

974/01

**MATHEMATICS C2**

**Pure Mathematics**

A.M. MONDAY, 22 May 2006

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^{0.4} \sqrt{1+x^4} \, dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. (a) Find all values of  $x$  between  $0^\circ$  and  $360^\circ$  satisfying

$$\tan x = -0.4. \quad [2]$$

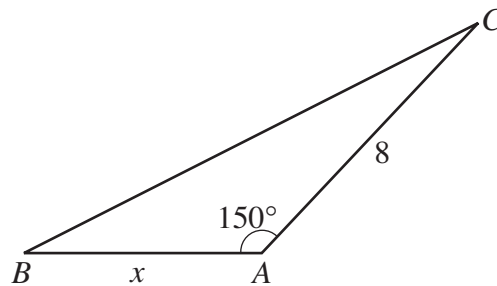
- (b) Find all values of  $x$  between  $0^\circ$  and  $180^\circ$  satisfying

$$\cos 3x = \frac{1}{2}. \quad [4]$$

- (c) Find all values of  $\theta$  between  $0^\circ$  and  $360^\circ$  satisfying

$$2 \cos^2 \theta + 3 \sin \theta = 0. \quad [5]$$

3. The diagram below shows the triangle  $ABC$  with  $AB = x$  cm,  $AC = 8$  cm and  $\widehat{BAC} = 150^\circ$ .



Given that the area of the triangle  $ABC$  is  $10 \text{ cm}^2$ ,

- (a) find  $x$ , [3]  
 (b) calculate the length of the longest side of the triangle  $ABC$ , giving your answer correct to two decimal places. [3]

4. (a) An arithmetic series has first term  $a$  and common difference  $d$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{n}{2} [2a + (n-1)d]. \quad [3]$$

- (b) The sum of the first twenty terms of an arithmetic series is 540 and the sum of the first thirty terms of the series is 1260.

- (i) Find the first term and the common difference of the series.  
 (ii) Calculate the **50th term** of the series. [6]

5. The second term of a geometric series is nine times the fourth term of the series.

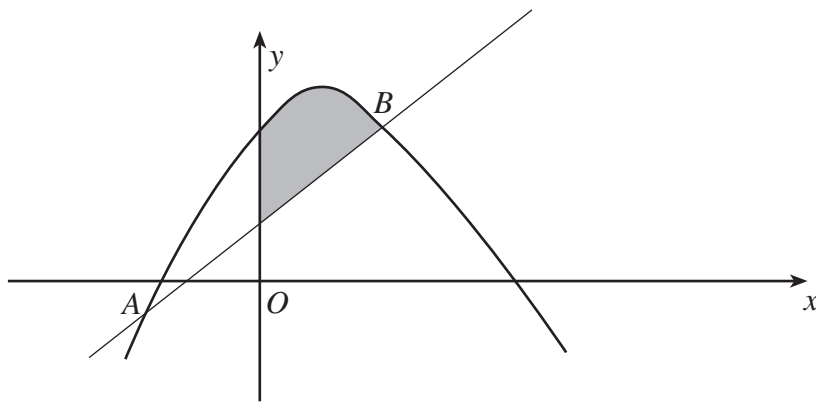
(a) Find the possible values of the common ratio. [4]

(b) Given that the common ratio is positive and the sum to infinity of the series is 12, find the third term of the series. [3]

6. Find

$$\int \left( 4x^{\frac{1}{3}} - 3x^{-3} + 5 \right) dx. \quad [3]$$

7.



The diagram shows the curve  $y = 7 + 2x - x^2$  and the line  $y = x + 1$  intersecting at the points  $A$  and  $B$ .

(a) Find the coordinates of  $B$ . [4]

(b) Evaluate the area of the shaded region. [8]

8. (a) Given that  $x > 0$ , show that

$$\log_a (x^n) = n \log_a x. \quad [3]$$

(b) Solve the equation

$$5^{3x+1} = 6,$$

giving your answer correct to four decimal places. [4]

**TURN OVER.**

9. A circle  $C$  has centre  $D$  and equation

$$x^2 + y^2 + 2x - 8y + 8 = 0.$$

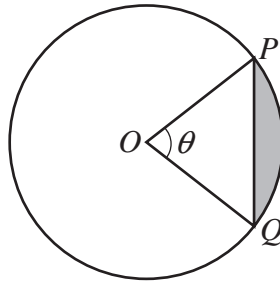
(a) Find the coordinates of  $D$  and the radius of  $C$ . [3]

(b) A line is drawn through the point  $P(4, 6)$  so that it touches the circle  $C$  at the point  $T$ .

(i) Show that  $PT = \sqrt{20}$ .

(ii) Find the equation of the circle centre  $P$  which passes through the point  $T$ . [5]

10.



The diagram shows two points  $P$  and  $Q$  on a circle, with centre  $O$  and radius 4 cm, such that  $\widehat{POQ} = \theta$  radians. The perimeter and the area of the **sector**  $POQ$  are  $x$  cm and  $A$  cm<sup>2</sup>, respectively. Given that  $x = 3A$ ,

(a) show that  $\theta = 0.4$ , [4]

(b) calculate the area of the shaded region, giving your answer correct to three decimal places. [4]