 8y - 5 = 0$	A1	AO3	
	This is the equation of a circle			
	$x^2 + y^2 + 2x + \frac{8}{3}y = \frac{5}{3}$	M1	AO1	
	$(x+1)^2 + \left(y + \frac{4}{3}\right)^2 = \frac{5}{3} + 1 + \frac{16}{9}$	m1	AO1	
	Centre = $\left(-1, -\frac{4}{3}\right)$	A1	AO1	
	Radius = $\sqrt{\frac{5}{3} + 1 + \frac{16}{9}}$	M1	AO1	
	= 2.11 $\left(\frac{2\sqrt{10}}{3}\right)$	A1	AO1	
		[9]		

Qu. No.	Solution	Mark	AO	Notes
6.(a)	Reflection matrix = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	AO1	
	Translation matrix = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$	B1	AO1	
	Rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	AO1	
	$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1	AO1	
	$= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	A1	AO1	
	$= \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	A1	AO1	
(b)	Fixed points satisfy $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	M1	AO1	
	$-x+2=x; y+1=y$	A1	AO1	
	The second equation is inconsistent so there are no fixed points.	A1	AO2	
		[9]		

Qu. No.	Solution	Mark	AO	Notes
7.(a)	Putting $z = x + \mathrm{i}y$ and $w = u + \mathrm{i}v$, $u + \mathrm{i}v = (x + \mathrm{i}y)(x + 1 + \mathrm{i}y)$ v = imaginary part = $y(x + 1) + xy$ $= y(1 + 2x)$ u = real part = $x(x + 1) - y^2$	M1 A1 A1 A1	AO1 AO1 AO1 AO1	
(b)	$u = x(x+1) - (x+1)^2$ $= -x - 1$ $v = (x+1)(2x+1)$ $= -u(-2u - 1)$ $= 2u^2 + u$	M1 A1 M1 A1 A1 **[9]**	AO3 AO3 AO3 AO3 AO3	
8.(a)(i)	$\mathbf{AB} = (2\mathbf{i}+3\mathbf{j}+\mathbf{k}) - (\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ $= \mathbf{i}+\mathbf{j}-2\mathbf{k}$	M1 A1	AO1 AO1	
(ii)	Equation of line is $\mathrm{r} = \mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-2\mathbf{k})$	B1	AO1	
(b)(i)	The line cuts the plane where $1 + \lambda + 3(2 + \lambda) - 2(3 - 2\lambda) = 5$ $\lambda = \frac{1}{2}$ Point of intersection = $\left(\frac{3}{2}, \frac{5}{2}, 2\right)$	M1 A1 A1	AO1 AO1 AO1	
(ii)	Direction of normal = $(1,3,-2)$ If θ denotes the angle between L and the normal, $\cos\theta = \frac{(1,3,-2).(1,1,-2)}{\lvert(1,3,-2)\rvert\,\lvert(1,1,-2)\rvert}$ $= \frac{8}{\sqrt{14}\sqrt{6}}$ $\theta = 29.2(0593\ldots)^{\circ}$ Angle between L and plane = 60.8°. **[12]**	B1 M1 A1 A1 A1 A1	AO1 AO1 AO1 AO1 AO1 AO1	

GCE

FURTHER MATHEMATICS

UNIT 2: FURTHER STATISTICS A

SAMPLE ASSESSMENT MATERIALS

(1 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variable X has mean14 and standard deviation 5. The independent random variable Y has mean 12 and standard deviation 3. The random variable W is given by $W = XY$. Find the value of

   (a) E(W), [1]

   (b) Var(W). [6]

2. The queueing times, T minutes, of customers at a local Post Office are modelled by the probability density function

$$f(t) = \frac{1}{2500}t(100 - t^2) \qquad \text{for } 0 \le t \le 10,$$
$$f(t) = 0 \qquad \text{otherwise.}$$

   (a) Determine the mean queueing time. [3]

   (b) (i) Find the cumulative distribution function, $F(t)$, of T.

   (ii) Find the probability that a randomly chosen customer queues for more than 5 minutes.

   (iii) Find the median queueing time. [10]

3. A class of 8 students sit examinations in History and Geography. The marks obtained by these students are given below.

Student	A	B	C	D	E	F	G	H
History mark	73	59	83	49	57	82	67	60
Geography mark	55	51	58	59	44	66	49	67

   (a) Calculate Spearman's rank correlation coefficient for this data set. [6]

   (b) Hence determine whether or not, at the 5% significance level, there is evidence of a positive association between marks in History and marks in Geography. [2]

   (c) Explain why it might not have been appropriate to use Pearson's product moment correlation coefficient to test association using this data set. [1]

4. A year 12 student wishes to study at a Welsh university. For a randomly chosen year between 2000 and 2017 she collected data for seven universities in Wales from the Complete University Guide website. The data are for the variables:

- 'Entry standards' – the average UCAS tariff score of new undergraduate students;
- 'Student satisfaction' – a measure of student views of the teaching quality at the university taken from the National Student Survey (maximum 5);
- 'Graduate prospects' – a measure of the employability of a university's first degree graduates (maximum 100);
- 'Research quality' – a measure of the quality of the research undertaken in the university (maximum 4).

(a) Pearson's product-moment correlation coefficients, for each pairing of the four variables, are shown in the table below.
Discuss the correlation between graduate prospects and the other three variables. [2]

Variable	Entry standards	Student satisfaction	Graduate prospects	Research quality
Entry standards	1			
Student satisfaction	-0.030	1		
Graduate prospects	0.772	0.236	1	
Research quality	0.866	0.066	0.827	1

(b) Calculate the equation of the least squares regression line to predict 'Entry standards'(y) from 'Research quality'(x), given the summary statistics:

$$\sum x = 22.24,\ \sum y = 2522,\ S_{xx} = 1.0542,\ S_{yy} = 20193.5, S_{xy} = 122.72.$$

[5]

(c) The data for one of the Welsh universities are missing. This university has a research quality of 3.00. Use your equation to predict the entry standard for this university. [2]

5. The manager of a hockey team studies last season's results and puts forward the theory that the number of goals scored per match by her team can be modelled by a Poisson distribution with mean 2.0. The number of goals scored during the season are summarised below.

Goals scored	0	1	2	3	4 or more
Frequency	6	11	15	10	8

(a) State suitable hypotheses to carry out a goodness of fit test. [1]

(b) Carry out a χ^2 goodness of fit test on this data set, using a 5% level of significance and draw a conclusion in context. [9]

6. Customers arrive at a shop such that the number of arrivals in a time interval of t minutes follows a Poisson distribution with mean $0.5t$.

(a) Find the probability that exactly 5 customers arrive between 11 a.m. and 11.15 a.m. [3]

(b) A customer arrives at exactly 11 a.m.

(i) Let the next customer arrive at T minutes past 11 a.m. Show that

$$P(T > t) = \mathrm{e}^{-0.5t}.$$

(ii) Hence find the probability density function, $f(t)$, of T.

(iii) Hence, giving a reason, write down the mean and the standard deviation of the time between the arrivals of successive customers. [7]

7. The Pew Research Center's Internet Project offers scholars access to raw data sets from their research.

One of the Pew Research Center's projects was on teenagers and technology. A random sample of American families was selected to complete a questionnaire. For each of their children, between and including the ages of 13 and 15, parents of these families were asked:

Do you know your child's password for any of [his/her] social media accounts?

Responses to this question were received from 493 families. The table below provides a summary of their responses.

	Age (years)			**Total**
Parent know password	**13**	**14**	**15**	
Yes	76	75	67	218
No	66	103	106	275
Total	142	178	173	493

(a) A test for significance is to be undertaken to see whether there is an association between whether a parent knows any of their child's social media passwords and the age of the child.

(i) Clearly state the null and alternative hypotheses.

(ii) Obtain the expected value that is missing from the table below, indicating clearly how it is calculated from the data values given in the table above.

Expected values:

	Age (years)		
Parent knows password	**13**	**14**	**15**
Yes	62.79	78.71	76.50
No		99.29	96.50

(iii) Obtain the two chi-squared contributions that are missing from the table below.

Chi-squared contributions:

	Age (years)		
Parent knows password	**13**	**14**	**15**
Yes		0.175	1.180
No	2.203		0.935

The following output was obtained from the statistical package that was used to undertake the analysis:

Pearson chi-squared (2) = 7.409 *p*-value = 0.0305

(iv) Indicate how the degrees of freedom have been calculated for the chi-squared statistic.

(v) Interpret the output obtained from the statistical test in terms of the initial hypotheses.

[10]

(b) Comment on the nature of the association observed, based on the contributions to the test statistic calculated in (a). [2]

AS Further Mathematics Unit 2: Further Statistics A
General instructions for marking GCE Mathematics

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3. Premature Approximation
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- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
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- 'C' marks are awarded for drawing curves

AS Further Mathematics Unit 2: Further Statistics A

Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1.(a)	$E(W) = E(X)E(Y) = 168$	B1	AO1	
(b)	$E(X^2) = (E(X))^2 + \text{Var}(X)$	M1	AO1	
	$= 221$	A1	AO1	
	$E(Y^2) = 153$	A1	AO1	
	$\text{Var}(W) = E(W^2) - [E(W)]^2$			
	$= E(X^2)E(Y^2) - (E(X)E(Y))^2$	M1	AO3	
	$= 221\times153 - 168^2 (= 5589)$	A1	AO1	
	SD = 74.8 (74.75961….)	A1	AO1	
		[7]		
2(a)	$E(T) = \frac{1}{2500}\int_0^{10} t^2(100 - t^2)\,\mathrm{d}t$	M1	AO3	
	$= \frac{1}{2500}\left[\frac{100t^3}{3} - \frac{t^5}{5}\right]_0^{10}$	A1	AO1	
	$= 5.33(333\ldots)$	A1	AO1	
(b)(i)	$F(t) = \frac{1}{2500}\int_0^{t} u(100 - u^2)\,\mathrm{d}u$	M1	AO3	
	$= \frac{1}{2500}\left[50u^2 - \frac{u^4}{4}\right]_0^{t}$	A1	AO1	
	$= \frac{1}{2500}\left(50t^2 - \frac{t^4}{4}\right)$ (for $0 \le t \le 10$)	A1	AO1	
	$= 1$ for $t > 10$	B1	AO1	Allow omission of $t < 0$
	($F(t) = 0$ for $t < 0$)			
(ii)	$P(T > 5) = 1 - F(5)$	M1	AO3	
	$= 0.563$ (0.5625)	A1	AO1	
(iii)	The median m satisfies $F(m) = 0.5$	M1	AO3	
	$m^4 - 200m^2 + 5000 = 0$	A1	AO3	
	$m^2 = \frac{200 \pm \sqrt{40000 - 20000}}{2}$	A1	AO1	
	(= 29.289...)			
	$m = 5.41(1961\ldots)$	A1	AO1	
		[13]		

