WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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975/01

MATHEMATICS C3

Pure Mathematics

P.M. WEDNESDAY, 24 May 2006

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_1^2 \sqrt{\ln x}\, dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\cos(a+b) \equiv \cos a + \cos b$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$7 - \sec^2\theta = \tan^2\theta + \tan\theta\,.$$ [6]

3. *(a)* Given that $x = \cos t$, $y = \sin 2t$, find $\frac{dy}{dx}$ in terms of t. [4]

(b) Given that

$$x^4 + 2x^2y + y^2 = 21,$$

find $\frac{dy}{dx}$ in terms of x and y. [4]

4. *(a)* (i) Find $\int_0^a (e^{2x} - 1)\, dx$.

(ii) Given that

$$\int_0^a (e^{2x} - 1)\, dx = \frac{1}{2}(9 - a)$$

show that

$$e^{2a} - a - 10 = 0.$$ [4]

(b) Show that the equation

$$e^{2a} - a - 10 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$a_{n+1} = \frac{1}{2}\ln(a_n + 10)$$

with $a_0 = 1{\cdot}2$ can be used to find α. Find and record the values of a_1, a_2, a_3, a_4.

Write down the value of a_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

5. *(a)* Differentiate each of the following with respect to x,

(i) $\tan^{-1} 4x$ (ii) $\ln(1+x^2)$ (iii) $x^2 e^{3x}$ [7]

(b) By first writing $\cot x = \dfrac{\cos x}{\sin x}$, show that $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [3]

6. Solve the following.

(a) $3|x| + 4 = 6 - 2|x|$ [2]

(b) $|7x - 5| \geqslant 3$ [3]

7. *(a)* Find (i) $\displaystyle\int \frac{7}{(5x+2)^4}\,dx$, (ii) $\displaystyle\int \frac{2}{(8x+7)}\,dx$. [4]

(b) Evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos 3x\,dx$. [4]

8. The function f has domain $x \geqslant 1$ and is defined by

$$f(x) = x - \frac{1}{x}.$$

(a) Show that $f'(x)$ is always positive. Deduce the least value of $f(x)$. [3]

(b) Find the range of f. [1]

(c) The function g has domain $[0, \infty)$ and is defined by

$$g(x) = 3x^2 + 2.$$

Solve the equation

$$gf(x) = \frac{3}{x^2} + 8.$$ [4]

9. Given that $f(x) = e^x$, sketch the graphs of $y = f(x)$ and $y = 2f(x) - 1$ on the same diagram. Label the coordinates of the points of intersection with the y-axis and indicate the behaviour of the graphs for large positive and negative values of x. [5]

10. The function f has domain $[0, \infty)$ and is defined by

$$f(x) = \sqrt{x+1}.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Write down the domain and range of f^{-1}. [2]

(c) Sketch the graph of $y = f^{-1}(x)$. Using the same diagram, sketch the graph of $y = f(x)$. [3]