WJEC
CBAC

GCE AS/A level

982/01

MATHEMATICS M3
Mechanics 3

A.M. WEDNESDAY, 17 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A body, of mass 9 kg, is projected along a straight horizontal track with an initial speed of 20 ms^{-1}. At time t s the body experiences a resistance of magnitude $(0{\cdot}2 + 0{\cdot}03v)$ N where v ms^{-1} is its speed.

(a) Show v satisfies the differential equation [3]

$$900\ \frac{dv}{dt} = -(20 + 3v).$$

(b) Find an expression for t in terms of v. [5]

(c) Calculate, to the nearest second, the time taken for the body to come to rest. [2]

2. At time $t = 0$, a particle P is projected from a point O so that it moves in a straight line with Simple Harmonic Motion with centre O. Two seconds later P comes to rest for the first time at the point A, where $OA = 24$ cm.

(a) Determine the speed of projection. [6]

(b) The point B is between O and A, such that $OB = 15$ cm. Find the value of t when P is at B for the **third** time. [4]

(c) Calculate the speed of P when $t = 1{\cdot}5$ s. [4]

(d) Find the speed of P when it is at a distance 20 cm from O. [3]

3. A cyclist, of mass 65 kg, starts from rest and rides his bicycle, of mass 10 kg, along a straight horizontal road. The cyclist produces a constant forward force of 180 N and experiences a variable resistance to motion of magnitude $3v^2$ N, where v ms^{-1} is the speed of the bicycle. Show that v satisfies the differential equation

$$25v\frac{dv}{dx} = 60 - v^2,$$

where x is the distance from the start of motion.
Calculate the speed of the cyclist when he has cycled a distance of 20 m. Give your answer correct to two decimal places. [10]

4. Two spheres P and Q, of mass 5 kg and 3 kg respectively, rest on a smooth table. They are connected by a light inextensible string which is initially slack. An impulse of magnitude 1·2 Ns is applied to Q in the direction PQ.

(a) Determine the speed with which Q begins to move. [2]

(b) Find the speed with which P moves after the string tightens, and determine the impulsive tension in the string. [6]

(c) Calculate the loss in energy when the string tightens. [4]

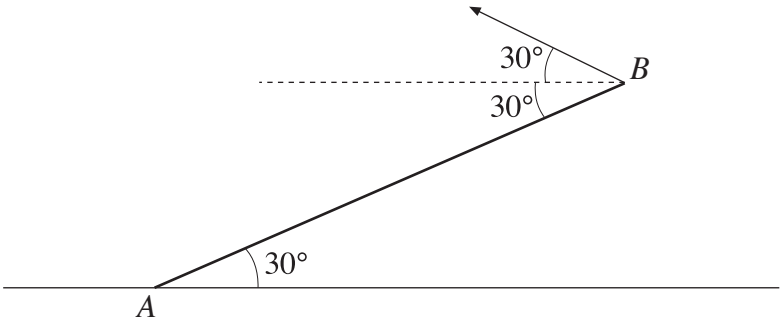
5. A particle P, of mass 2 kg, moves along a horizontal x-axis so that at time t s, its speed is v ms^{-1}. The particle moves under the action of a force $(156 - 52x)$ N, where x is the x-co-ordinate of P, and a resistive force of magnitude $4v$ N. Initially, the particle P is at the origin O and its velocity is 3 ms^{-1}.

(a) Show that x satisfies the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x = 78. \qquad [2]$$

(b) Find an expression for x in terms of t and determine the value of x when $t = 0{\cdot}5$. [12]

6. The diagram shows a uniform rod AB of mass 15 kg with its lower end A resting on a rough horizontal floor. A string is attached to the end B of the rod and applies a force on the rod at B in the direction shown in the diagram. The rod is in equilibrium when it is inclined at an angle of 30° to the floor. The coefficient of friction between the rod and the floor is μ.



Find the least possible value for μ. [12]