Qu. No.	Solution	Mark	AO	Notes
3(a)	The ranks are S A B C D E F G H H 3 6 1 8 7 2 4 5 G 5 6 4 3 8 2 7 1	M1 A1 A1	AO3 AO1 AO1	Attempt to find ranks Correct values for 1st row Correct values for 2nd row
	$\sum d^2 = 64$	B1	AO1	
	$r_s = 1 - \frac{6 \times 64}{8 \times 63}$ = 0.238(095238…)	M1 A1	AO1 AO1	
(b)	5% 1-tail crit value = 0.6429 This suggests that there is no positive association between marks in History and marks in Geography.	B1 B1	AO1 AO3	
(c)	Because the data might not follow a bivariate normal distribution.	B1 **[9]**	AO2	
4(a)	The evidence suggests that good graduate prospects are associated with: strong research quality high entry standards.	B1 B1	AO2 AO2	Or The evidence suggests that good graduate prospects are not associated with student satisfaction
(b)	Gradient $= \frac{122.72}{1.0542}$ = 116.4(105)	M1 A1	AO2 AO1	
	Intercept $= \frac{2522}{7} - 116.4105 \times \frac{22.24}{7}$ = -9.5(67)	M1 A1	AO2 AO1	Allow for using 116.4 giving -9.5(337)
	y = 116.4(105) x – 9.5(67)	B1	AO1	FT 'their' gradient and intercept
(c)	116.4 × 3 – 9.6 = 339.6	M1 A1 **[9]**	AO3 AO1	Accept 358.7(988...) if using exact values throughout

Qu. No.	Solution	Mark	AO	Notes
5(a)	H_0 : The data can be modelled by the Poisson distribution with mean 2. H_1 : The data cannot be modelled by the Poisson distribution with mean 2.	B1	AO3	
(b)	The expected frequencies are Goals scored: 0 | 1 | 2 | 3 | 4 or more Obs: 6 | 11 | 15 | 10 | 8 Exp: 6.767 | 13.534 | 13.534 | 9.022 | 7.144	B1 B1	AO3 AO3	For at least 1 correct For all correct
	Use of χ^2 stat = $\sum \frac{O^2}{E} - N$	M1	AO3	
	$= \frac{6^2}{6.767} + \frac{11^2}{13.534} + \ldots + \frac{8^2}{7.144} - 50$	A1	AO2	
	$= 0.93$	A1	AO1	
	DF = 4	B1	AO1	
	5% crit val = 9.488	B1	AO1	
	Since 0.93 < 9.488 (Accept H_0)	B1	AO2	
	We conclude that the data can be modelled by the Poisson distribution with mean 2.	B1 **[10]**	AO3	
6(a)	The number of arrivals X is Poi(7.5)	B1	AO3	
	$P(X = 5) = \frac{e^{-7.5} \times 7.5^5}{5!}$	M1	AO1	Or straight from the calculator
	$= 0.109(3745\ldots)$	A1	AO1	
(b)(i)	$P(T > t)$ = P(No customers arrive between 11am and t mins after 11am)			
	$= e^{-0.5t}$	B1	AO2	
(ii)	The cumulative distribution function of T is			
	$F(t) = P(T \le t)$	M1	AO3	
	$= 1 - P(T > t) = 1 - e^{-0.5t}$	A1	AO2	
	Let $f(t)$ denote the probability density function of T			
	$f(t) = F'(t)$	M1	AO2	
	$= 0.5e^{-0.5t}$	A1	AO1	
(iii)	This is the exponential distribution.	B1	AO2	
	Therefore mean = standard deviation = 1/0.5 = 2	B1 **[10]**	AO2	

<table>
<tr><th>Qu. No.</th><th>Solution</th><th>Mark</th><th>AO</th><th>Notes</th></tr>
<tr><td>7(a)(i)</td><td>H_0: There is no association between parents knowing their child's social media passwords and age of child
H_1: There is an association between parents knowing their child's social media passwords and age of child</td><td>B1</td><td>AO3</td><td>Or H_0: Parents knowing their child's social media passwords is independent of age
H_1: Parents knowing their child's social media passwords is not independent of age</td></tr>
<tr><td>(ii)</td><td>Expected values
<table><tr><th></th><th colspan="3">Age (years)</th></tr><tr><th>Parent knows password</th><th>13</th><th>14</th><th>15</th></tr><tr><td>Yes</td><td>62.79</td><td>78.71</td><td>76.50</td></tr><tr><td>No</td><td>79.21</td><td>99.29</td><td>96.50</td></tr></table>
$142\times\frac{275}{493}=79.21$ OR $275\times\frac{142}{493}=79.21$</td><td>M1
A1</td><td>AO2
AO1</td><td>Or any equivalent correct method</td></tr>
<tr><td>(iii)</td><td>Chi-squared contributions
<table><tr><th></th><th colspan="3">Age (years)</th></tr><tr><th>Parent knows password</th><th>13</th><th>14</th><th>15</th></tr><tr><td>Yes</td><td>2.779</td><td>0.175</td><td>1.180</td></tr><tr><td>No</td><td>2.203</td><td>0.139</td><td>0.935</td></tr></table></td><td>M1
A1</td><td>AO2
AO1</td><td>M1A0 for one correct χ^2 contribution
FT 'their observed values'</td></tr>
<tr><td>(iv)</td><td>2 degrees of freedom from (3 – 1) × (2 – 1)</td><td>B1</td><td>AO1</td><td></td></tr>
<tr><td>(v)</td><td>Since p-value < 0.05, Reject H_0

Strong evidence to suggest there is an association between parents knowing their child's social media passwords and age</td><td>B1
B1

B1
B1</td><td>AO1
AO2

AO2
AO3</td><td>B1 for < 0.05
B1 for Reject H_0

B1 for strong evidence
B1 for relating back to hypothesis</td></tr>
<tr><td>(b)</td><td>Largest contribution for 13-year-olds especially for yes
It seems more parents than expected know passwords for their 13-year-old children.</td><td>E1

E1

[12]</td><td>AO2

AO2</td><td></td></tr>
</table>

GCE

FURTHER MATHEMATICS

UNIT 3: FURTHER MECHANICS A

SAMPLE ASSESSMENT MATERIALS

(1 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. By burning a charge, a cannon fires a cannon ball of mass 12 kg horizontally. As the cannon ball leaves the cannon, its speed is 600 ms^{-1}. The recoiling part of the cannon has a mass of 1600 kg.

   (a) Determine the speed of the recoiling part immediately after the cannon ball leaves the cannon. [3]

   (b) Find the energy created by the burning of the charge. State any assumption you have made in your solution and briefly explain how the assumption affects your answer. [5]

   (c) Calculate the constant force needed to bring the recoiling part to rest in 1.2 m. State, with a reason, whether your answer is an overestimate or an underestimate of the actual force required. [4]

2. A particle *P*, of mass 3 kg, is attached to a fixed point *O* by a light inextensible string of length 4 m. Initially, particle *P* is held at rest at a point which is $2\sqrt{3}$ m horizontally from *O*. It is then released and allowed to fall under gravity.

   (a) Show that the speed of *P* when it first begins to move in a circle is $\sqrt{3g}$. [4]

   (b) In the subsequent motion, when the string first makes an angle of 45° with the downwards vertical,

      (i) calculate the speed *v* of *P*,

      (ii) determine the tension in the string. [8]

3. At time $t = 0$ s, the position vector of an object *A* is $\mathbf{i}$ m and the position vector of another object *B* is $3\mathbf{i}$ m. The constant velocity vector of *A* is $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ ms^{-1} and the constant velocity vector of *B* is $\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ ms^{-1}. Determine the value of *t* when *A* and *B* are closest together and find the least distance between *A* and *B*. [9]

4. Relative to a fixed origin O, the position vector $\mathbf{r}$ m at time t s of a particle P, of mass 0.4 kg, is given by

$$\mathbf{r} = e^{2t}\mathbf{i} + \sin(2t)\mathbf{j} + \cos(2t)\mathbf{k}.$$

(a) Show that the velocity vector $\mathbf{v}$ and the position vector $\mathbf{r}$ are never perpendicular to each other. [6]

(b) Given that the speed of P at time t is v ms^{-1}, show that

$$v^2 = 4e^{4t} + 4.$$ [2]

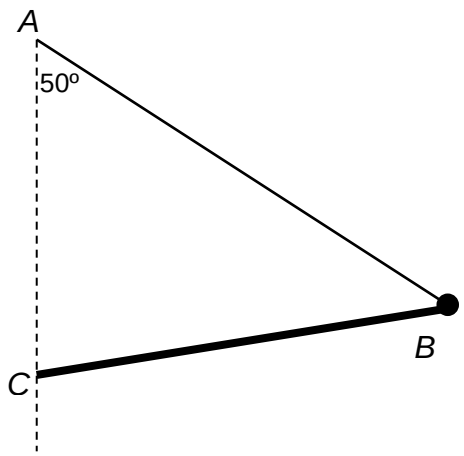
(c) Find the kinetic energy of P at time t. [1]

(d) Calculate the work done by the force acting on P in the interval $0 < t < 1$. [2]

(e) Determine an expression for the rate at which the force acting on P is working at time t. [2]

5. A particle of mass m kg is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set in motion such that it moves in a horizontal circle of radius 2 m with constant speed 4.8 ms^{-1}.
Calculate the angle the string makes with the vertical. [6]

6.



A particle of mass 5 kg is attached to a string *AB* and a rod *BC* at the point *B*. The string *AB* is light and elastic with modulus λ N and natural length 2 m. The rod *BC* is light and of length 2 m. The end *A* of the string is attached to a fixed point and the end C of the rod is attached to another fixed point such that *A* is vertically above *C* with *AC* = 2 m. When the particle rests in equilibrium, *AB* makes an angle of 50º with the downward vertical.

(a) Determine, in terms of λ, the tension in the string *AB*. [3]

(b) Calculate, in terms of λ, the energy stored in the string *AB*. [2]

(c) Find, in terms of λ, the thrust in the rod *BC*. [4]

7. A vehicle of mass 6000 kg is moving up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{6}{49}$. The vehicle's engine exerts a constant power of P W. The constant resistance to motion of the vehicle is R N. At the instant the vehicle is moving with velocity $\frac{16}{5}$ ms^{-1}, its acceleration is 2 ms^{-2}. The maximum velocity of the vehicle is $\frac{16}{3}$ ms^{-1}.

