



**GCE AS/A level**

0975/01

**MATHEMATICS – C3**  
**Pure Mathematics**

A.M. FRIDAY, 24 May 2013

1½ hours

#### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

#### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

#### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \ln(x^3 + 6) dx.$$

Show your working and give your answer correct to three decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_1^3 \ln \sqrt{x^3 + 6} dx. \quad [1]$$

2. (a) Find all values of  $\theta$  in the range  $0^\circ \leq \theta \leq 360^\circ$  satisfying

$$4\cot^2\theta - 8 = 2 \operatorname{cosec}^2\theta - 5 \operatorname{cosec}\theta. \quad [6]$$

- (b) Find all values of  $\phi$  in the range  $0^\circ \leq \phi \leq 360^\circ$  satisfying

$$\sec\phi + 2\tan\phi = 0. \quad [3]$$

3. The curve  $C$  is defined by

$$x^3y^2 = 128.$$

- (a) Find an expression for  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [3]

The point  $P$  lies on  $C$  and has coordinates  $(a, b)$ .

- (b) Given that the value of  $\frac{dy}{dx}$  at the point  $P$  is 3,

(i) show that  $b = -2a$ ,

- (ii) find the value of  $a$  and the value of  $b$ . [4]

4. Given that, for  $t > 0$ ,

$$x = \ln t, y = 5t^4,$$

- (a) find and simplify an expression for  $\frac{dy}{dx}$  in terms of  $t$ , [4]

- (b) find the value of  $t$  for which  $\frac{d^2y}{dx^2} = 0.648$ . [4]

5. Differentiate each of the following with respect to  $x$ , simplifying your answer wherever possible.

(a)  $(7 - 9x^2)^5$  (b)  $\tan^{-1} 6x$  [2], [2]

(c)  $e^{4x} \tan 2x$  (d)  $\frac{3 + \sin x}{2 + \cos x}$  [3], [3]

6. (a) Find

(i)  $\int \cos\left(3x + \frac{\pi}{2}\right) dx$ , (ii)  $\int e^{3-4x} dx$ ,

(iii)  $\int \frac{7}{8x+5} dx$ . [6]

(b) Evaluate  $\int_1^2 \frac{9}{(2x-1)^4} dx$ . [4]

7. (a) Show, by counter-example, that the statement

$$\text{'If } |a + 1| = |b + 1|, \text{ then } a = b\text{'}$$

is false. [2]

- (b) Solve the inequality

$$|x^2 - 10| \leq 6. \quad [4]$$

8. You may assume that the equation

$$x^2 + e^x - 3 = 0$$

has a root  $\alpha$  between  $-2$  and  $-1$ .

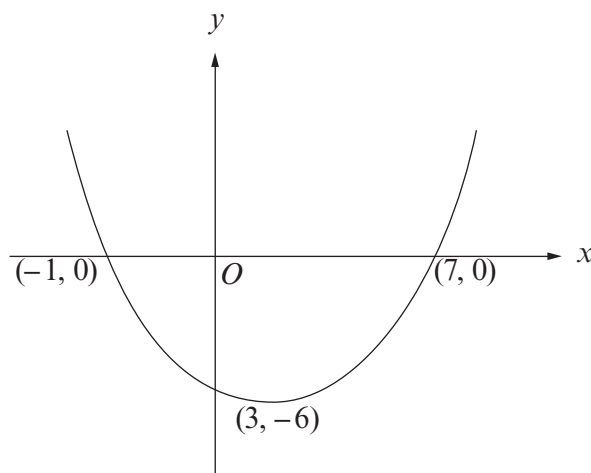
The recurrence relation

$$x_{n+1} = -(3 - e^{x_n})^{\frac{1}{2}}$$

with  $x_0 = -1.5$  can be used to find  $\alpha$ . Find and record the values of  $x_1, x_2, x_3, x_4$ . Write down the value of  $x_4$  correct to five decimal places and prove that this is the value of  $\alpha$  correct to five decimal places. [5]

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9. The diagram shows a sketch of the graph of  $y = f(x)$ . The graph passes through the points  $(-1, 0)$  and  $(7, 0)$  and has a minimum point at  $(3, -6)$ .



Sketch the graph of  $y = -\frac{2}{3}f(x+4)$ , indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the  $x$ -axis. [3]

10. The function  $f$  has domain  $(-\infty, 10]$  and is defined by

$$f(x) = e^{5-\frac{x}{2}} + 6.$$

- (a) Find an expression for  $f^{-1}(x)$ . [4]  
 (b) Write down the domain of  $f^{-1}$ . [2]

11. The functions  $f$  and  $g$  have domains  $(0, \infty)$  and  $(0, \frac{\pi}{4}]$  respectively and are defined by

$$\begin{aligned} f(x) &= \ln x, \\ g(x) &= \tan x. \end{aligned}$$

- (a) (i) Write down the domain of  $fg$ .  
 (ii) Write down the range of  $fg$ . [3]
- (b) (i) Solve the equation  $fg(x) = -0.4$ . Give your answer correct to two decimal places.  
 (ii) Giving a reason, write down a value for  $k$  so that  $fg(x) = k$  has no solution. [3]