

982/01

MATHEMATICS M3

Mechanics 3

P.M. THURSDAY, 15 June 2006

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9.8 ms^{-2} .

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle is moving along the x -axis. At time $t = 0$ the particle is at the origin O , its velocity is 12 ms^{-1} and its acceleration is -16 ms^{-2} . At time t s, the particle is x m from O and its velocity $v \text{ ms}^{-1}$ satisfies the equation

$$v = \frac{B}{x + A},$$

where A and B are constants.

- (a) Find, in terms of A , B and x , an expression for the acceleration of the particle at time t s. [3]
 (b) Show that $A = 9$ and $B = 108$. [3]
 (c) Find an expression for t in terms of x . [4]

2. Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 10x = 5t - 14,$$

such that $x = 4 \frac{1}{2}$ and $\frac{dx}{dt} = 3 \frac{1}{2}$ when $t = 0$. [12]

3. A ball of mass 0.4 kg is thrown vertically upwards from a point O with initial speed 17 ms^{-1} . When the ball is at a height of x m above O and its speed is $v \text{ ms}^{-1}$, the air resistance acting on the ball has magnitude $0.01 v^2 \text{ N}$.

- (a) Show that, as the ball is ascending, v satisfies the differential equation

$$40v \frac{dv}{dx} = -(392 + v^2). \quad [3]$$

- (b) Find, correct to two decimal places, the greatest height of the ball. [7]
 (c) State, with a reason, whether the speed of the ball when it returns to O is
 (i) greater than 17 ms^{-1} ,
 (ii) less than 17 ms^{-1} ,
 (iii) equal to 17 ms^{-1} . [2]

4. A particle P is moving in a straight line with Simple Harmonic Motion about centre O , with period 4 s. The maximum speed of P is $3\pi \text{ ms}^{-1}$. The point A is 4.8 m from O .

- (a) Find the amplitude of the motion. [4]
 (b) Find the speed of P when it is at A . [3]
 (c) Calculate the time taken by P to move directly from O to A . [3]
 (d) Determine the magnitude of the maximum acceleration of P . [2]
 (e) Find the distance travelled by the particle P in 12 s. [2]

5. Particle A , of mass 2 kg , and particle B , of mass 3 kg , are connected by a light inextensible string of length $l\text{ m}$. Initially, both particles are lying at rest on a smooth horizontal surface a distance $l\text{ m}$ apart, with the string just slack. Particle B is given a blow of impulse 40 N s in a direction away from A at an angle α to the line joining the initial positions of A and B .



Immediately after the blow, the speed of particle A is 4 ms^{-1} .

- (a) Determine the value of α . [6]
- (b) Calculate the magnitude and direction of the velocity of B immediately after the blow. [6]
6. A uniform ladder, of mass 37.5 kg and length 8 m , rests with its top end against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle of 60° to ground. The coefficient of friction between the ladder and the ground is μ . A person, of mass 75 kg , climbs the ladder.
- (a) Given that $\mu = 0.25$, determine how far the person can ascend before the ladder slips. [10]
- (b) Given that the person is able to ascend to the top of the ladder, determine the minimum value of μ . [4]
- (c) State **one** modelling assumption you have made in your solution. [1]