Determine the value of P and the value of R. [9]

AS Further Mathematics Unit 3: Further Mechanics A
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ($\surd$ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

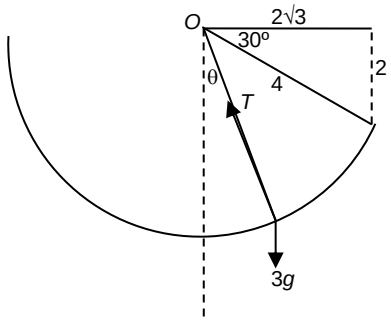
5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

AS Further Mathematics Unit 3: Further Mechanics A

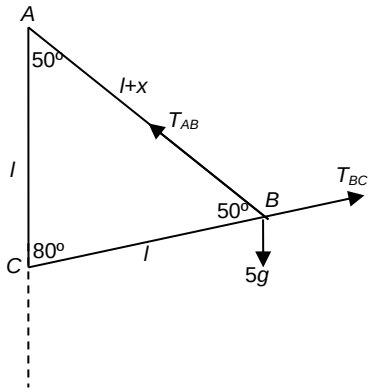
Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	Conservation of momentum	M1	AO3	Dimensionally correct
	$12 \times 600 = 1600 \times v$	A1	AO2	
	$v = \frac{9}{2}$ (ms^{-1})	A1	AO1	allow -ve
(b)	Energy considerations	M1	AO3	
	$E = 0.5 \times 12 \times 600^2 + 0.5 \times 1600 \times 4.5^2$	A1	AO2	both expressions correct, Ft v in (a)
	$E = 2160000 + 16200$			
	$E = \underline{2176200 \text{ (J)}}$	A1	AO1	cao
	Energy dissipated by eg sound of cannon firing ignored.	B1	AO3	oe
	In actual fact, quite a lot of energy would be dissipated as sound or heat or in overcoming the friction in the barrel of the cannon.	E1	AO3	
(c)	Work-energy principle $F \times d = E$	M1	AO3	Used
	$F \times 1.2 = 16200$			
	$F = \underline{13500 \text{ (N)}}$	A1	AO2	cao
	We have not taken into account friction or other resistance to motion which would stop the recoiling part anyway even if no external force is applied.	E1	AO3	
	So the calculated force is an over estimate of the force required.	B1	AO3	
		[12]		

Question Number	Solution	Mark	AO	Notes
2.				
(a)	For *P* falling freely $v^2 = u^2 + 2as$, $u = 0$, $s = 2$, $a = g$ $v^2 = 2g \times 2$	M1	AO3	
	$v = 2\sqrt{g}$	A1	AO1	
	When string tightens, *P* has vertical speed $2\sqrt{g}$. The component of this along the string is destroyed and *P* begins to move in a vertical circle with initial speed $2\sqrt{g}\cos 30^\circ$	m1	AO3	
	$= 2\sqrt{g} \times \frac{\sqrt{3}}{2} = \sqrt{3g}$	A1	AO1	
(b) (i)	Conservation of energy	M1	AO3	KE and PE in dim correct equation
	$\frac{1}{2}mv^2 = \frac{1}{2}m \times 3g + mg \times 4(\cos 45^\circ - \sin 30^\circ)$	A1 A1	AO1 AO1	KE PE
	$v^2 = 3g - 4g + 8g\cos 45^\circ$ $v^2 = 45.63717(165)$			
	$v = 6.76\ \text{ms}^{-1}$ $(6.7555289\ldots)$	A1	AO1	
(ii)	N2L towards centre	M1	AO3	dim correct equation
	$T - mg\cos 45^\circ = \frac{mv^2}{4}$	A1	AO2	
	$T = 3g\left(\frac{1}{\sqrt{2}}\right) + \frac{3}{4}(45.63717)$	m1	AO1	substitute for v^2
	$T = 55.02$ $(55.0168\ldots)$	A1	AO1	
		[12]		

Question Number	Solution	Mark	AO	Notes
3.	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$	M1	AO3	Used
	$\mathbf{r}_A = (1 + 2t)\mathbf{i} + 5t\mathbf{j} - 4t\mathbf{k}$ $\mathbf{r}_B = (3 + t)\mathbf{i} + 3t\mathbf{j} - 5t\mathbf{k}$	A1	AO2	either correct, any form
	$\mathbf{r}_B - \mathbf{r}_A = (2 - t)\mathbf{i} - 2t\mathbf{j} - t\mathbf{k}$	M1	AO3	
	$AB^2 = x^2 + y^2 + z^2$	M1	AO1	
	$AB^2 = (2 - t)^2 + 4t^2 + t^2$	A1	AO1	
	$(AB^2 = 6t^2 - 4t + 4)$			cao
	Differentiate	M1	AO2	
	$\frac{dAB^2}{dt} = 2(2 - t)(-1) + 10t \ (= 12t - 4)$			at least 1 power reduced
	$-4 + 2t + 10t = 0$	m1	AO2	
	$t = \frac{1}{3}$	A1	AO1	equating to 0
	(least distance)^2 = $(2 - \frac{1}{3})^2 + 5(\frac{1}{3})^2$			cao
	least distance = $\sqrt{\frac{10}{3}}$ = <u>1.83 (m)</u>	A1 **[9]**	AO1	cao

Question Number	Solution	Mark	AO	Notes
4. (a)	$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	M1	AO2	
	$\mathbf{v} = 2\mathrm{e}^{2t}\,\mathbf{i} + 2\cos(2t)\,\mathbf{j} - 2\sin(2t)\,\mathbf{k}$	A1	AO1	correct differentiation of any one term
		A1	AO1	all correct
	$\mathbf{v}.\mathbf{r} = 2\mathrm{e}^{4t} + 2\cos(2t)\sin(2t) - 2\sin(2t)\cos(2t)$	M1	AO2	
		A1	AO1	correct dot product
	$\mathbf{v}.\mathbf{r} = 2\mathrm{e}^{4t}$ which is never 0 Hence $\mathbf{v}$ and $\mathbf{r}$ are never perpendicular to each other	E1	AO2	
(b)	$v^2 = (2\mathrm{e}^{2t})^2 + (2\cos(2t))^2 + (-2\sin(2t))^2$	M1	AO2	
	$v^2 = 4\mathrm{e}^{4t} + 4\cos^2(2t) + 4\sin^2(2t)$ $v^2 = 4\mathrm{e}^{4t} + 4$	A1	AO1	
(c)	KE = $0.5 \times 0.4 \times (4\mathrm{e}^{4t} + 4)$ KE = $0.8(\mathrm{e}^{4t} + 1)$	B1	AO1	
(d)	WD = change in KE	M1	AO1	
	WD = $0.8(\mathrm{e}^4 + 1) - 0.8(1 + 1)$ WD = $0.8(\mathrm{e}^4 - 1) = 42.9$ (J)	A1	AO1	
(e)	Rate of work = $\dfrac{\mathrm{d}}{\mathrm{d}t}$(KE)	M1	AO2	
	Rate of work = $\dfrac{\mathrm{d}}{\mathrm{d}t}(0.8(\mathrm{e}^{4t} + 1))$			
	Rate of work = $3.2\,\mathrm{e}^{4t}$ (W)	A1	AO1	
		[13]		

Question Number	Solution	Mark	AO	Notes
5.	Resolve vertically $T\cos\theta = mg$ N2L towards centre $T\sin\theta = \frac{mv^2}{r}$ $T\sin\theta = \frac{m\times 4.8^2}{2}$ $\tan\theta = \frac{4.8^2}{2\times 9.8}$ $\theta = 49.61(2371\ldots)^{\circ}$	M1 A1 M1 A1 m1 A1 **[6]**	AO3 AO2 AO3 AO2 AO1 AO1	
6.				
(a)	$AB = 2\times 2\cos 50^{\circ}$ Hooke's Law $T_{AB} = \frac{\lambda}{2}(4\cos 50^{\circ} - 2) = \lambda(2\cos 50^{\circ} - 1)$ $T_{AB} = 0.286\lambda$ (N)	B1 M1 A1	AO3 AO2 AO1	
(b)	$EE = \frac{1}{2}\frac{\lambda\left(4\cos 50^{\circ} - 2\right)^2}{2}$ $EE = 0.0816\lambda$ (J)	M1 A1	AO1 AO1	
(c)	For vertical equilibrium $T_{AB}\cos 50^{\circ} + T_{BC}\cos 80^{\circ} = mg$ $T_{BC}\cos 80^{\circ} = 49 - \lambda 0.286\cos 50^{\circ}$ $T_{BC} = 282.180 - 1.057\lambda$ (N) OR For horizontal equilibrium $T_{AB}\sin 50^{\circ} = T_{BC}\sin 80^{\circ}$ $T_{BC} = \lambda(2\cos 50^{\circ} - 1)\times\frac{\sin 50^{\circ}}{\sin 80^{\circ}}$ $T_{BC} = 0.222\lambda$ (N)	M1 A1 m1 A1 (M1) (A1) (m1) (A1) **[9]**	AO3 AO2 AO1 AO1 (AO3) (AO2) (AO1) (AO1)	resolve vertically Resolve horizontally

Question Number	Solution	Mark	AO	Notes
7.	N2L	M1	AO3	dim correct, all forces
	$T - mg\sin\alpha - R = ma$	A1	AO2	correct equation
	$T = \frac{P}{v}$	B1	AO3	used si
	$\frac{5P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 6000 \times 2$	A1	AO1	correct equation in P & R
	$\frac{5P}{16} - R = 19200$			
	N2L with $a = 0$	M1	AO3	dim correct, all forces
	$T - mg\sin\alpha - R = 0$	A1	AO2	correct equation
	$\frac{3P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 0$	A1	AO1	correct equation in P & R
	$\frac{3P}{16} - R = 7200$			
	Solving simultaneously	m1	AO1	eliminating one variable, Dep. on both M's
	$\frac{2P}{16} = 12000$			
	$P = 96000; R = 10800$	A1 **[9]**	AO1	both answers cao

GCE

FURTHER MATHEMATICS

UNIT 4: FURTHER PURE MATHEMATICS B

SAMPLE ASSESSMENT MATERIALS

(2 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Evaluate the integral

$$\int_0^{\infty} \frac{\mathrm{d}x}{(1+x)^5}.$$ [3]

