ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Solve the equation

$$\cosh 2\theta = 6\sinh\theta - 3.$$

Give your answers in the form $\ln\left(p + \sqrt{q}\right)$, where p, q are positive integers. [7]

2. Find the first three non-zero terms of the Maclaurin series of $\ln(2 - e^x)$. [9]

3. Use the substitution $x = 2\sinh u$ to evaluate the integral

$$\int_0^2 \frac{\mathrm{d}x}{\left(x^2 + 4\right)^{\frac{3}{2}}} .$$

Give your answer correct to two decimal places. [8]

4. The region R is bounded by the x-axis, the line $x = a$ and that part of the curve $y^2 = 4ax$ lying between the points $(0, 0)$ and $(a, 2a)$. Show that the curved surface area of the solid formed when R is rotated through $360°$ about the x-axis is

$$\frac{8\left(2\sqrt{2} - 1\right)}{3}\pi a^2 .$$ [7]

5. *(a)* Sketch the curve having polar equation

$$r = 2 + \cos\theta \quad (0 \leqslant \theta \leqslant \pi).$$ [1]

(b) Determine the area of the region enclosed between the curve, the initial line and the line

$$\theta = \frac{\pi}{2} .$$ [6]

(c) Find the polar coordinates of the point on the curve at which the tangent is parallel to the initial line. [7]
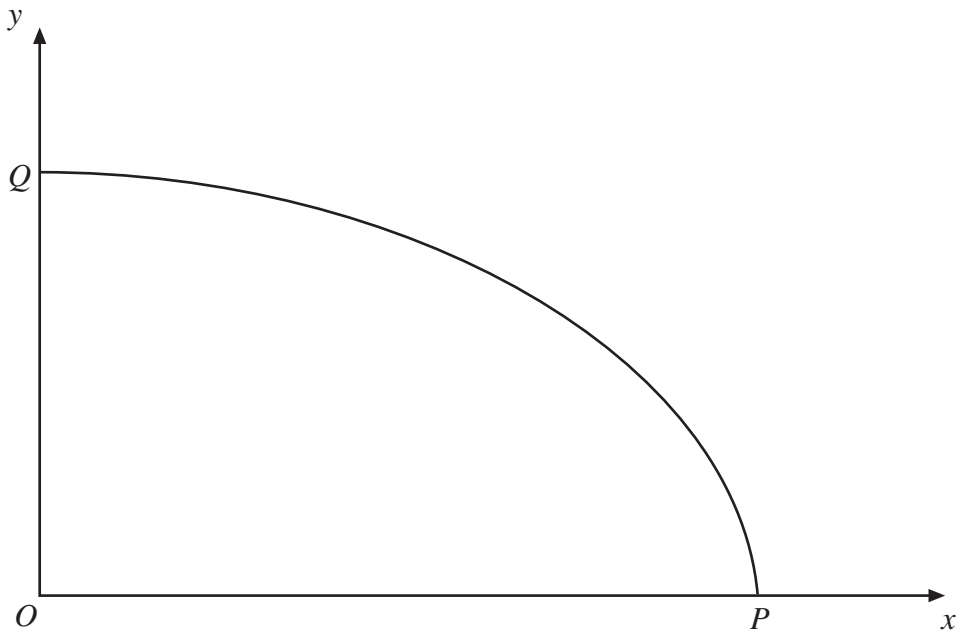
6. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx \ .$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \frac{1}{n-1} - I_{n-2}$$ [5]

(b) Evaluate I_4, giving your answer in terms of π . [5]

7.

The above diagram shows a sketch of part of the graph of the function

$$f(x) = 2\cosh x - x\sinh x.$$

The graph cuts the x-axis at P and the y-axis at Q.

(a) (i) Show that $f'(0)$ and $f''(0)$ are both zero.

  (ii) Giving a reason, state whether or not Q is a stationary point of inflection. [5]

(b) The x-coordinate of P is denoted by α.

  (i) Show that $\alpha\tanh\alpha = 2$.

  (ii) Show that α lies between 2 and 2·1.

  (iii) Consider the following iterative sequence based on the above equation in α.

$$\alpha_{n+1} = \frac{2}{\tanh \alpha_n}; \quad \alpha_0 = 2.05.$$

   Show, by evaluating an appropriate derivative, that this sequence is convergent.

  (iv) Use this sequence to find the value of α correct to four decimal places. [9]

(c) Find the area of the region enclosed between the graph and the two coordinate axes. [6]