

GCE AS/A level

0979/01

MATHEMATICS – FP3 Further Pure Mathematics

P.M. MONDAY, 24 June 2013 1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet:
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer all questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question. You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S13-0979-01)

1. Determine the two positive roots of the equation

$$\cosh 2x - 7\cosh x + 7 = 0,$$

giving your answers correct to two decimal places.

[6]

- 2. Jim wants to evaluate the real cube roots of several positive numbers but his calculator only performs the basic arithmetic operations add, subtract, multiply, divide.
 - (a) He therefore decides to determine $\sqrt[3]{a}$ by applying the Newton-Raphson method to the equation $x^3 a = 0$ where a > 0.
 - (i) Show that this gives the iterative sequence

$$x_{n+1} = \frac{2x_n^3 + a}{3x_n^2}.$$

- (ii) Taking $x_0 = 2$, use this method to find $\sqrt[3]{10}$ correct to four decimal places. [5]
- (b) Huw suggests that an alternative method for determining $\sqrt[3]{a}$ could be to rearrange the equation $x^3 a = 0$ in the form

$$x = \frac{a}{x^2}$$

and to define the iterative sequence

$$x_{n+1} = \frac{a}{x_n^2}.$$

Show, however, that this sequence diverges for all values of a.

[4]

3. The function *f* is defined by

$$f(x) = \ln(2e^x - 1)$$
.

(a) Show that

$$f''(x) = \frac{-2e^x}{(2e^x - 1)^2}.$$
 [3]

- (b) Determine the Maclaurin series for f(x) as far as the term in x^3 . [6]
- **4.** Determine the value of the integral

$$\int_1^2 \sqrt{3+2x-x^2} \, \mathrm{d}x,$$

giving your answer correct to three significant figures.

[10]

5. The integral I_n is defined, for $n \ge 0$, by

$$I_n = \int_0^1 x^n \sinh x \, \mathrm{d}x.$$

(a) Show that, for $n \ge 2$,

$$I_n = \cosh 1 - n \sinh 1 + n(n-1)I_{n-2}.$$
 [5]

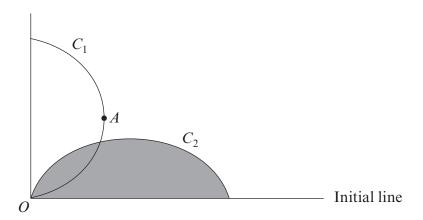
(b) Evaluate I_4 , giving your answer in the form

$$a \cosh 1 + b \sinh 1 + c$$
,

where a, b, c are integers.

[5]

6.



The diagram shows sketches, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$, of the curve C_1 having polar equation $r = \sin^2 \theta$ and the curve C_2 having polar equation $r = 1 - \sin \theta$.

- (a) Find the polar coordinates of the point A on C_1 at which the tangent is perpendicular to the initial line. [8]
- (b) Find the area of the shaded region enclosed between C_2 and the initial line. [6]

TURN OVER

- 7. (a) (i) Assuming the derivatives of $\cosh x$ and $\sinh x$, show that the derivatives of $\operatorname{cosech} x$ and $\operatorname{coth} x$ are respectively $-\operatorname{cosech} x \operatorname{coth} x$ and $-\operatorname{cosech}^2 x$.
 - (ii) Hence show that

$$\frac{d}{dx} \left[\ln(\operatorname{cosech} x + \operatorname{coth} x) \right] = -\operatorname{cosech} x.$$
 [6]

(b) (i) Show that the length L of the arc joining the points (1, 0) and (e, 1) on the graph of $y = \ln x$ is given by

$$\int_1^e \frac{\sqrt{1+x^2}}{x} \, \mathrm{d}x.$$

(ii) Use the substitution $x = \sinh u$ to show that

$$L = \int_{\sinh^{-1} 1}^{\sinh^{-1} e} (\operatorname{cosech} u + \sinh u) du.$$

(iii) Use the result in (a)(ii) to determine the value of L correct to three significant figures. [11]