WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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977/01

MATHEMATICS FP1

Further Pure Mathematics

P.M. TUESDAY, 28 June 2005

($1\frac{1}{2}$ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The complex number z is represented by the point P on an Argand diagram. Given that

$$|z + 1| = 2|z - 2\text{i}|$$

find, in its simplest form, the Cartesian equation of the locus of P. [5]

2. Find an expression, in terms of n, for

$$\sum_{r=1}^{n} 4r\left(r^2 - 1\right) .$$

Give your answer as a product of linear factors. [6]

3. Differentiate $\dfrac{1}{x^2 + x}$ from first principles. [6]

4. The transformation T of the plane is equivalent to a reflection in the line $y = x$ followed by the translation in which the point (x, y) is translated to the point $(x + 1, y + 2)$.

(a) Find the 3×3 matrix representing T. [4]

(b) Show that T has no fixed points. [3]

5. Use mathematical induction to prove that

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}$$

for all positive integers n. [7]

6. Given that $1 + \text{i}$ is a root of the equation

$$x^3 + 2x^2 + \lambda x + \mu = 0,$$

(a) find the values of the real constants λ and μ,

(b) find all the other roots of the equation. [10]

7. The complex numbers z and w are represented, respectively, by the points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{z}\ .$$

(a) Show that

$$x = \frac{u}{u^2 + v^2}$$

and obtain an expression for y in terms of u and v. [5]

(b) The point P moves along the circle $x^2 + y^2 = 2$. Find the equation of the locus of Q in the (u, v) plane. [3]

8. The roots of the cubic equation

$$x^3 - 2x^2 + 3x + 3 = 0$$

are denoted by α, β, γ. Find the cubic equation whose roots are $\frac{1}{\beta\gamma}$, $\frac{1}{\gamma\alpha}$ and $\frac{1}{\alpha\beta}$. [11]

9. *(a)* The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 3 & 5 & \lambda \end{bmatrix}\ .$$

Find the value of λ for which $\mathbf{A}$ is singular. [3]

(b) Consider the system of equations

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 3 & 5 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}\ .$$

(i) Given that $\lambda = 5$, find the general solution of this system of equations.

(ii) You are now given that $\lambda = 3$. By first finding the inverse of the matrix $\mathbf{A}$, solve this system of equations. [12]