



**GCE AS/A level**

981/01

**MATHEMATICS M2**  
**Mechanics 2**

A.M. MONDAY, 13 June 2011

1½ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves along the  $x$ -axis and its velocity  $v \text{ ms}^{-1}$  at time  $t$  s is given by

$$v = 12 \sin 3t - 8 \cos 2t.$$

(a) Find an expression for the acceleration of the particle at time  $t$  s. [3]

(b) Given that at time  $t = 0$  the particle is at the origin  $O$ , find an expression for the displacement of the particle from  $O$  at time  $t$  s. [5]

2. A particle of mass  $0.5 \text{ kg}$  is attached to one end of a light inextensible string of length  $0.6 \text{ m}$ . The other end of the string is fixed at a point  $O$  on a smooth horizontal surface. The particle moves on the surface in a circle with centre  $O$ , so that the string is taut and the angular velocity of the particle about  $O$  is  $5$  radians per second.

(a) Calculate the speed of the particle. [2]

(b) Find the tension in the string. [2]

3. A particle  $P$ , of mass  $2 \text{ kg}$ , is moving under the action of a force  $\mathbf{F} \text{ N}$  so that its velocity  $\mathbf{v} \text{ ms}^{-1}$  at time  $t$  s is given by

$$\mathbf{v} = 2\mathbf{i} + 6t\mathbf{j} + 4t^3\mathbf{k}.$$

(a) Find an expression for  $\mathbf{F}$  at time  $t$  s. [3]

(b) Determine the value of  $\mathbf{F} \cdot \mathbf{v}$  when  $t = 1$  and state the units of your answer. [4]

4. A car of mass  $800 \text{ kg}$  is travelling against a constant resistance to motion of  $540 \text{ N}$ .

(a) Find the power of the engine when the car is travelling on a level racing track at a constant speed of  $60 \text{ ms}^{-1}$ . [4]

(b) With the engine working at  $32.4 \text{ kW}$  and the resistance to motion unchanged, the car ascends a hill inclined at an angle  $\alpha$  to the horizontal where  $\sin \alpha = \frac{1}{16}$ .

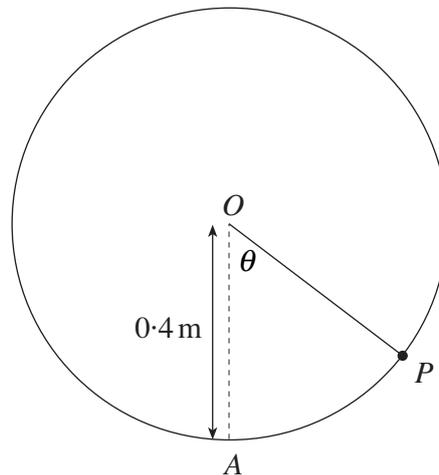
Find the acceleration of the car when its velocity is  $15 \text{ ms}^{-1}$ . [5]

5. A light elastic string, of natural length  $1.6 \text{ m}$  and modulus of elasticity  $80 \text{ N}$ , has one end attached to a fixed point  $A$  and the other end attached to a particle  $P$ , of mass  $4 \text{ kg}$ . Initially,  $P$  is held at a point  $0.5 \text{ m}$  vertically below the point  $A$ . The particle  $P$  is released from rest and allowed to fall.

(a) Calculate the tension in the string when the length of the string is  $2 \text{ m}$ . [2]

(b) Determine the speed of  $P$  when the length of the string is  $2 \text{ m}$ . [8]

6. A stone is thrown from the top of a vertical cliff, 100 m above sea level. The initial velocity of the stone is  $6.5 \text{ ms}^{-1}$  at an angle  $\alpha$  above the horizontal, where  $\tan \alpha = \frac{5}{12}$ .
- (a) Find the time taken for the stone to reach the sea. Give your answer correct to two decimal places. [5]
- (b) Calculate the horizontal distance from the bottom of the cliff to the point where the stone hits the sea. [2]
- (c) Calculate the magnitude and direction of the velocity with which the stone hits the sea. [7]
7. At time  $t$ , the position vectors relative to a fixed origin  $O$ , of two particles  $A$  and  $B$  are given by  $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k})$  and  $\mathbf{OB} = 5\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} + t(3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$ .
- (a) Find the speed of particle  $A$ . [3]
- (b) Show that the distance  $AB$  at time  $t$  is given by  $AB^2 = 5t^2 - 30t + 211$ . Determine the time at which the particles  $A$  and  $B$  are closest together. [7]
8. The diagram shows a particle  $P$ , of mass 3 kg, attached by a light inextensible string of length 0.4 m to a fixed point  $O$ . Initially,  $P$  is projected from the point  $A$ , which is vertically below  $O$ , with a horizontal speed of  $4 \text{ ms}^{-1}$ .



- (a) The speed of  $P$  when  $OP$  makes an angle  $\theta$  with  $OA$  is  $v \text{ ms}^{-1}$ . Show that  $v^2 = 8.16 + 7.84 \cos \theta$ . [4]
- (b) Find an expression, in terms of  $\theta$ , for the tension in the string when  $OP$  makes an angle  $\theta$  with  $OA$ . [4]
- (c) Determine whether or not  $P$  describes complete circles. [3]
- (d) Would your conclusion to (c) be different if the string was replaced by a light rigid rod? Justify your answer. [2]