(b) By putting $u = \ln x$, determine whether or not the following integral has a finite value.

$$\int_2^{\infty} \frac{\mathrm{d}x}{x \ln x}.$$ [4]

2. Evaluate the integral

$$\int_0^{1} \frac{\mathrm{d}x}{\sqrt{2x^2 + 4x + 6}}.$$ [6]

3. The curve C has polar equation $r = 3(2 + \cos\theta)$, $0 \leq \theta \leq \pi$. Determine the area enclosed between C and the initial line. Give your answer in the form $\frac{a}{b}\pi$, where a and b are positive integers whose values are to be found. [5]

4. Find the three cube roots of the complex number $2 + 3\mathrm{i}$, giving your answers in Cartesian form. [9]

5. Find all the roots of the equation

$$\cos\theta + \cos 3\theta + \cos 5\theta = 0$$

lying in the interval $[0, \pi]$. Give all the roots in radians in terms of π. [8]

6. The matrix $\mathbf{M}$ is given by

$$\mathbf{M}=\begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{bmatrix}.$$

(a) Find

(i) the adjugate matrix of $\mathbf{M}$,

(ii) hence determine the inverse matrix $\mathbf{M}^{-1}$. [5]

(b) Use your result to solve the simultaneous equations

$$\begin{aligned} 2x + y + 3z &= 13 \\ x + 3y + 2z &= 13 \\ 3x + 2y + 5z &= 22 \end{aligned}$$ [2]

7. The function f is defined by

$$f(x) = \frac{8x^2 + x + 5}{(2x+1)(x^2+3)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Hence evaluate

$$\int_2^3 f(x)\mathrm{d}x,$$

giving your answer correct to three decimal places. [6]

8. The curve $y = 1 + x^3$ is denoted by C.

(a) A bowl is designed by rotating the arc of C joining the points (0,1) and (2,9) through four right angles about the y-axis. Calculate the capacity of the bowl. [5]

(b) Another bowl with capacity 25 is to be designed by rotating the arc of C joining the points with y coordinates 1 and a through four right angles about the y-axis. Calculate the value of a. [5]

9. (a) Use mathematical induction to prove de Moivre's Theorem, namely that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta,$$

where n is a positive integer. [7]

(b) (i) Use this result to show that

$$\sin 5\theta = a\sin^5\theta - b\sin^3\theta + c\sin\theta,$$

where a, b and c are positive integers to be found.

(ii) Hence determine the value of $\lim_{\theta \to 0} \frac{\sin 5\theta}{\sin\theta}$ [7]

10. Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\tan x = \sin x, \qquad 0 < x < \frac{\pi}{2}.$$

(a) Find an integrating factor for this differential equation. [4]

(b) Solve the differential equation given that $y = 0$ when $x = \frac{\pi}{4}$, giving your answer in the form $y = f(x)$. [7]

11. (a) Show that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \qquad \text{where } -1 < x < 1. \qquad [4]$$

(b) Given that

$$a\cosh x + b\sinh x \equiv r\cosh(x + \alpha), \qquad \text{where } a > b > 0,$$

show that

$$\alpha = \frac{1}{2}\ln\left(\frac{a+b}{a-b}\right)$$

and find an expression for r in terms of a and b. [7]

(c) Hence solve the equation

$$5\cosh x + 4\sinh x = 10,$$

giving your answers correct to three significant figures. [6]

12. The function f is given by

$$f(x) = \mathrm{e}^x \cos x .$$

(a) Show that $f''(x) = -2\mathrm{e}^x \sin x$. [2]

(b) Determine the Maclaurin series for $f(x)$ as far as the x^4 term. [6]

(c) Hence, by differentiating your series, determine the Maclaurin series for $\mathrm{e}^x \sin x$ as far as the x^3 term. [4]

(d) The equation

$$10\mathrm{e}^x \sin x - 11x = 0$$

has a small positive root. Determine its approximate value, giving your answer correct to three decimal places. [4]

A2 Further Mathematics Unit 4: Further Pure Mathematics B
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
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MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
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F.T. = follow through ($\surd$ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

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- 'E' marks are awarded for explanation
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- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

A2 Further Mathematics Unit 4: Further Pure Mathematics B

Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1.(a)	$\int_0^{\infty} \frac{\mathrm{d}x}{(1+x)^5} = -\frac{1}{4}\left[\frac{1}{(1+x)^4}\right]_0^{\infty}$	M1	AO1	
	$= -\frac{1}{4}(0-1)$	A1	AO1	
	$= \frac{1}{4}$	A1	AO1	
(b)	$\mathrm{d}u = \frac{\mathrm{d}x}{x}; [2,\infty) \to [\ln 2, \infty)$	B1	AO1	
	Integral $= \int_{\ln 2}^{\infty} \frac{\mathrm{d}u}{u}$	M1	AO1	
	$= [\ln u]_{\ln 2}^{\infty}$	A1	AO1	
	$\to \infty$ because $\ln u \to \infty$	A1	AO1	
		[7]		
2.	Attempting to complete the square	M1	AO3	Award M0 for unsupported working
	Integral $= \int_0^1 \frac{\mathrm{d}x}{\sqrt{2(x+1)^2+4}}$	A1	AO3	
	$= \frac{1}{\sqrt{2}} \int_0^1 \frac{\mathrm{d}x}{\sqrt{(x+1)^2+2}}$	A1	AO3	
	$= \frac{1}{\sqrt{2}}\left[\sinh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)\right]_0^1$	A1	AO3	
	$= \frac{1}{\sqrt{2}}\left(\sinh^{-1}\left(\frac{2}{\sqrt{2}}\right) - \sinh^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$	A1	AO3	
	$= 0.345 \ \ (0.344882\ldots)$	A1	AO3	
		[6]		

Qu. No.	Solution	Mark	AO	Notes
3.	Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta$	M1	AO1	
	$= \frac{9}{2}\int_0^{\pi}(4 + 4\cos\theta + \cos^2\theta)\mathrm{d}\theta$	A1	AO1	
	$= \frac{9}{2}\int_0^{\pi}\left(\frac{9}{2} + 4\cos\theta + \frac{\cos 2\theta}{2}\right)$	A1	AO1	
	$= \frac{9}{2}\left[\frac{9}{2}\theta + 4\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi}$	A1	AO1	
	$= \frac{81\pi}{4}$	A1 **[5]**	AO1	
4.	$\lvert z\rvert = \sqrt{13}$	B1	AO3	
	$\arg(z) = \tan^{-1}1.5 = 0.98279...$	B1	AO3	
	$z = \sqrt{13}(\cos 0.98279.. + \mathrm{i}\sin 0.98279..)$	M1	AO3	
	First cube root $= 13^{1/6}(\cos 0.32759... + \mathrm{i}\sin 0.32759...)$	m1	AO3	
	$= 1.45 + 0.493\mathrm{i}$	A1	AO3	
	Second cube root $= 13^{1/6}(\cos(0.32759... + 2\pi/3) + \mathrm{i}\sin(0.32759... + 2\pi/3))$	M1	AO3	
	$= -1.15 + 1.01\mathrm{i}$	A1	AO3	
	Third cube root $= 13^{1/6}(\cos(0.32759... + 4\pi/3) + \mathrm{i}\sin(0.32759... + 4\pi/3))$	M1	AO3	
	$= -2.99 - 1.50\mathrm{i}$	A1 **[9]**	AO3	

Qu. No.	Solution	Mark	AO	Notes
5.	Rewrite the equation in the form $\cos 3\theta + 2\cos 2\theta \cos 3\theta = 0$ $\cos 3\theta(1 + 2\cos 2\theta) = 0$ Either $\cos 3\theta = 0$ $3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ $\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ Or $\cos 2\theta = -\frac{1}{2}$ $2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	M1 A1 M1 A1 A1 M1 A1 A1 **[8]**	AO1 AO1 AO1 AO1 AO1 AO1 AO1 AO1	
6.(a)(i)	$\text{adj}(\mathbf{M}) = \begin{bmatrix} 11 & 1 & -7 \\ 1 & 1 & -1 \\ -7 & -1 & 5 \end{bmatrix}$	M1 A1	AO1 AO1	Award M1 if at least 5 correct
(ii)	$\det(\mathbf{M}) = 2\times(15–4) + 1\times(6–5) + 3\times(2–9)$ $= 2$ $\mathbf{M}^{-1} = \frac{1}{2}\begin{bmatrix} 11 & 1 & -7 \\ 1 & 1 & -1 \\ -7 & -1 & 5 \end{bmatrix}$	M1 A1 B1	AO1 AO1 AO1	
(b)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 11 & 1 & -7 \\ 1 & 1 & -1 \\ -7 & -1 & 5 \end{bmatrix}\begin{bmatrix} 13 \\ 13 \\ 22 \end{bmatrix}$ $= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	M1 A1 **[7]**	AO1 AO1	

Qu. No.	Solution	Mark	AO	Notes
7.(a)	Let $\dfrac{8x^2+x+5}{(2x+1)(x^2+3)} = \dfrac{A}{2x+1} + \dfrac{Bx+C}{x^2+3}$	M1	AO1	
	$= \dfrac{A(x^2+3)+(Bx+C)(2x+1)}{(2x+1)(x^2+3)}$	A1	AO1	A1 each constant
	$A = 2$, $B = 3$, $C = -1$	A1 A1	AO1 AO1	
(b)	Integral $= \left(\int_2^3 \dfrac{2}{2x+1} + \dfrac{3x}{x^2+3} - \dfrac{1}{x^2+3}\right)\mathrm{d}x$	M1	AO1	Award M0 for work unsupported
	$= \left[\ln(2x+1) + \dfrac{3}{2}\ln(x^2+3) - \dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)\right]_2^3$	A1 A1 A1	AO1 AO1 AO1	A1 each integral
	$= \ln 7 + \dfrac{3}{2}\ln 12 - \dfrac{1}{\sqrt{3}}\tan^{-1}\sqrt{3} - \ln 5 - \dfrac{3}{2}\ln 7 + \dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$	A1	AO1	
	$= 1.035$	A1	AO1	
		[10]		
8.(a)	Capacity $= \pi\int_1^9 x^2\mathrm{d}y$	M1	AO3	
	$= \pi\int_1^9 (y-1)^{2/3}\mathrm{d}y$	A1	AO3	
	$= \pi\left[\dfrac{3}{5}(y-1)^{5/3}\right]_1^9$	A1	AO3	
	$= \dfrac{3\pi}{5}(32-0)$	A1	AO3	
	$= 60.3(1857\ldots.)$	A1	AO3	
(b)	Capacity $= \pi\int_1^a (y-1)^{2/3}\mathrm{d}y$	M1	AO3	
	$= \pi\left[\dfrac{3}{5}(y-1)^{5/3}\right]_1^a$	A1	AO3	
	$= \dfrac{3\pi}{5}(a-1)^{5/3}$	A1	AO3	
	Attempting to solve $\dfrac{3\pi}{5}(a-1)^{5/3} = 25$	M1	AO3	
	$a = 5.72$ $(5.71610...)$	A1	AO3	
		[10]		

