

973/01

**MATHEMATICS C1**

**Pure Mathematics**

A.M. MONDAY, 22 May 2006

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Calculators are **not** allowed for this paper.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points  $A, B, C, D$  have coordinates  $(3, 2), (-4, 3), (5, 6), (4, -1)$ , respectively.

(a) Show that the lines  $AC$  and  $BD$  are perpendicular. [5]

(b) Show that the line  $AC$  has equation

$$2x - y - 4 = 0$$

and find the equation of the line  $BD$ . [4]

(c) Find the coordinates of  $E$ , the point of intersection of  $AC$  and  $BD$ . [2]

(d) Find the length of  $AE$ . [2]

2. Simplify each of the following, expressing your answers in surd form.

(a)  $\frac{5 - \sqrt{3}}{\sqrt{3} + 1}$ , [4]

(b)  $(2 + \sqrt{3})(4 - \sqrt{12})$ . [4]

3. The curve  $C$  has equation  $y = x^2 - 4x + 7$ . The point  $A$  has coordinates  $(1, 4)$ .

(a) Find the equation of the tangent to  $C$  at  $A$ . [4]

(b) Find the equation of the normal to  $C$  at the point  $A$ . [2]

4. (a) Given that the equation

$$kx^2 - 4x + k - 3 = 0$$

has equal roots, find the values of  $k$ . [5]

(b) Express  $x^2 + 8x + 2$  in the form  $(x + a)^2 + b$ . Hence write down the least value of  $x^2 + 8x + 2$ . [3]

5. The polynomial

$$f(x) \equiv px^3 - x^2 + qx - 6$$

has  $x - 3$  as a factor. When  $f(x)$  is divided by  $x - 2$ , the remainder is  $-20$ .

(a) Show that  $p = 2$  and find the value of  $q$ . [6]

(b) Factorise  $f(x)$ . [3]

6. (a) Expand  $(a + b)^4$ . Hence expand  $\left(3x - \frac{1}{3x}\right)^4$ , simplifying each term of the expansion. [4]  
 (b) The coefficient of  $x^2$  in the expansion of  $(1 + 2x)^n$  is 40. Given that  $n$  is a positive integer, find the value of  $n$ . [2]

7. (a) Given that  $y = x^2 - 3x + 4$ , show from first principles that

$$\frac{dy}{dx} = 2x - 3. \quad [5]$$

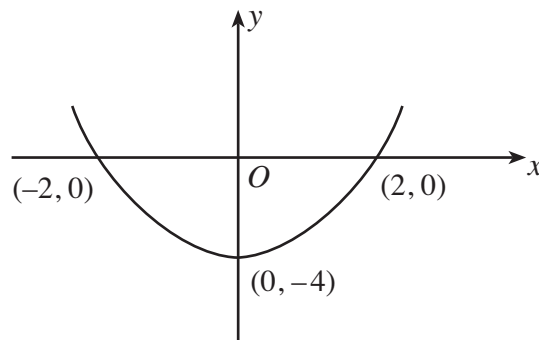
- (b) Differentiate  $\frac{2}{x^2} + 7\sqrt{x}$  with respect to  $x$ . [2]

8. Solve the following inequalities.

(a)  $1 - 5x < x + 8$  [2]

(b)  $(x + 8)(x + 1) < 3x$  [4]

9.



The diagram shows the graph of  $y = f(x)$ . The curve passes through the points  $(2, 0)$  and  $(-2, 0)$ , and has a minimum point at  $(0, -4)$ .

Sketch on separate diagrams the graphs of

(a)  $y = f(x) + 4$ , [2]

(b)  $y = f(x + 2)$ , [3]

indicating the coordinates of the points of intersection with the  $x$ -axis and the coordinates of the stationary points.

10. The curve  $C$  has equation

$$y = x^3 - 3x^2 - 9x + 2.$$

Find the coordinates of the stationary points of  $C$  and determine the nature of each of these stationary points. [7]