

974/01

**MATHEMATICS C2**

**Pure Mathematics**

A.M. MONDAY, 21 May 2007

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + \sin x} \, dx,$$

giving the value correct to three decimal places.

[4]

2. (a) Find all values of  $x$  between  $0^\circ$  and  $180^\circ$  satisfying

$$\tan 3x = \sqrt{3}.$$

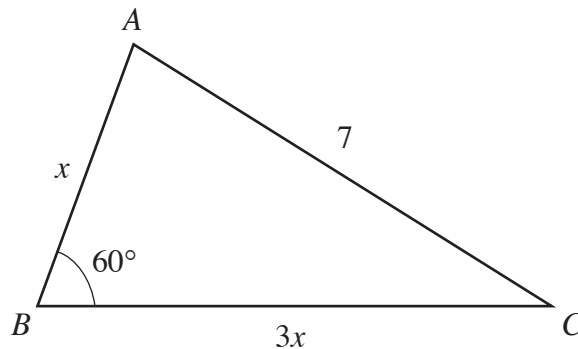
[4]

- (b) Find all values of  $\theta$  in the interval  $0^\circ$  to  $360^\circ$  satisfying

$$4\cos^2\theta - \cos\theta = 2\sin^2\theta.$$

[6]

3. The diagram below shows the triangle  $ABC$  with  $AB = x$  cm,  $BC = 3x$  cm,  $AC = 7$  cm and  $\hat{ABC} = 60^\circ$ .



- (a) Show that  $x = \sqrt{7}$ .

[3]

- (b) Find  $\hat{ACB}$ .

[2]

4. The third term of an arithmetic series is four times the sixth term of the series. The sum of the first twenty terms of the series is 350.

- (a) Find the first term and the common difference of the series.

[6]

- (b) Given that the  $n$ th term of the series is 125, find the value of  $n$ .

[2]

5. (a) A geometric series has first term  $a$  and common ratio  $r$ . Prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

Given that  $|r| < 1$ , write down the sum to infinity of the series. [4]

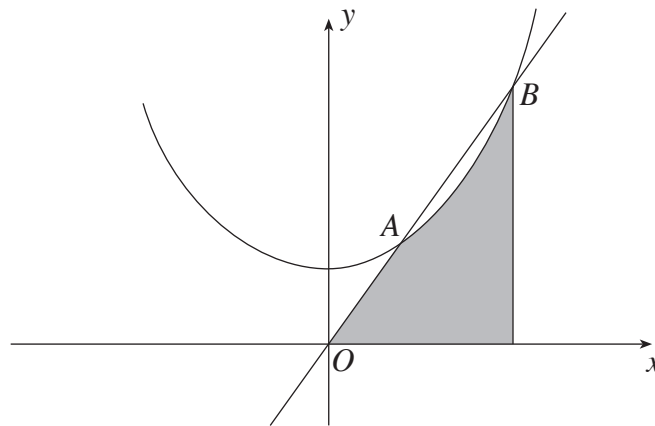
- (b) The sum to infinity of a geometric series with first term  $a$  and common ratio  $r$  is 10. The sum to infinity of a second geometric series with first term  $a$  and common ratio  $2r$  is 15.

(i) Find the value of  $r$ . [4]

(ii) Find the sum of the first four terms of the **first** series, giving your answer correct to two decimal places. [3]

6. (a) Find  $\int \left( 2x^{\frac{3}{2}} + \frac{9}{x^4} \right) dx$ . [2]

(b)



The diagram shows a sketch of the curve  $y = x^2 + 2$  and the line  $y = 3x$ . The line and the curve intersect at the points  $A$  and  $B$ .

(i) Find the coordinates of the points  $A$  and  $B$ . [4]

(ii) Evaluate the area of the shaded region. [7]

7. (a) (i) Given that  $p > 0$ ,  $q > 0$ , show that  $\log_a pq = \log_a p + \log_a q$ .

(ii) Given that

$$\log_a x + \log_a (3x + 4) = 2 \log_a (3x - 4), \text{ where } x > \frac{4}{3},$$

find the value of  $x$ . [8]

- (b) Solve  $3^x = 11$ , giving your answer correct to three decimal places. [2]

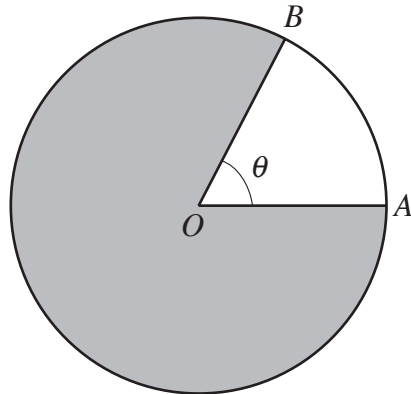
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8. The circle  $x^2 + y^2 + 4x - 16y + 18 = 0$  has centre  $A$  and radius  $r$ .

(a) Find the coordinates of  $A$  and the value of  $r$ . [3]

(b) The line  $y = x + 2$  and the circle  $x^2 + y^2 + 4x - 16y + 18 = 0$  intersect at the points  $B$  and  $C$ . Find the coordinates of  $B$  and  $C$ . [4]

9.



The diagram shows two points  $A$  and  $B$  on a circle, with centre  $O$  and radius 6 cm, such that  $\widehat{AOB} = \theta$  radians. Given that the circumference of the circle is 24 cm more than twice the length of the arc  $AB$ ,

(a) show that  $\theta = \pi - 2$ , [4]

(b) calculate the area of the shaded region. [3]