Qu. No.	Solution	Mark	AO	Notes
9.(a)	Putting $n = 1$, the proposition gives $\cos\theta + \mathrm{i}\sin\theta = \cos\theta + \mathrm{i}\sin\theta$ which is true	B1	AO2	
	Let the proposition be true for $n = k$, ie $[\cos\theta + \mathrm{i}\sin\theta]^k = \cos k\theta + \mathrm{i}\sin k\theta$	M1	AO2	
	Consider (for $n = k + 1$) $(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos\theta + \mathrm{i}\sin\theta)^k(\cos\theta + i\sin\theta)$	M1	AO2	
	$= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$	A1	AO2	
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\sin k\theta\cos\theta + \sin\theta\cos k\theta)$	A1	AO2	
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1	AO2	
	Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n =1 the proposition is proved by induction.	A1	AO2	
(b)(i)	Consider $\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$	M1	AO2	
	$= \mathrm{i}(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$ + real terms It follows equating imaginary terms that	A1	AO2	
	$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$	A1	AO2	
	$= 5(1-\sin^2\theta)^2\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$	A1	AO2	
	$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$	A1	AO2	
(ii)	$\dfrac{\sin 5\theta}{\sin\theta} = 16\sin^4\theta - 20\sin^2\theta + 5$	M1	AO1	
	$\to 5$ as $\theta \to 0$	A1	AO1	
		[14]		

Qu. No.	Solution	Mark	AO	Notes
10.(a)	Integrating factor $= e^{\int 2\tan x dx}$	M1	AO1	
	$= e^{2\ln\sec x}$	A1	AO1	
	$= e^{\ln\sec^2 x}$	A1	AO1	
	$= \sec^2 x$	A1	AO1	
(b)	Applying the integrating factor,			
	$\sec^2 x\frac{dy}{dx} + 2y\tan x\sec^2 x = \sin x\sec^2 x$	M1	AO1	
	$= \frac{\sin x}{\cos^2 x}$ (or $\sec x\tan x$)	A1	AO1	
	Integrating,			
	$y\sec^2 x = \sec x + C$	A1 A1	AO1 AO1	A1 each side
	$0 = \sqrt{2} + C$	M1	AO1	
	$C = -\sqrt{2}$	A1	AO1	
	The solution is $y = \cos x - \sqrt{2}\cos^2 x$	A1 **[11]**	AO1	

Qu. No.	Solution	Mark	AO	Notes
11.(a)	Let $y = \tanh^{-1} x$ so $x = \tanh y$			
	$= \dfrac{e^{y} - e^{-y}}{e^{y} + e^{-y}}$	M1	AO2	
	$xe^{y} + xe^{-y} = e^{y} - e^{-y}$	A1	AO2	
	$e^{2y} = \dfrac{1+x}{1-x}$	A1	AO2	
	$y = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$	A1	AO2	
(b)	$a\cosh x + b\sinh x \equiv r\cosh(x + \alpha)$	M1	AO2	
	$= r\cosh x\cosh\alpha + r\sinh x\sinh\alpha$	A1	AO2	
	Equating like terms,			
	$r\cosh\alpha = a$			
	$r\sinh\alpha = b$	A1	AO2	
	Dividing,			
	$\tanh\alpha = \dfrac{b}{a}$	M1	AO2	
	$\alpha = \tanh^{-1}\left(\dfrac{b}{a}\right)$	A1	AO2	
	$= \dfrac{1}{2}\ln\left(\dfrac{1+b/a}{1-b/a}\right) = \dfrac{1}{2}\ln\left(\dfrac{a+b}{a-b}\right)$			
	Squaring and subtracting the above equations,			
	$r^{2}(\cosh^{2}\alpha - \sinh^{2}\alpha) = a^{2} - b^{2}$	M1	AO1	
	$r = \sqrt{a^{2} - b^{2}}$	A1	AO1	
(c)	Here $r = 3$	B1	AO1	
	$\alpha = \dfrac{1}{2}\ln 9 = \ln 3$	B1	AO1	
	The equation simplifies to $3\cosh(x + \ln 3) = 10$	B1	AO1	
	$x + \ln 3 = (\pm)\cosh^{-1}\left(\dfrac{10}{3}\right)$	M1	AO1	
	$x = 0.775$	A1	AO1	
	or $x = -2.97$	A1	AO1	
		[17]		

Qu. No.	Solution	Mark	AO	Notes
12.(a)	$f'(x) = e^x \cos x - e^x \sin x$	B1	AO2	
	$f''(x) = e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x$	B1	AO2	
	$= -2e^x \sin x$			
(b)	$f'''(x) = -2e^x \sin x - 2e^x \cos x$	B1	AO1	
	$f^{(4)}(x) = -2e^x \sin x - 2e^x \cos x - 2e^x \cos x + 2e^x \sin x$	B1	AO1	
	$(= -4e^x \cos x)$			
	$f(0) = 1,\ f'(0) = 1,\ f''(0) = 0$	B1	AO1	
	$f'''(0) = -2,\ f^{(4)}(0) = -4$	B1	AO1	
	The Maclaurin series is			
	$e^x \cos x = 1 + x - \frac{2x^3}{6} - \frac{4x^4}{24} + \ldots$	M1	AO1	
	$= 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \ldots$	A1	AO1	
(c)	Valid attempt at differentiating both sides,	M1	AO1	
	$e^x \cos x - e^x \sin x = 1 - x^2 - \frac{2x^3}{3} + \ldots$	A1	AO1	
	$e^x \sin x = 1 + x - \frac{x^3}{3} - 1 + x^2 + \frac{2x^3}{3} + \ldots$	A1	AO1	
	$= x + x^2 + \frac{x^3}{3} + \ldots$	A1	AO1	
(d)	Replacing $e^x \sin x$ by its series,			
	$10\left(x + x^2 + \frac{x^3}{3}\right) - 11x = 0$	M1	AO3	
	$10x^3 + 30x^2 - 3x = 0$	A1	AO3	
	$x = \frac{-30 + \sqrt{900 + 120}}{20}$	m1	AO3	
	$= 0.097$	A1	AO3	
		[16]		

GCE

FURTHER MATHEMATICS

UNIT 5: FURTHER STATISTICS B

SAMPLE ASSESSMENT MATERIALS

(1 hour 45 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. Alun does the crossword in the Daily Bugle every day. The time that he takes to complete the crossword, X minutes, is modelled by the normal distribution $\mathrm{N}(32,4^2)$. You may assume that the times taken to complete the crossword on successive days are independent.

   (a) (i) Find the upper quartile of X and explain its meaning in context.

   (ii) Find the probability that the total time taken by Alun to complete the crosswords on five randomly chosen days is greater than 170 minutes. [7]

   (b) Belle also does the crossword every day and the time that she takes to complete the crossword, Y minutes, is modelled by the normal distribution $\mathrm{N}(18, 2^2)$. Find the probability that, on a randomly chosen day, the time taken by Alun to complete the crossword is more than twice the time taken by Belle to complete the crossword. [6]

2. A factory manufactures a certain type of string. In order to ensure the quality of the product, a random sample of 10 pieces of string is taken every morning and the breaking strength of each piece, in Newtons, is measured. One morning, the results are as follows.

   68·1 70·4 68·6 67·7 71·3 67·6 68·9 70·2 68·4 69·8

   You may assume that this is a random sample from a normal distribution with unknown mean μ and unknown variance σ^2.

   (a) Determine a 95% confidence interval for μ. [9]

   (b) The factory manager is given these results and he asks 'Can I assume that the confidence interval that you have given me contains μ with probability 0.95?' Explain why the answer to this question is no and give a correct interpretation. [2]

3. A motoring organisation wishes to determine whether or not the petrol consumption of two different car models A and B are the same. A trial is therefore carried out in which 6 cars of each model are given 10 litres of petrol and driven at a predetermined speed around a track until the petrol is used up. The distances travelled, in miles, are shown below

Model A:	86.3	84.2	85.8	83.1	84.7	85.3
Model B:	84.9	85.9	84.8	86.5	85.2	85.5

It is proposed to use a test with significance level 5% based on the Mann-Whitney statistic U.

(a) State suitable hypotheses. [2]

(b) Find the critical region for the test. [3]

(c) Determine the value of U for the above data and state your conclusion in context. You must justify your answer. [4]

4. (a) In an opinion poll of 1800 people, 1242 said that they preferred red wine to white wine. Calculate a 95% confidence interval for the proportion of people in the population who prefer red wine to white wine. [6]

(b) In another opinion poll of 1000 people on the same subject, the following confidence interval was calculated.

[0.672, 0.732].

Determine

(i) the number of people in the sample who stated that they prefer red wine to white wine,

(ii) the confidence level of the confidence interval, giving your answer as a percentage correct to three significant figures. [6]

5. A new species of animal has been found on an uninhabited island. A zoologist wishes to investigate whether or not there is a difference in the mean weights of males and females of the species. She traps some of the animals and weighs them with the following results.

Males (kg)	5·3, 4·6, 5·2, 4·5, 4·3, 5·5, 5·0, 4·8
Females (kg)	4·9, 5·0, 4·1, 4·6, 4·3, 5·3, 4·2, 4·5, 4·8, 4·9

You may assume that these are random samples from normal populations with a common standard deviation of 0·5 kg.

(a) State suitable hypotheses for this investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [9]

6. A medical student is investigating two different methods, A and B, of measuring a patient's blood pressure. He believes that Method B gives, on average, a higher reading than Method A so he defines the following hypotheses.

H_0 : There is on average no difference in the readings obtained using Methods A and B;

H_1 : The reading obtained using Method B is on average higher than the reading obtained using Method A.

He selects 10 patients at random and he measures their blood pressures using both methods. He obtains the following results.

