



GCE AS/A level

977/01

MATHEMATICS FP1
Further Pure Mathematics

P.M. FRIDAY, 18 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Differentiate $\frac{1}{1+x^2}$ from first principles. [6]

2. The complex number $z = 2 - i$. The complex conjugate of z is denoted by \bar{z} . Find the modulus and argument of the complex number

$$z - \frac{5\bar{z}}{z}. \quad [7]$$

3. The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} 2 & \lambda & 3 \\ 1 & 2 & \lambda \\ 4 & 5 & 5 \end{bmatrix}$$

(a) Find the values of λ for which \mathbf{A} is singular. [4]

(b) Given that $\lambda = 3$,

(i) find the inverse of \mathbf{A} ,

(ii) **hence** solve the equations

$$\begin{aligned} 2x + 3y + 3z &= 2 \\ x + 2y + 3z &= -1 \\ 4x + 5y + 5z &= 4. \end{aligned} \quad [6]$$

4. The roots of the quadratic equation

$$x^2 + 2x + 3 = 0$$

are denoted by α, β . Find the quadratic equation whose roots are

$$\alpha - \frac{1}{\beta^2}, \beta - \frac{1}{\alpha^2}. \quad [8]$$

5. Use mathematical induction to prove that $4^{2n} - 1$ is divisible by 15 for all positive integers n . [6]

6. (a) Express $\frac{1}{r(r+2)}$ in partial fractions. [3]

(b) Hence show that

$$\sum_{r=1}^n \frac{1}{r(r+2)} = \frac{3}{4} - \frac{(2n+3)}{2(n+1)(n+2)}. \quad [4]$$

7. The function f is defined for $x > 0$ by

$$f(x) = x^{-2x}.$$

- (a) Obtain and simplify an expression for $f'(x)$. [5]
- (b) Find the stationary value of $f(x)$ and show that it is a maximum. [5]

8. The transformation T in the plane consists of an anti-clockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x - 3, y + 1)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

[3]

- (b) Find the coordinates of the fixed point of T . [4]
- (c) The image of the line L under T has equation $x + 2y = 3$. Find the equation of L . [4]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{1-z}.$$

- (a) Obtain expressions for u and v in terms of x and y . [4]
- (b) The point P moves along the line $y = 1 - x$. Find the equation of the locus of Q . [2]
- (c) Find the coordinates of the points in the z -plane which are transformed to points with the same coordinates in the w -plane. [4]