WJEC
CBAC

GCE AS/A level

974/01

MATHEMATICS C2

Pure Mathematics

A.M. THURSDAY, 27 May 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^2 \sqrt{1+\frac{1}{x}}\ \mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$12\cos^2\theta - 5\sin\theta = 10.$$ [6]

(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan 2x = -1{\cdot}6.$$ [2]

(c) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 180°$ satisfying

$$\tan\phi + 2\sin\phi = 0.$$ [4]

3. *(a)* The triangle ABC is such that $AB = 11$ cm and $B\widehat{A}C = 110°$. Given that the area of the triangle ABC is 31 cm^2, find the length of BC. [4]

(b) The triangle XYZ is such that $XY = 2$ cm, $YZ = (2\sqrt{3} - 1)$ cm and $Y\widehat{X}Z = 60°$. Find an expression for $\sin X\widehat{Z}Y$ in the form $\frac{m+\sqrt{3}}{n}$, where m, n are integers whose values are to be found. [3]

4. Find $\int \left(3\sqrt{x} - \frac{6}{x^4} - 1\right) \mathrm{d}x.$ [3]

5. *(a)* An arithmetic series has first term a and common difference d.
Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}[2a + (n-1)d].$$ [3]

(b) The first term of an arithmetic series is 4 and the common difference is 2. The sum of the first n terms of the arithmetic series is 460.
Write down an equation satisfied by n. Hence find the value of n. [3]

(c) The fifth term of another arithmetic series is 9. The sum of the sixth term and the tenth term of this series is 42. Find the first term and the common difference of the arithmetic series. [5]

6. *(a)* Find the sum to infinity of the geometric series

$$40 - 24 + 14{\cdot}4 - \ldots$$ [3]

(b) Another geometric series has first term a and common ratio r. The fourth term of this geometric series is 8. The sum of the third, fourth and fifth terms of the series is 28.

(i) Show that r satisfies the equation

$$2r^2 - 5r + 2 = 0.$$

(ii) Given that $|r| < 1$, find the value of r and the corresponding value of a. [6]

7. The region R is bounded by the curve $y = 3x + \frac{1}{5}x^3$, the x-axis and the lines $x = 1$, $x = 3$. Find the area of R. [5]

8. *(a)* Given that $x > 0$, show that

$$\log_a x^n = n \log_a x.$$ [3]

(b) Solve the equation

$$6^{2y-1} = 4.$$

Show your working and give your answer correct to three decimal places. [3]

(c) Given that $\log_a 4 = \frac{1}{2}$, find the value of a. [2]

9. The circle C has centre A and equation

$$x^2 + y^2 - 8x + 2y + 7 = 0.$$

(a) Find the coordinates of A and the radius of C. [3]

(b) The point P has coordinates $(7, -2)$.

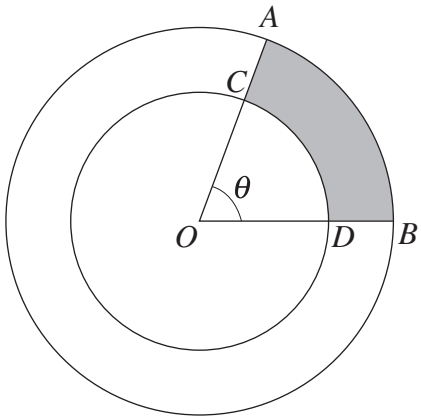
(i) Verify that P lies on C.

(ii) Given that the point Q is such that PQ is a diameter of C, find the coordinates of Q. [4]

(c) The line L has equation $y = 2x - 4$. Find the coordinates of the points of intersection of L and C. [4]

TURN OVER

10.



The diagram shows two concentric circles with common centre O. The radius of the larger circle is R cm and the radius of the smaller circle is r cm. The points A and B lie on the larger circle and are such that $A\hat{O}B = \theta$ radians. The smaller circle cuts OA and OB at the points C and D respectively. The length of the arc AB is L cm **greater** than the length of the arc CD. The area of the shaded region is K cm^2.

(a) (i) Write down an expression for L in terms of R, r and θ.

(ii) Write down an expression for K in terms of R, r and θ. [2]

(b) Use your results to part *(a)* to find an expression for r in terms of R, K and L. [3]