Patient	A	B	C	D	E	F	G	H	I	J
Method A	121	133	119	142	151	139	161	148	151	125
Method B	126	131	127	152	145	151	157	155	160	126

(a) Carry out an appropriate Wilcoxon signed rank test on this data set, using a 5% significance level. [6]

(b) State what conclusion the medical student should reach, justifying your answer. [2]

7. The discrete random variable X has the following probability distribution, where θ is an unknown parameter belonging to the interval $\left(0, \frac{1}{3}\right)$.

Value of X	1	3	5
Probability	θ	$1 - 3\theta$	2θ

(a) Obtain an expression for $E(X)$ in terms of θ and show that

$$\text{Var}(X) = 4\theta(3 - \theta).$$ [4]

In order to estimate the value of θ, a random sample of n observations on X was obtained and $\overline{X}$ denotes the sample mean.

(b) (i) Show that

$$V = \frac{\overline{X} - 3}{2}$$

is an unbiased estimator for θ.

(ii) Find an expression for the variance of V. [4]

(c) Let Y denote the number of observations in the random sample that are equal to 1.

Show that

$$W = \frac{Y}{n}$$

is an unbiased estimator for θ and find an expression for $\text{Var}(W)$. [5]

(d) Determine which of V and W is the better estimator, explaining your method clearly. [4]

A2 Further Mathematics Unit 5: Further Statistics B
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ($\surd$ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

A2 Further Mathematics Unit 5: Further Statistics B

Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1(a)(i)	Upper quartile = $\mu + 0.6745\sigma$ = 32 + 0.6745 × 4 = 34.7 This is the time that is exceeded on 25% of the days.	M1 A1 E1	AO3 AO1 AO2	
(ii)	Let $T = X_1 + X_2 + X_3 + X_4 + X_5$ Then $E(T) = 160$ $\text{Var}(T) = 5\text{Var}(X)$ $\text{Var}(T) = 80$ $P(T > 170) = 0.132$	 B1 M1 A1 B1	 AO3 AO3 AO1 AO1	
(b)	Consider $U = X - 2Y$ $E(U) = -4$ $\text{Var}(U) = \text{Var}(X) + 4\text{Var}(Y)$ = 32 We require P(U > 0) = 0.240	M1 A1 M1 A1 M1 A1 **[13]**	AO3 AO1 AO3 AO1 AO3 AO1	
2(a)	$\Sigma x = 691,\ \Sigma x^2 = 47762.32$ $\hat{\mu} = 69.1$ $s^2 = \sum \frac{x^2}{n-1} - \frac{\left(\sum x\right)^2}{n(n-1)}$ = 1.58 DF = 9 t value = 2.262 Standard error = $\frac{s}{\sqrt{n}} = \frac{\sqrt{1.58}}{\sqrt{10}}$ Confidence limits = $\bar{x} \pm t \times \frac{s}{\sqrt{n}}$ $= 69.1 \pm 2.262 \times \frac{\sqrt{1.58}}{\sqrt{10}}$ leading to [68.2,70.0]	 B1 M1 A1 B1 B1 B1 M1 A1 A1	 AO1 AO3 AO1 AO1 AO1 AO1 AO3 AO1 AO1	
(b)	The value of μ either lies in the interval or it does not, there is no question of a probability being involved. EITHER The confidence interval is an observed value of a random interval which contains μ with probability 0.95. OR If the process is carried out a large number of times, we would expect 95% of the confidence intervals obtained to contain μ. **[11]**	E1 E1 **[11]**	AO2 AO2 **[11]**	

Qu. No.	Solution	Mark	AO	Notes
3(a)	H_0 : The petrol consumptions of models A and B are the same	B1	AO3	B0 for saying that the **mean** petrol consumption is the same
	H_1 : The petrol consumptions of models A and B are not the same	B1	AO3	For correctly identifying the alternative hypothesis as two-sided
(b)	From tables upper crit value = 31 Therefore lower crit value = 36 – 31 = 5 The critical region is $(U \geq 31) \cup (U \leq 5)$	B1 B1 B1	AO1 AO2 AO2	
(c)	Use of the formula $U = \sum \sum z_{ij}$ U = 1 + 6 + 2 +6 + 6 + 3 = 24	M1 A1	AO3 AO1	
	The conclusion is that there is no difference in petrol consumption of the two models because 24 is not in the critical region.	B1 B1 **[9]**	AO3 AO2	
4(a)	$\hat{p} = \frac{1242}{1800} = 0.69$	B1	AO3	
	ESE = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= \sqrt{\frac{0.69 \times 0.31}{1800}}$ = 0.0109(0107…)	M1 A1	AO1 AO1	
	95% confidence limits are $\hat{p} \pm z \times \text{ESE}$ $0.69 \pm 1.96 \times 0.0109..$ giving [0.669,0.711]	M1 A1 A1	AO3 AO2 AO1	
(b)(i)	$\hat{p} = \frac{0.672 + 0.732}{2} = 0.702$ Number of people = 0.702 × 1000 = 702	B1 B1	AO3 AO1	
(ii)	$0.732 - 0.672 = 2z\sqrt{\frac{0.702 \times 0.298}{1000}}$ z = 2.07417.. Prob from tables = 0.98077 or 0.98097 from calc Confidence level = 96.2%	M1 A1 A1 A1 **[12]**	AO3 AO1 AO1 AO2	

Qu. No.	Solution	Mark	AO	Notes
5(a)	$H_0 : \mu_M = \mu_F; H_1 : \mu_M \neq \mu_F$	B1	AO3	
(b)	Let X= male weight, Y=female weight			
	$(\sum x = 39.2; \sum y = 46.6)$			
	$\bar{x} = 4.9;$	B1	AO1	
	$\bar{y} = 4.66$	B1	AO1	
	SE of diff of means= $\sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{10}}$	M1	AO2	
	$= 0.237\ldots$	A1	AO1	Award m0 if no working seen
	Test statistic = $\frac{4.9 - 4.66}{0.237...}$	m1	AO1	
	$= 1.01$	A1	AO1	From calculator, prob = 0.1558 FT 'their' test statistic
	Prob from tables = 0.1562	A1	AO1	From calculator, p-value = 0.3116
	p-value = 0.3124	B1	AO2	FT 'their' p-value
	Insufficient evidence to conclude that there is a difference in mean weight between males and females.	B1	AO3	
		[10]		
6(a)	The differences are 5 –2 8 10 –6 12 –4 7 9 1	B1	AO3	
	The signs may be omitted at this stage. The ranks are 4 2 7 9 5 10 3 6 8 1	M1 A1	AO3 AO1	Attempting to rank absolute values All correct
	W = Sum of positive ranks = 4 + 7 + 9 +10 + 6 + 8 + 1 = 45 The critical value is 44.	M1 A1 B1	AO3 AO1 AO1	
(b)	The conclusion at this significance level is that Method B gives on average a higher reading than Method A because 45 > 44	B1 E1 **[8]**	AO3 AO2	

Qu. No.	Solution	Mark	AO	Notes
7(a)	$E(X) = \theta + 3(1-3\theta) + 5\times 2\theta$	M1	AO1	
	$= 2\theta + 3$	A1	AO1	
	Var(X) = $\theta + 9(1-3\theta) + 25\times 2\theta - (2\theta+3)^2$	M1	AO2	
	$= \theta + 9 - 27\theta + 50\theta - 4\theta^2 - 12\theta - 9$ $= 4\theta(3-\theta)$	A1	AO2	
(b)(i)	Consider $E(V) = \dfrac{E(\bar{X}) - 3}{2}$	M1	AO2	
	$= \dfrac{2\theta + 3 - 3}{2} = \theta$ (Therefore V is unbiased)	A1	AO2	
(ii)	Var(V) = $\dfrac{\text{Var}(\bar{X})}{4}$	M1	AO3	
	$= \dfrac{\theta(3-\theta)}{n}$	A1	AO1	
(c)	Y is B(n,θ) So $E(Y) = n\theta$	M1 A1	AO3 AO2	
	$E(W) = E\left(\dfrac{Y}{n}\right) = \theta$ (Therefore W is unbiased)	A1	AO2	
	Var(W) = $\dfrac{\text{Var}(Y)}{n^2}$	M1	AO2	
	$= \dfrac{\theta(1-\theta)}{n}$	A1	AO1	
(d)	$\dfrac{\text{Var}(V)}{\text{Var}(W)} = \dfrac{\theta(3-\theta)}{n} \div \dfrac{\theta(1-\theta)}{n}$	M1	AO3	
	$= \dfrac{(3-\theta)}{(1-\theta)}$	A1	AO1	
	It follows that W is the better estimator since it has the smaller variance	B1 B1 **[17]**	AO2 AO2	

GCE

FURTHER MATHEMATICS

UNIT 6: FURTHER MECHANICS B

SAMPLE ASSESSMENT MATERIALS

(1 hour 45 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. A ball of mass 0.4 kg is thrown vertically upwards from a point O with initial speed 17 ms^{-1}. When the ball is at a height of x m above O and its speed is v ms^{-1}, the air resistance acting on the ball has magnitude $0.01v^2$ N.

   (a) Show that, as the ball is ascending, v satisfies the differential equation

   $$40v\frac{\mathrm{d}v}{\mathrm{d}x} = -(392 + v^2)\,.$$ [3]

   (b) Find an expression for v in terms of x. [7]

   (c) Calculate, correct to two decimal places, the greatest height of the ball. [2]

   (d) State, with a reason, whether the speed of the ball when it returns to O is greater than 17 ms^{-1}, less than 17 ms^{-1} or equal to 17 ms^{-1}. [2]

2. (a) Prove that the centre of mass of a uniform solid cone of height h and base radius b is at a height of $\frac{1}{4}h$ above its base. [4]

   (b) A uniform solid cone C_1 has height 3 m and base radius 2 m. A smaller cone C_2 of height 2 m and base radius 1 m is contained symmetrically inside C_1. The bases of C_1 and C_2 have a common centre and the axis of C_2 is part of the axis of C_1. If C_2 is removed from C_1, show that the centre of mass of the remaining solid is at a distance of $\frac{11}{5}$ m from the vertex of C_1. [6]

   (c) The remaining solid is suspended from a string which is attached to a point on the outer curved surface at a distance of $\frac{1}{3}\sqrt{13}$ m from the vertex of C_1. Given that the axis of symmetry is inclined at an angle of α to the vertical, find $\tan\alpha$. [5]

3. A body, of mass 9 kg, is projected along a straight horizontal track with an initial speed of 20 ms^{-1}. At time t s the body experiences a resistance of magnitude $(0.2 + 0.03v)$ N where v ms^{-1} is its speed.

