WJEC
CBAC

GCE AS/A level

977/01

MATHEMATICS FP1
Further Pure Mathematics

P.M. MONDAY, 15 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$S_n = \sum_{r=1}^{n} r(r+1)^2 ,$$

find an expression for S_n in terms of n, giving your answer as a product of linear factors. [6]

2. The roots of the quadratic equation

$$x^2 + 3x + 4 = 0$$

are denoted by α and β. Find the cubic equation whose roots are α, β and $\alpha\beta$. [8]

3. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix} .$$ [6]

(b) Hence solve the system of equations

$$\begin{aligned} x + 2y + 3z &= 13 \\ 2x + 3y + z &= 13 \\ 3x + 5y + 2z &= 22. \end{aligned}$$ [2]

4. The complex number z is given by

$$z = \frac{9 + 7\mathrm{i}}{3 - \mathrm{i}} .$$

(a) Express z in the form $x + \mathrm{i}y$, where x, y are real. [4]

(b) Find the modulus and argument of z. [2]

5. Use mathematical induction to prove that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1} .$$

for all positive integers n. [8]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 1 & 2 \\ 2 & -1 & \lambda \\ 3 & \lambda & 4 \end{bmatrix} .$$

(a) Show that $\lambda = 1$ is the only positive value of λ for which **A** is singular. [5]

(b) Consider the following equations.

$$\begin{aligned} x + y + 2z &= 2 \\ 2x - y + z &= -2 \\ 3x + y + 4z &= 2 \end{aligned}$$

(i) Show that these equations are consistent.

(ii) Find the general solution. [6]

7. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$|z - 1| = 2|z + 2|,$$

show that the locus of P is a circle, and find its radius and the coordinates of its centre. [7]

8. The transformation T in the plane consists of a reflection in the line $x + y = 0$ followed by a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & h \\ -1 & 0 & k \\ 0 & 0 & 1 \end{bmatrix} .$$ [3]

(b) Given that the image of the point (1, 2) under T is the point (2, 1),

(i) find the values of h and k,

(ii) find the equation of the image under T of the line $y = 3x + 2$. [8]

TURN OVER

9. The function f is defined for $x > 0$ by

$$f(x) = x^x e^{-2x}.$$

(a) Show that

(i) $\ln f(x) = x\ln x - 2x$,

(ii) $f'(x) = f(x)\,(a\ln x + b)$, where the values of the constants a and b are to be found. [4]

(b) Write down an expression for $f''(x)$ in terms of $f(x)$ and $f'(x)$. [1]

(c) Find the coordinates of the stationary point on the graph of f and determine whether this point is a maximum or a minimum. [5]