WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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977/01

MATHEMATICS FP1

Further Pure Mathematics

P.M. MONDAY, 23 January 2006

($1\frac{1}{2}$ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Express the complex number $(\sqrt{3}+\mathrm{i})$ in trigonometric form. Hence find the smallest positive integer n such that $(\sqrt{3}+\mathrm{i})^n$ is a real number. [7]

2. Show that the following matrix is non-singular for all values of the real constant λ.

$$\begin{bmatrix} 1 & -2 & \lambda \\ \lambda & 2 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

[6]

3. Differentiate $\dfrac{1}{1-x^2}$ from first principles. [6]

4. The complex number z and its complex conjugate $\bar{z}$ satisfy the equation

$$2z+\bar{z}=\frac{11+7\mathrm{i}}{1+\mathrm{i}}.$$

Find z in the form $x + \mathrm{i}y$. [6]

5. The transformations T_1 and T_2 in the plane are defined as follows.
T_1 : A translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$.
T_2 : An anti-clockwise rotation through $\dfrac{\pi}{2}$ about the origin.

The single transformation T is equivalent to T_1 followed by T_2.

(a) Find the 3 x 3 matrix representing T. [5]

(b) Find the coordinates of the fixed point of T. [4]

6. Consider the proposition P given by

‘ $\sum_{r=1}^{n}(2r+1)=(n+1)^2$ where n is a positive integer.’

(a) Show that if P is true for $n = k$, then it is true for $n = k + 1$. [5]

(b) Explain why it cannot be deduced, using mathematical induction, that P is true for all positive integers n. Show that P is in fact false. [2]

7. Consider the system of equations:

$$\begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 2 \\ 1 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ \mu \end{bmatrix}$$

(a) Use reduction to echelon form to find the value of λ for which the equations do not have a unique solution. [5]

(b) For this value of λ, find the value of μ for which the equations are consistent. Find the general solution of the equations in this case. [5]

8. The roots of the cubic equation

$$x^3 + px^2 + 47x + q = 0$$

are in arithmetic progression with common difference 1.
Given that all the roots of the equation are negative, find

(a) the roots of the equation, [5]

(b) the values of the constants p and q. [2]

9. The function f is defined on the domain $(0, \infty)$ by

$$f(x) = x^{\frac{1}{x}}.$$

(a) Find the coordinates of the stationary point on the graph of f. [7]

(b) Determine the nature of this stationary point. [2]

10. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{z+3}{z+1}.$$

The point P moves around the circle with equation $|z| = 1$. Find the Cartesian equation of the locus of Q. Identify this locus. [8]