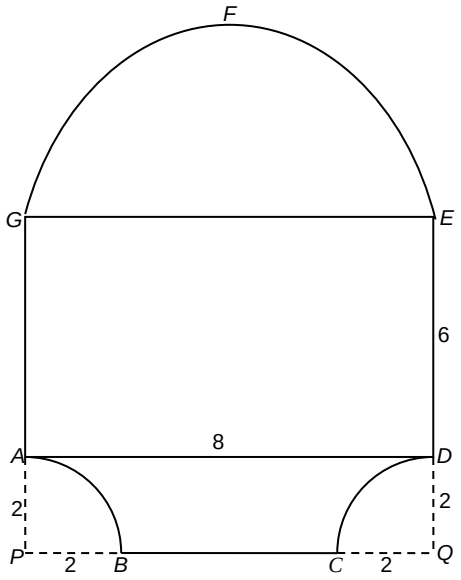
   (a) Show that v satisfies the differential equation

   $$900\frac{\mathrm{d}v}{\mathrm{d}t} = -(20 + 3v)\,.$$ [3]

   (b) Find an expression for t in terms of v. [5]

   (c) Calculate, to the nearest second, the time taken for the body to come to rest. [2]

4. The diagram shows a uniform lamina consisting of a rectangular section *GPQE* with a semi-circular section *EFG* of radius 4 cm. Quadrants *APB* and *CQD* each with radius 2 cm are removed. Dimensions in cm are as shown in the diagram.



(a) Write down the distance of the centre of mass of the lamina *ABCDEFG* from *AG*. [1]

(b) Determine the distance of the centre of mass of the lamina *ABCDEFG* from *BC*. [7]

(c) The lamina *ABCDEFG* is suspended freely from the point *E* and hangs in equilibrium. Calculate the angle *EG* makes with the vertical. [3]

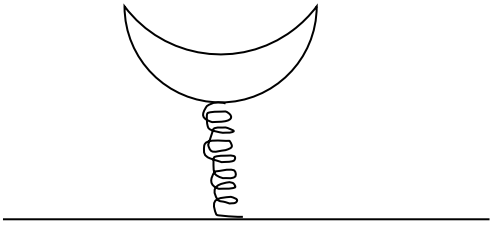
5. A particle *A*, of mass m kg, has position vector $11\mathbf{i} + 6\mathbf{j}$ and a velocity $2\mathbf{i} + 7\mathbf{j}$. At the same moment, second particle *B*, of mass $2m$ kg, has position vector $7\mathbf{i} + 10\mathbf{j}$ and a velocity $5\mathbf{i} + 4\mathbf{j}$.

   (a) If the particles continue to move with these velocities, prove that the particles will collide. Given that the particles coalesce after collision, find the common velocity of the particles after collision. [9]

   (b) Determine the impulse exerted by *A* on *B*. [2]

   (c) Calculate the loss of kinetic energy caused by the collision. [2]

6. The diagram shows a playground ride consisting of a seat *P*, of mass 12 kg, attached to a vertical spring, which is fixed to a horizontal board. When the ride is at rest with nobody on it, the compression of the spring is 0.05 m.



   The spring is of natural length 0.75 m and modulus of elasticity λ.

   (a) Find the value of λ. [2]

   The seat *P* is now pushed vertically downwards a further 0.05 m and is then released from rest.

   (b) Show that *P* makes Simple Harmonic oscillations of period $\frac{\pi}{7}$ and write down the amplitude of the motion. [5]

   (c) Find the maximum speed of *P*. [2]

   (d) Calculate the speed of *P* when it is at a distance 0.03 m from the equilibrium position. [3]

   (e) Find the distance of *P* from the equilibrium position 1.6 s after it is released. [3]

   (f) State one modelling assumption you have made about the seat and one modelling assumption you have made about the spring. [2]

A2 Further Mathematics Unit 6: Pure Mechanics B
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ( indicates correct working following an error and  indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

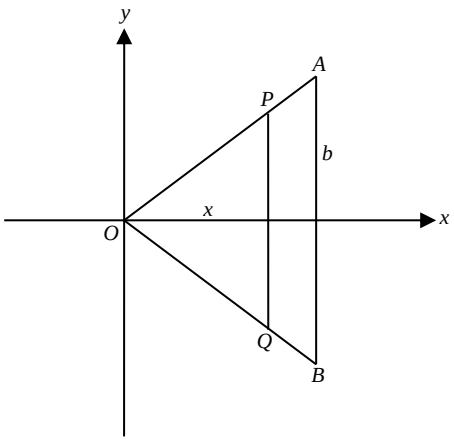
5. Marking codes

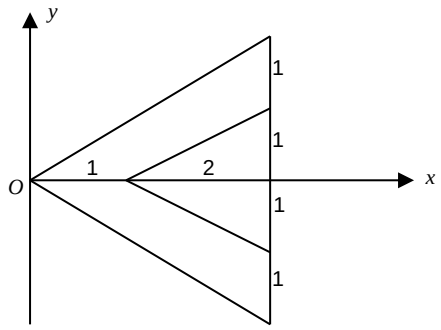
- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

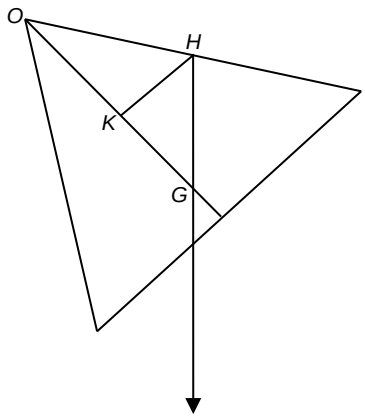
A2 Further Mathematics Unit 6: Further Mechanics B

Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	N2L on ball, upwards positive	M1	AO3	dim correct
	$-0.01v^2 - 0.4g = 0.4a$	A1	AO2	correct equation
	$0.4v\dfrac{dv}{dx} = -3.92 - 0.01v^2$			
	$40v\dfrac{dv}{dx} = -(392 + v^2)$	A1	AO2	convincing
(b)	$40\displaystyle\int \frac{v}{392+v^2}\,dv = -\int dx$	M1	AO2	separate variables
	$20\ln(392 + v^2) = -x + C$	A1 A1	AO1 AO1	$\ln(392 + v^2)$ everything correct
	When $t = 0$, $v = 17$, $x = 0$ $20\ln(392 + 17^2) = C$ $C = 20\ln(681)$	m1 A1	AO2 AO1	use of initial conditions
	$x = 20\ln(681) - 20\ln(392 + v^2)$			
	$x = 20\ln\left(\dfrac{681}{392+v^2}\right)$			
	$\dfrac{x}{20} = \ln\left(\dfrac{681}{392+v^2}\right)$			
	$\left(\dfrac{681}{392+v^2}\right) = e^{0.05x}$	m1	AO1	
	$681 = (392 + v^2)\,e^{0.05x}$			
	$v^2 = 681e^{-0.05x} - 392$			
	$v = \sqrt{681e^{-0.05x} - 392}$	A1	AO1	
(c)	At greatest height $v = 0$	M1	AO2	
	$x = 20\ln\left(\dfrac{681}{392}\right) = 11.05$	A1	AO1	cao
(d)	Speed of ball when it returns to O is less than 17 ms^{-1}.	B1	AO2	
	This is because energy is lost in overcoming air resistance.	E1	AO2	
		[14]		

Question Number	Solution	Mark	AO	Notes
2. (a)				
	Let ρ be mass per unit volume. By symmetry, c of m lies on Ox. Divide cone into slices parallel to base. Consider slice PQ, distance x from O and of thickness δx.	M1	AO2	
	By similar triangles, radius of slice is $\frac{bx}{h}$.			
	Mass of slice = $\frac{\pi b^2 x^2}{h^2}\rho\delta x$ acting x from O.			
	Mass of cone = $\frac{\pi b^2 h}{3}\rho$ acting at $\bar{x}$ from O.			
	Take moments about y axis	m1	AO2	
	$\frac{\pi b^2 h}{3}\rho\bar{x} = \int_0^h \frac{\pi b^2 x^2}{h^2}\times x\rho\,\mathrm{d}x$	A1	AO2	
	$\frac{1}{3}h\bar{x} = \frac{1}{h^2}\left[\frac{1}{4}x^4\right]_0^h$			
	$\bar{x} = \frac{3}{h^3}\frac{h^4}{4}$			
	$\bar{x} = \frac{3h}{4}$	A1	AO2	

Question Number	Solution	Mark	AO	Notes
2 (b)				
	Shape mass distance			
	C_1 $\frac{\pi}{3}(2)^2\times 3\rho$ $\frac{3}{4}\times 3$	B1	AO1	
	C_2 $\frac{\pi}{3}\times 1^2\times 2\rho$ $1+\frac{3}{4}\times 2$	B1	AO1	
	Rem. $\frac{\pi}{3}\rho(12-2)$ $\bar{h}$	B1	AO1	
	Take moments about y axis	M1	AO3	
	$\frac{\pi}{3}\rho\times 10\times\bar{h} = \frac{\pi}{3}\times 12\times\rho\times\frac{9}{4}$			
	$-\frac{\pi}{3}\times 2\rho\times\frac{5}{2}$	A1	AO1	
	$\bar{h}=\frac{11}{5}$	A1	AO1	

Question Number	Solution	Mark	AO	Notes
2. (c)	 Draw *HK* perpendicular to *OG*. *OH* = $\frac{\sqrt{13}}{3}$, *OG* = $\frac{11}{5}$ Angle *HOK* = θ, $\tan\theta = \frac{2}{3}$ $\sin\theta = \frac{2}{\sqrt{13}}$, $\cos\theta = \frac{3}{\sqrt{13}}$ $HK = OH\sin\theta = \frac{\sqrt{13}}{3} \times \frac{2}{\sqrt{13}} = \frac{2}{3}$ $KG = \frac{11}{5} - OH\cos\theta = \frac{11}{5} - \frac{\sqrt{13}}{3} \times \frac{3}{\sqrt{13}}$ $KG = \frac{6}{5}$ $\tan\alpha = \frac{2}{3} \div \frac{6}{5} = \frac{2}{3} \times \frac{5}{6} = \frac{5}{9}$	B1 B1 B1 B1 B1 **[15]**	AO3 AO3 AO3 AO3 AO3	

