GCE AS/A level

977/01

MATHEMATICS FP1

Further Pure Mathematics

P.M. MONDAY, 1 February 2010

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Show that 1 + 2i is a root of the equation $x^3 + x + 10 = 0$. [3]

(b) Determine the other two roots of the equation. [4]

2. The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(a) Find the inverse of **A**. [3]

(b) Find the 2 × 2 matrix **X** that satisfies the equation

$$\mathbf{AX} = \mathbf{B}.$$ [3]

3. The complex number z is given by

$$z = \frac{1 + 8i}{1 - 2i}.$$

(a) Express z in the form $x + iy$. [3]

(b) Find the modulus and argument of z. [3]

4. *(a)* Show that the following matrix is singular.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 4 & 5 & 7 \end{bmatrix}$$ [2]

(b) Consider the following equations

$$\begin{aligned} x + 2y + 2z &= 1 \\ 2x + y + 3z &= 3 \\ 4x + 5y + 7z &= \lambda \end{aligned}$$

(i) Find the value of λ for which these equations are consistent.

(ii) Find the general solution corresponding to this value of λ. [7]

5. Given that the cubic equation $x^3 - qx + r = 0$ has two equal roots, show that

$$4q^3 = 27r^2.$$ [6]

6. *(a)* Use mathematical induction to prove that

$$1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n! = (n+1)! - 1$$

for all positive integers n. [6]

(b) Given that

$$S_n = \sum_{r=1}^{n} r(3r+1),$$

obtain an expression for S_n in terms of n, simplifying your answer. [5]

7. The function f is defined for $0 < x < \frac{\pi}{2}$ by

$$f(x) = (\operatorname{cosec} x)^x.$$

(a) Obtain and simplify an expression for $f'(x)$. [4]

(b) (i) Show that $f(x)$ has a stationary value at $x = \alpha$, where

$$\alpha = \tan\alpha \ln(\operatorname{cosec}\alpha).$$

(ii) Use the recurrence relation

$$\alpha_{n+1} = \tan\alpha_n \ln(\operatorname{cosec}\alpha_n)$$

with $\alpha_0 = 0{\cdot}5$ to find the value of α correct to four decimal places. [5]

8. The transformation T in the plane consists of a reflection in the line $y = x$ followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y - 1)$ followed by a clockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$ [5]

(b) Show that T has no fixed points. [3]

(c) Find the equation of the image under T of the line $y = 2x + 1$. [5]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and $w = 1 + z^2$.

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point P moves along the line $y = 2x$. Find the equation of the locus of Q. [4]