

981/01

**MATHEMATICS M2**

**Mechanics 2**

P.M. WEDNESDAY, 22 June 2005

(1½ hours)

**NEW SPECIFICATION**

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. One end of a light elastic string, of natural length 0.8 m, is attached to a fixed point  $O$ , and the other end is attached to a particle of mass 5 kg. When the particle hangs in equilibrium vertically below  $O$ , the length of the string is 1.3 m.

(a) Calculate the modulus of elasticity of the string. [3]

(b) Determine the elastic energy stored in the string. [2]

2. A particle moves in a straight line such that its acceleration  $a \text{ ms}^{-2}$  is given by

$$a = 4 - 6t \quad \text{for } t \geq 0.$$

At time  $t = 0$ , the particle is at the point  $O$  and its velocity is  $4 \text{ ms}^{-1}$ .

(a) Find an expression for the velocity of the particle at time  $t$  s. [3]

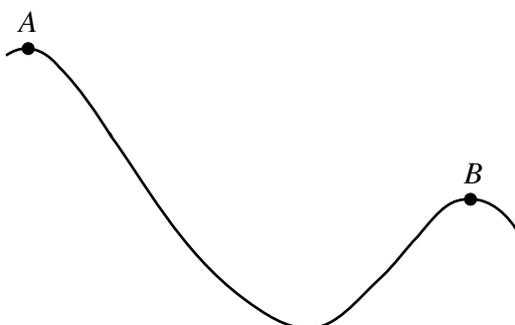
(b) Find an expression for the displacement of the particle from  $O$  at time  $t$  s. [3]

(c) Determine the time when the particle comes to rest instantaneously and the distance of the particle from  $O$  at this time. [3]

(d) Calculate the **speed** of the particle when  $t = 3$ , and determine whether or not the **speed** of the particle is increasing or decreasing at this time. [3]

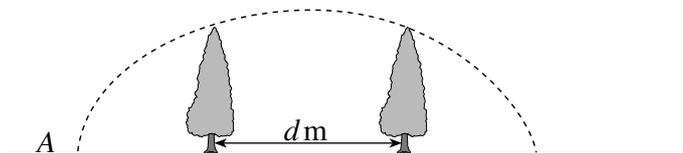
3. A car, of mass 1250 kg, is travelling up a hill inclined at an angle  $\alpha$  to the horizontal at a constant speed of  $7.5 \text{ ms}^{-1}$ . The car's engine is working at a rate of 30 kW and the resistance to motion of the car is 1550 N. Find the value of  $\alpha$ , giving your answer in degrees correct to one decimal place. [6]

4. The diagram shows two points  $A$  and  $B$  on a roller coaster ride in an amusement park.



The heights of  $A$  and  $B$  above ground level are 30 m and 22 m respectively. The length of the track between  $A$  and  $B$  is 88 m. The resistance to motion of the carriage may be assumed to have a constant magnitude of 132 N. A carriage, of total mass 240 kg, has speed  $2 \text{ ms}^{-1}$  at  $A$ . Calculate the speed of the carriage at  $B$ . [8]

5. A golfer hits a ball from the point  $A$  with initial velocity  $24.5 \text{ ms}^{-1}$  at an angle  $\alpha$  above the horizontal, where  $\sin \alpha = 0.8$ . The ball just clears the tops of two trees. The tops of the trees are both  $14.7 \text{ m}$  above the level of  $A$  and are a horizontal distance  $d \text{ m}$  apart.



- (a) (i) Find the time taken for the ball to reach the top of the first tree.  
(ii) Determine the value of  $d$ . [8]
- (b) Find the magnitude and direction of the velocity of the ball  $0.75 \text{ s}$  after it was hit. [6]

6. At time  $t \text{ s}$ , a particle  $P$  has position vector  $\mathbf{r} \text{ m}$  with respect to an origin  $O$  given by

$$\mathbf{r} = (2t - 5)\mathbf{i} + (t - 3)\mathbf{j} + (7 - 2t)\mathbf{k}.$$

- (a) Show that the distance of the particle from the origin at time  $t \text{ s}$  is given by

$$OP^2 = 9t^2 - 54t + 83,$$

and find the value of  $t$  when  $P$  is closest to  $O$ . [5]

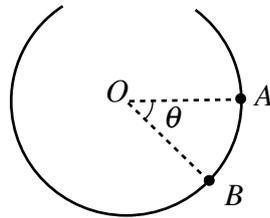
- (b) Find the velocity of  $P$  and determine its magnitude. [3]
- (c) Show that, when  $P$  is closest to  $O$ , the direction of the velocity of  $P$  is perpendicular to the line  $OP$ . [3]

7. A rider on a motorcycle is travelling at a constant speed of  $42 \text{ ms}^{-1}$  in a horizontal circle on a track banked at an angle of  $25^\circ$  to the horizontal. There is no tendency to sideslip at this speed. The total mass of the motorcycle and the rider is  $600 \text{ kg}$ .  
Modelling the motorcycle and the rider as a particle,

- (a) calculate the normal reaction of the track on the motorcycle, [3]
- (b) determine the radius of the circle. [4]

**TURN OVER.**

8. A marble, of mass  $0.3 \text{ kg}$ , moves on the inner surface of a spherical bowl of radius  $0.4 \text{ m}$ . Initially, the marble is held at  $A$ , on the inner surface of the bowl, where  $OA$  is horizontal, as shown in the diagram.



The marble is then projected with a speed of  $2 \text{ ms}^{-1}$  vertically downwards. When the marble is at the point  $B$ ,  $\widehat{AOB} = \theta$ .

- (a) Find, in terms of  $\theta$ , an expression for  $v^2$ , where  $v \text{ ms}^{-1}$  is the speed of the marble at  $B$ . [4]
- (b) Show that the reaction  $R \text{ N}$  of the surface of the bowl on the marble is given by

$$R = 3 + 8.82 \sin \theta \quad . \quad [4]$$

- (c) Find the greatest possible value of  $\theta$ , and briefly describe the subsequent motion of the marble. [4]