Question Number	Solution	Mark	AO	Notes
3. (a)	Using N2L	M1	AO3	
	$-0.2 - 0.03v = 9\dfrac{\mathrm{d}v}{\mathrm{d}t}$	A1	AO2	
	$900\dfrac{\mathrm{d}v}{\mathrm{d}t} = -(20 + 3v)$	A1	AO2	
(b)	$900\int \dfrac{\mathrm{d}v}{20+3v} = -\int \mathrm{d}t$	M1	AO2	sep. var.
	$900 \times \dfrac{1}{3}\ln(20 + 3v) = -t\ (+ C)$	A1 A1	AO1 AO1	$\ln(20 + 3v)$ all correct
	When t =0, v = 20 C = 300 ln 80	m1	AO2	used
	Therefore $t = 300\ln(80) - 300\ln(20 + 3v)$ $t = 300\ln\left(\dfrac{80}{20+3v}\right)$	A1	AO1	
(c)	When body is at rest, v=0 T = 300 ln(80) – 300 ln(20) $T = 300\ln(4)$	m1	AO2	used
	T = <u>416 s</u>	A1	AO1	cao
		[10]		

Question Number	Solution	Mark	AO	Notes
4. (a)	$\bar{x}$ = 4 (cm)	B1	AO1	
(b)	Shape mass distance			
	GPQE 64 4	B1	AO1	
	EFG 8π $8+\frac{16}{3\pi}$	B1	AO3	
	APB π $\frac{8}{3\pi}$			
	CQD π $\frac{8}{3\pi}$	B1	AO1	either *APB* or *CQD*
	ABCDEFG $64+6\pi$ $\bar{y}$	B1	AO1	areas
	Moments about *BC*	M1	AO3	
	$(64+6\pi)\bar{y} = 64\times 4 + 8\pi\times(8+\frac{16}{3\pi})$			
	$-2\pi\times\frac{8}{3\pi}$	A1	AO1	
	$\bar{y}$ = 5.967 (cm) (correct to 3 d.p.)	A1	AO1	
(c)	If hanging in equilibrium, vertical passes through centre of mass.	M1	AO3	correct triangle
	$\theta=\tan^{-1}\left(\frac{8-5\cdot 967}{4}\right)$	A1	AO1	
	θ = 26.94(1954…)°	A1	AO1	
		[11]		

Question Number	Solution	Mark	AO	Notes
5. (a)	$\mathbf{r}_A = 11\mathbf{i} + 6\mathbf{j} + (2\mathbf{i} + 7\mathbf{j})t$ $\mathbf{r}_B = 7\mathbf{i} + 10\mathbf{j} + (5\mathbf{i} + 4\mathbf{j})t$ If particles collide, $\mathbf{r}_A = \mathbf{r}_B$ for some value of *t.* For **i** component $11 + 2t = 7 + 5t$ $t = \frac{4}{3}$ For **j** component $6 + 7t = 10 + 4t$ $t = \frac{4}{3}$ Since the value for *t* for both components are equal, the particles collide. Conservation of momentum $m(2\mathbf{i} + 7\mathbf{j}) + 2m(5\mathbf{i} + 4\mathbf{j}) = 3m(x\mathbf{i} + y\mathbf{j})$ $12\mathbf{i} + 15\mathbf{j} = 3x\mathbf{i} + 3y\mathbf{j}$ $x = 4, y = 5$ $x\mathbf{i} + y\mathbf{j} = 4\mathbf{i} + 5\mathbf{j}$ (Ns)	M1 A1 M1 A1 A1 M1 A1 m1 A1	AO3 AO1 AO2 AO2 AO2 AO3 AO2 AO2 AO1	
(b)	**I** = change in momentum $\mathbf{I} = 2m(4\mathbf{i} + 5\mathbf{j}) - 2m(5\mathbf{i} + 4\mathbf{j})$ $\mathbf{I} = m(-2\mathbf{i} + 2\mathbf{j})$ $\mathbf{I} = 2m(-\mathbf{i} + \mathbf{j})$ (Ns)	M1 A1	AO3 AO1	used
(c)	Loss in KE = $\frac{1}{2}m(4 + 49) + \frac{1}{2}2\text{m}(25 + 16)$ $- \frac{1}{2} \times 3m(16 + 25)$ Loss in KE = $6m$ (J)	 M1 A1 **[13]**	 AO3 AO1	

Question Number	Solution	Mark	AO	Notes
6. (a)	At equilibrium			
	$12g = \dfrac{\lambda \times 0 \cdot 05}{0 \cdot 75}$	M1	AO3	use of Hooke's Law
	λ = <u>1764 (N)</u>	A1	AO1	
(b)	Consider a displacement x from the equilibrium position. Apply N2L			
	$12g - T = 12\,\ddot{x}$	M1	AO3	
	$12g - \dfrac{\lambda(0 \cdot 05 + x)}{0 \cdot 75} = 12\ \ddot{x}$	A1	AO3	ft λ
	$\ddot{x} = -(14)^2 x$			
	Therefore is SHM (with $\omega = 14$).	A1	AO2	
	Amplitude = <u>0.05 (m)</u>	B1	AO1	
	Period = $\dfrac{2\pi}{\omega} = \dfrac{\pi}{7}$ s	B1	AO1	
(c)	Maximum speed = $a\omega$	M1	AO3	used
	$= 0.05 \times 14$			
	= <u>0.7 (ms^{-1})</u>	A1	AO1	ft a
(d)	Use of $v^2 = \omega^2(a^2 - x^2)$			
	with $\omega = 14$, $a = 0.05$(c), $x = 0.03$	M1	AO3	
	$v^2 = 14^2(0.05^2 - 0.03^2)$	A1	AO2	ft a
	$=14^2 \times 0.04^2$			
	v = <u>0.56 (ms^{-1})</u>	A1	AO1	cao
(e)	Displacement from Origin = x			
	$x = 0.05\cos(14t)$	M1	AO3	(Accept ±)
	When $t = 1.6$			
	$x = 0.05\cos(14 \times 1.6)$	A1	AO2	ft a (Accept ±)
	$x = (-)$<u>0.046 (m)</u>	A1	AO1	cao
(f)	The seat is modelled as a particle.	B1	AO3	
	The spring is assumed to be light.	B1	AO3	
		[17]		

APPENDIX

ASSESSMENT OBJECTIVE WEIGHTINGS

GCE FURTHER MATHEMATICS

Level	AO1	AO2	AO3	TOTAL
AS	108	49	53	210
	51%	23%	25%	
Total mark for assessment objectives must be in the range	95 - 115	42 - 63	42 - 63	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

Level	AO1	AO2	AO3	TOTAL
A2 - Unit 4 & Unit 5	103	43	54	200
	52%	22%	27%	
Total mark for assessment objectives must be in the range	90 - 110	40 - 60	40 - 60	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

Level	AO1	AO2	AO3	TOTAL
A2 - Unit 4 & Unit 6	101	47	52	200
	51%	24%	26%	
Total mark for assessment objectives must be in the range	90 - 110	40 - 60	40 - 60	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

Level	AO1	AO2	AO3	TOTAL
A LEVEL (Unit 5 option)	211	92	107	410
	51%	22%	26%	
Total mark for assessment objectives must be in the range	185 - 225	82 - 123	82 - 123	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

Level	AO1	AO2	AO3	TOTAL
A LEVEL (Unit 6 option)	209	96	105	410
	51%	23%	26%	
Total mark for assessment objectives must be in the range	185 - 225	82 - 123	82 - 123	
	(45% - 55%)	(20% - 30%)	(20% - 30%)	

AS FURTHER MATHEMATICS
Unit 1: Further Pure Mathematics A (70 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	0	7	0	**7**
2	5	0	6	**11**
3	6	0	0	**6**
4	2	5	0	**7**
5	5	0	4	**9**
6	8	1	0	**9**
7	4	0	5	**9**
8	12	0	0	**12**
9				**0**
10				**0**
				0
				0
				0
				0
TOTAL	**42**	**13**	**15**	**70**
Total mark for assessment objectives must be in the range	39 - 45	11 - 17	11 - 17	

AS FURTHER MATHEMATICS
Unit 2: Further Statistics A (70 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	6	0	1	**7**
2	8	0	5	**13**
3	6	1	2	**9**
4	4	4	1	**9**
5	3	2	5	**10**
6	3	5	2	**10**
7	4	6	2	**12**
8				**0**
9				**0**
10				**0**
				0
				0
				0
				0
TOTAL	**34**	**18**	**18**	**70**
Total mark for assessment objectives must be in the range	28 - 35	16 - 22	16 - 22	

AS FURTHER MATHEMATICS
Unit 3: Further Mechanics A (70 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	2	3	7	**12**
2	7	1	4	**12**
3	4	3	2	**9**
4	8	5	0	**13**
5	2	2	2	**6**
6	5	2	2	**9**
7	4	2	3	**9**
8				**0**
9				**0**
10				**0**
				0
				0
				0
				0
TOTAL	**32**	**18**	**20**	**70**
Total mark for assessment objectives must be in the range	28 - 35	16 - 22	16 - 22	

A2 FURTHER MATHEMATICS
Unit 4: Further Pure Mathematics B (120 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	7	0	0	**7**
2	0	0	6	**6**
3	5	0	0	**5**
4	0	0	9	**9**
5	8	0	0	**8**
6	7	0	0	**7**
7	10	0	0	**10**
8	0	0	10	**10**
9	2	12	0	**14**
10	11	0	0	**11**
11	8	9	0	**17**
12	10	2	4	**16**
				0
				0
				0
				0
				0
				0
TOTAL	**68**	**23**	**29**	**120**
Total mark for assessment objectives must be in the range	63 - 74	20 - 31	20 - 31	

A2 FURTHER MATHEMATICS
Unit 5: Further Statistics B (80 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	6	1	6	**13**
2	7	2	2	**11**
3	2	3	4	**9**
4	6	2	4	**12**
5	6	2	2	**10**
6	3	1	4	**8**
7	5	9	3	**17**
8				**0**
9				**0**
10				**0**
				0
				0
				0
				0
TOTAL	**35**	**20**	**25**	**80**
Total mark for assessment objectives must be in the range	28 - 36	20 - 28	20 - 28	

A2 FURTHER MATHEMATICS
Unit 6: Further Mechanics B (80 marks)

Question Number	AO1	AO2	AO3	TOTAL
1	6	7	1	**14**
2	5	4	6	**15**
3	4	5	1	**10**
4	8	0	3	**11**
5	4	5	4	**13**
6	6	3	8	**17**
7				**0**
8				**0**
9				**0**
10				**0**
				0
				0
				0
				0
TOTAL	**33**	**24**	**23**	**80**
Total mark for assessment objectives must be in the range	28 - 36	20 - 28	20 - 28	