



**GCE AS/A level**

**974/01**

**MATHEMATICS C2**

**Pure Mathematics**

A.M. TUESDAY, 13 January 2009

**1½ hours**

#### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

#### **INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

#### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^1 \frac{1}{1+x^4} dx .$$

Show your working and give your answer correct to three decimal places.

[4]

2. (a) Find all values of  $\theta$  between  $0^\circ$  and  $360^\circ$  satisfying

$$6 \cos^2 \theta + \sin \theta = 4.$$

[6]

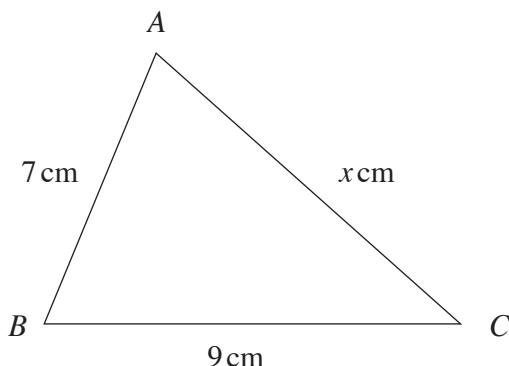
- (b) Find all values of  $x$  between  $0^\circ$  and  $180^\circ$  satisfying

$$\tan 3x = -1.54.$$

[3]

3. The diagram below shows a sketch of the triangle  $ABC$  with  $AB = 7 \text{ cm}$ ,  $AC = x \text{ cm}$ ,  $BC = 9 \text{ cm}$  and

$$\cos B\hat{A}C = \frac{2}{7} .$$



- (a) Write down and simplify a quadratic equation satisfied by  $x$ . Hence evaluate  $x$ .

[3]

- (b) (i) Express  $\sin B\hat{A}C$  in the form  $\frac{\sqrt{m}}{n}$ , where  $m, n$  are integers whose values are to be found.

- (ii) Express  $\sin A\hat{C}B$  in the form  $\frac{\sqrt{p}}{3}$ , where  $p$  is an integer whose value is to be found. [4]

4. (a) The thirteenth term of an arithmetic series is 51. The ninth term of the series is five times the second term. Find the first term and common difference of the arithmetic series. [5]
- (b) The first term of another arithmetic series is 5 and the twentieth term is 62. Find the sum of the first twenty terms of this arithmetic series. [2]
5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r} . \quad [3]$$

- (b) Find the sum of the first eighteen terms of the geometric series

$$10 + 9 + 8\cdot 1 + \dots$$

Give your answer correct to the nearest whole number. [3]

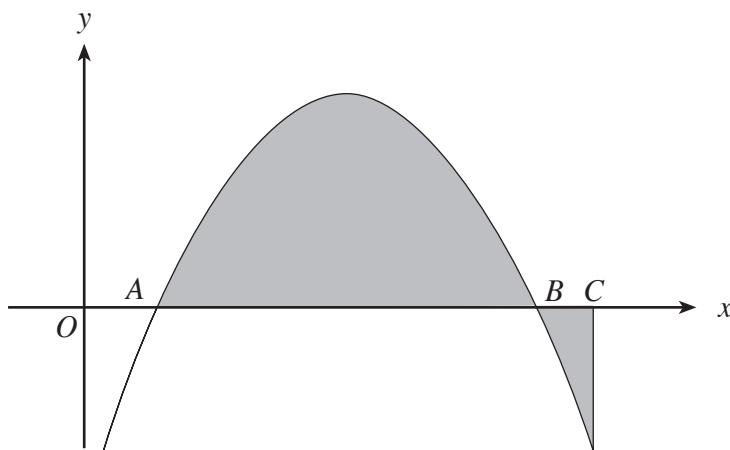
- (c) The second term of another geometric series is  $-4$ . The sum to infinity of the series is 9.
- (i) Show that  $r$ , the common ratio of the series, satisfies the equation

$$9r^2 - 9r - 4 = 0.$$

- (ii) Find the value of  $r$ , giving a reason for your answer. [6]

6. (a) Find  $\int \left( \frac{3}{x^2} - 2\sqrt{x} \right) dx$ . [2]

- (b)



The diagram shows a sketch of the curve  $y = 5x - 4 - x^2$ .

The curve intersects the  $x$ -axis at the points  $A$  and  $B$ . The point  $C$  has coordinates  $(5, 0)$ .

- (i) Find the  $x$ -coordinates of the points  $A$  and  $B$ . [3]
- (ii) Find the **total** area of the shaded regions. [7]

# TURN OVER

7. (a) Given that  $x > 0, y > 0$ , show that

$$\log_a xy = \log_a x + \log_a y. \quad [3]$$

- (b) Solve the equation

$$\log_9 x = -\frac{1}{2}. \quad [2]$$

- (c) Solve the equation

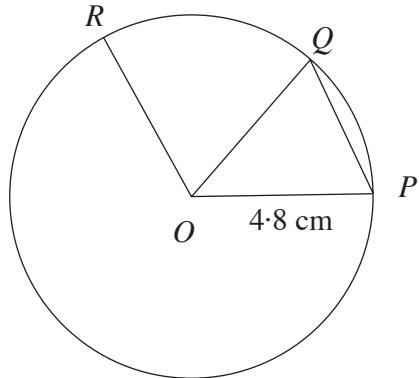
$$\log_a(4x + 7) = \log_a x + 2\log_a 3. \quad [4]$$

8. The circle  $C_1$  has centre  $A$  and equation

$$x^2 + y^2 + 4x - 2y - 20 = 0.$$

- (a) Find the coordinates of  $A$  and the radius of  $C_1$ . [3]
- (b) The line  $L$  has equation  $y = -x + 6$ . Find the coordinates of the points of intersection of  $L$  and  $C_1$ . [4]
- (c) The circle  $C_2$  has centre  $(10, 6)$  and radius  $r$ . Given that  $C_1$  and  $C_2$  touch externally, find the value of  $r$ . [3]

9.



The diagram shows three points  $P, Q$  and  $R$  on a circle with centre  $O$  and radius 4.8 cm.

- (a) Given that  $\hat{POQ} = 0.7$  radians, find the area of triangle  $POQ$ . Give your answer correct to two decimal places. [2]
- (b) The length of the arc  $RQ$  is  $L$  cm and the area of sector  $ROQ$  is  $A$  cm<sup>2</sup>. Given that  $A = kL$ , find the value of  $k$ . [3]