

974/01

MATHEMATICS C2

Pure Mathematics

P.M. MONDAY, 10 January 2005

(1½ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for

$$\int_0^1 \sqrt{1+x^3} \, dx.$$

Show your working and give your answer correct to two decimal places. [4]

2. Use the substitution $3^x = u$ to solve the equation

$$3^{2x} - 3^{x+2} + 14 = 0,$$

giving your answers correct to three decimal places. [6]

3. (a) Find all values of x in the range $0^\circ \leq x \leq 360^\circ$ satisfying

$$2 \sin^2 x + \cos x - 1 = 0. [6]$$

- (b) Find all values of x in the range 0° to 180° satisfying

$$\tan 3x = 1. [4]$$

4. The lengths of the three sides of a triangle are 8.5 cm, 6.8 cm and 9.4 cm. Find, correct to one decimal place,

(a) the largest angle of the triangle,

(b) the area of the triangle. [5]

5. In an arithmetic series the sum of the first term and the fifth term is zero. The thirteenth term is 20.

(a) Find the first term and the common difference of the series. [5]

(b) Calculate the sum of the first twenty terms of the series. [2]

6. The circles C_1 and C_2 are given

by $(x + 1)^2 + (y + 2)^2 = 25$

and $x^2 + y^2 - 10x - 5y + 25 = 0$, respectively.

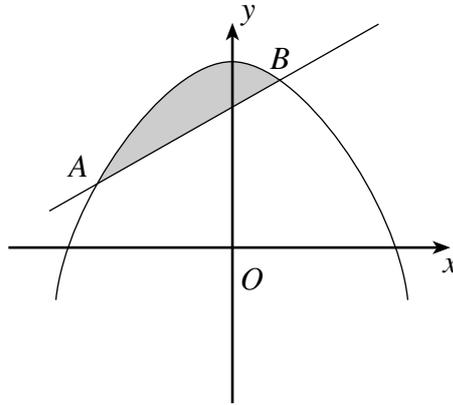
(a) Write down the radius and the coordinates of the centre of C_1 . [2]

(b) Find the radius and the coordinates of the centre of C_2 . [3]

(c) Show that C_1 and C_2 touch. [3]

7. (a) Integrate $3\sqrt{x} - \frac{6}{x^3}$ with respect to x . [2]

(b)



The diagram shows a sketch of the curve $y = 16 - x^2$ and the line $y = 2x + 13$. The line and the curve intersect at the points A and B .

- (i) Find the coordinates of A and B . [3]
 (ii) Find the area of the shaded region. [7]

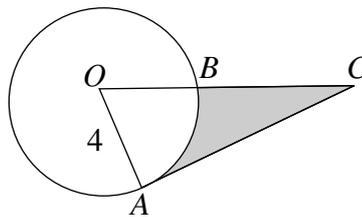
8. (a) A geometric series has first term a and common ratio r . Prove that the sum of the first n terms is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}.$$

Given that $|r| < 1$, write down the sum to infinity of the series. [4]

- (b) The sum to infinity of a geometric series is equal to 4. The sum of the first two terms of the series is 3. Find the common ratio, given that it is positive. [5]

9.



In the diagram the points A and B lie on a circle with centre O and radius 4 cm. The tangent to the circle at A intersects the line OB produced at the point C . The length of the arc AB is 3 cm.

- (a) Find \widehat{AOB} in radians. [2]
 (b) Calculate the area of the shaded region. Give your answer correct to two decimal places. [4]

10. (a) Show that if $x > 0$,

$$\log_a x^k = k \log_a x. \quad [3]$$

- (b) Solve the equation

$$\log_{10} (x^2 + 48) = \log_{10} x + 2 \log_{10} 4. \quad [5]$$