GCE AS/A level

977/01

MATHEMATICS FP1

Further Pure Mathematics

A.M. THURSDAY, 22 January 2009

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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1. *(a)* Differentiate 2^x. [3]

(b) Differentiate $\dfrac{x}{x+1}$ from first principles. [6]

2. Given that

$$S_n = 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 ,$$

obtain an expression for S_n in terms of n, giving your answer as a product of linear factors. [6]

3. Given that the roots of the cubic equation

$$x^3 + 4x^2 + 3x + 2 = 0$$

are α, β, γ, determine the cubic equation with roots $\beta\gamma$, $\gamma\alpha$, $\alpha\beta$. [7]

4. *(a)* Given that

$$2z - \mathrm{i}\bar{z} = 1 + 4\mathrm{i},$$

find an expression for the complex number z in the form $x + \mathrm{i}y$. [7]

(b) Find the modulus and argument of the complex number

$$\frac{1+3\mathrm{i}}{2-\mathrm{i}} .$$ [6]

5. The rotation T in the plane has matrix

$$\begin{bmatrix} 0{\cdot}6 & 0{\cdot}8 & 2 \\ -0{\cdot}8 & 0{\cdot}6 & 3 \\ 0 & 0 & 1 \end{bmatrix} .$$

(a) Find the coordinates of the fixed point of T. [4]

(b) Determine the centre and the angle of this rotation. [4]

6. Use mathematical induction to show that

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n & 2n^2 \\ 0 & 1 & 2n \\ 0 & 0 & 1 \end{bmatrix}$$

for all positive integers n. [8]

7. Given that **A** is a 2 × 2 matrix and k is a constant, show that

$$\det(k\mathbf{A}) = k^2 \det(\mathbf{A}).$$ [4]

8. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(1 - z).$$

(a) Show that

$$v = y(1 - 2x)$$

and find an expression for u in terms of x and y. [4]

(b) The point P moves along the line $y = x$. Find the Cartesian equation of the locus of Q. [4]

9. The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} \lambda+1 & 1 & \lambda \\ 1 & 2 & \lambda \\ 2 & \lambda & 1 \end{bmatrix}.$$

(a) (i) Find and simplify an expression for the determinant of **A**.

(ii) Show that **A** is singular when $\lambda = 1$ but there are no other real values of λ for which **A** is singular. [5]

(b) Now consider the system of equations

$$\mathbf{AX} = \mathbf{B}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}.$$

(i) Given that $\lambda = 1$, show that these equations are consistent and find their general solution.

(ii) Given that $\lambda = -1$, find the inverse matrix $\mathbf{A}^{-1}$ and **hence** solve these equations. [7]