GCE AS/A level

975/01

# MATHEMATICS C3
## PURE MATHEMATICS

P.M. WEDNESDAY, 9 June 2010

$1\frac{1}{2}$ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_0^{0\cdot8} \frac{1}{1+\mathrm{e}^{2x}}\,\mathrm{d}x.$$

Show your working and give your answer correct to four decimal places. [4]

**2.** *(a)* Show, by counter-example, that the statement

$$\cos\theta + \cos4\theta \equiv \cos2\theta + \cos3\theta$$

is false. [2]

*(b)* Find all values of $\theta$ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\tan^2\theta = \sec\theta + 8.$$ [6]

**3.** *(a)* Given that

$$y^4 + 4x^2y = 3x^3 - 5x,$$

find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$. [4]

*(b)* Given that $x = 4t + \cos 2t$, $y = \sin 3t$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{2}}$ when $t = \frac{\pi}{12}$. [5]

**4.** Show that the equation

$$4x^3 - 2x - 5 = 0$$

has a root $\alpha$ between 1 and 2.

The recurrence relation

$$x_{n+1} = \left(\frac{2x_n + 5}{4}\right)^{\frac{1}{3}},$$

with $x_0 = 1\cdot2$, may be used to find $\alpha$. Find and record the values of $x_1, x_2, x_3, x_4$. Write down the value of $x_4$ correct to five decimal places and prove that this value is the value of $\alpha$ correct to five decimal places. [7]

**5.** *(a)* Differentiate **each** of the following with respect to $x$, simplifying your answer wherever possible.

(i) $(7+2x)^{13}$ (ii) $\sin^{-1}5x$ (iii) $x^3e^{4x}$ [7]

*(b)* By first writing $\tan x = \dfrac{\sin x}{\cos x}$, show that

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

[3]

**6.** *(a)* Find

(i) $\displaystyle\int \sqrt{7x-9}\ dx$, (ii) $\displaystyle\int e^{\frac{x}{6}}\ dx$, (iii) $\displaystyle\int \frac{4}{5x-1}\ dx$. [6]

*(b)* Evaluate $\displaystyle\int_2^4 \frac{8}{(3x-4)^3}\ dx$. [4]

**7.** *(a)* Solve the inequality $|3x+1| \leqslant 5$. [3]

*(b)* The function $f$ is defined by $f(x) = |x|$.

(i) Sketch the graph of $y = f(x)$.

(ii) On a separate set of axes, sketch the graph of $y = f(x-3)+2$. On your sketch, indicate the coordinates of the point on the graph where the value of the $y$-coordinate is least and the coordinates of the point where the graph crosses the $y$-axis. [4]

**8.** The function $g$ is defined by $g(x) = 3\ln(4x^2+9)+2x-7$.

*(a)* Show that $g'(x) = \dfrac{2(2x+3)^2}{4x^2+9}$. [3]

*(b)* (i) Show that the graph of $y = g(x)$ has one stationary point.

(ii) Find the nature of this stationary point. [4]

# TURN OVER

**9.** The function $f$ has domain $[1, \infty)$ and is defined by

$$f(x) = \ln(3x - 2) + 5 .$$

*(a)* Find an expression for $f^{-1}(x)$. [4]

*(b)* State the domain of $f^{-1}$. [1]

**10.** The functions $f$ and $g$ have domains $[-3, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \sqrt{x+4} ,$$
$$g(x) = 2x^2 - 3.$$

*(a)* Write down the range of $f$ and the range of $g$. [2]

*(b)* Find an expression for $gf(x)$. Simplify your answer. [2]

*(c)* Solve the equation $fg(x) = 17$. [4]