GCE AS/A level

0977/01

S15-0977-01

MATHEMATICS – FP1
Further Pure Mathematics

P.M. TUESDAY, 16 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Differentiate $\dfrac{1}{x^2 - x}$ from first principles. [7]

2. The transformation T in the plane consists of a reflection in the line $y = x$ followed by a reflection in the line $y = -x$.

(a) Determine the 2×2 matrix which represents T. [4]

(b) Identify the single transformation that is equivalent to T. [1]

3. *(a)* The complex number z satisfies the equation

$$2z - \mathrm{i}\bar{z} = \frac{2 + \mathrm{i}}{1 - \mathrm{i}},$$

where $\bar{z}$ denotes the complex conjugate of z. Express z in the form $x + \mathrm{i}y$. [6]

(b) Find the modulus and the argument of the complex number $-20 - 21\mathrm{i}$. [3]

4. *(a)* The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

Show that $\mathbf{M}$ is singular. [3]

(b) (i) Find the value of μ for which the following system of equations is consistent.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ \mu \end{bmatrix}$$

(ii) For this value of μ, find the general solution to this system of equations. [7]

5. The roots of the cubic equation

$$x^3 - 4x^2 - 8x + k = 0$$

are in geometric progression. Determine the value of k. [5]

6. The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 4 \\ 3 & 3 & 6 \\ 2 & 2 & 3 \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 3 & -2 & 0 \\ -3 & -1 & 6 \\ 0 & 2 & -3 \end{bmatrix}.$$

(a) Evaluate the matrix $\mathbf{AB}$. [2]

(b) Hence, or otherwise, find the inverse matrix $\mathbf{A}^{-1}$. [2]

(c) Hence solve the simultaneous equations

$$3x + 2y + 4z = 14$$
$$3x + 3y + 6z = 18$$
$$2x + 2y + 3z = 11$$

[2]

7. *(a)* Express

$$\frac{2}{n(n+2)}$$

in partial fractions. [3]

(b) Given that

$$S_n = \sum_{r=1}^{n} \frac{2}{r(r+2)},$$

obtain an expression for S_n in the form

$$\frac{an^2 + bn}{2(n+1)(n+2)},$$

where a and b are positive integers whose values are to be determined. [5]

8. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

(a) Show that

$$\mathbf{A}^2 = 2\mathbf{A} - \mathbf{I},$$

where $\mathbf{I}$ denotes the 2×2 identity matrix. [2]

(b) Using mathematical induction, prove that

$$\mathbf{A}^n = n\mathbf{A} - (n-1)\mathbf{I}$$

for all positive integers n. [6]

TURN OVER

9. The function f is defined on the domain $(0, \pi)$ by

$$f(x) = 2^x \sin x.$$

(a) Obtain an expression for $f'(x)$. [4]

(b) Determine the x-coordinate of the stationary point on the graph of f, giving your answer correct to 2 decimal places. [4]

10. The complex number z is represented by the point $P(x, y)$ in the Argand diagram and

$$|z + 3| = k|z - \mathrm{i}|,$$

where k is a real positive constant.

(a) When $k \neq 1$, the locus of P is a circle. Find, in terms of k,

(i) the equation of the circle,

(ii) the coordinates of the centre of the circle. [7]

(b) (i) Write down the equation of the locus of P when $k = 1$.

(ii) Give a geometric interpretation of this locus. [2]

END OF PAPER