

974/01

**MATHEMATICS C2**

**Pure Mathematics**

P.M. WEDNESDAY, 10 January 2007

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for

$$\int_1^2 \sqrt{2+x^3} \, dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. (a) Find the values of  $x$  in the range  $0^\circ \leq x \leq 360^\circ$  satisfying

$$10 \sin^2 x - 3 \sin x = 4 \cos^2 x + 1. \quad [6]$$

- (b) Find the values of  $x$  in the range  $0^\circ \leq x \leq 180^\circ$  satisfying

$$\tan(2x + 30^\circ) = \sqrt{3}. \quad [3]$$

3. (a) A geometric series has first term  $a$  and common ratio  $r$ . Write down the  $n$ th term and prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

Given that  $|r| < 1$ , write down the sum to infinity of the series. [5]

- (b) The sum of the first term and the second term of a geometric series is equal to twice the sum of the second term and the third term of the series.

(i) Given that the common ratio of the series is positive, find the value of the common ratio. [4]

(ii) The sum to infinity of the series is 12. Find, correct to two decimal places, the sum of the first eight terms of the series. [4]

4. In an arithmetic series, the eighth term is twice the third term. The twentieth term of the series is 11. Find the common difference and the first term of the series. [5]

5. A circle  $C_1$  with centre  $A$  has equation

$$x^2 + y^2 - 6x + 8y - 75 = 0.$$

- (a) Find the coordinates of  $A$  and the radius of  $C_1$ . [3]

- (b) A second circle  $C_2$  has centre  $B(-6, 8)$  and radius 5.

(i) Show that  $C_1$  and  $C_2$  touch.

(ii) Given that the circles touch at the point  $P(-3, 4)$ , find the equation of the common tangent. [7]

6. The triangle  $ABC$  is such that  $AB = 6$  cm,  $AC = 10$  cm and  $\widehat{BAC}$  is an **obtuse** angle. The area of triangle  $ABC$  is  $15\sqrt{3}$  cm<sup>2</sup>.

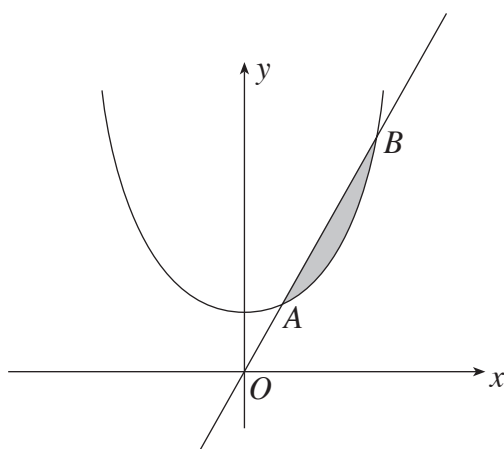
(a) Find the size of  $\widehat{BAC}$ . [3]

(b) Calculate the length of  $BC$ . [3]

7. (a) Find

$$\int \left( \sqrt{x} + \frac{2}{x^2} \right) dx. \quad [2]$$

(b)



The diagram shows a sketch of the curve  $y = x^2 + 3$  and the line  $y = 4x$ . The line and the curve intersect at the points  $A$  and  $B$ .

(i) Showing your working, find the coordinates of  $A$  and  $B$ .

(ii) Evaluate the area of the shaded region. [10]

8. (a) Given that  $x > 0$ ,  $y > 0$ , show that  $\log_a(xy) = \log_a x + \log_a y$ . [3]

(b) Express  $\log_a 36 + \frac{1}{2} \log_a 256 - 2 \log_a 48$  as a single logarithm. [4]

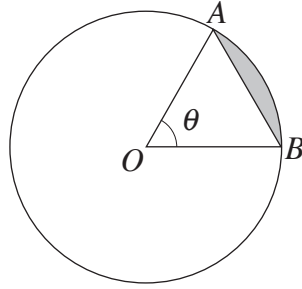
(c) Solve the equation

$$2^{x+1} = 5,$$

giving your answer correct to three decimal places. [2]

**TURN OVER.**

9.



The diagram shows two points  $A$  and  $B$  on a circle with centre  $O$  and radius 3 cm, such that  $\widehat{AOB} = \theta$  radians. The perimeter of the **sector**  $AOB$  is 10 cm.

- (a) Find the value of  $\theta$ . [3]
- (b) Find the area of the shaded segment, giving your answer correct to three decimal places. [4]