WJEC
CBAC

GCE AS/A level

982/01

MATHEMATICS M3
Mechanics 3

A.M. THURSDAY, 23 June 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A vehicle P, of mass 800 kg, on a straight horizontal road passes the point O with velocity $5\,\text{ms}^{-1}$. At time t s later its velocity is $v\,\text{ms}^{-1}$ and the vehicle is subject to a resistance given by $(4000 + 1600v)$ N.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -(5 + 2v) \ .$$ [2]

(b) (i) Find the time when P is at rest.

(ii) Find an expression for v in terms of t. [9]

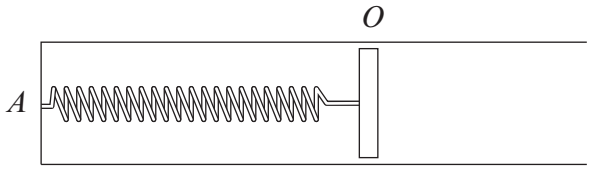
2. A particle, of mass 8 kg, moves along the x-axis. At time $t = 0$, the particle is at O and its velocity is $3\,\text{ms}^{-1}$. At time t s, the velocity of the particle is $v\,\text{ms}^{-1}$ and it moves under the action of a propulsive force of magnitude $4v$ N and a resistive force of magnitude $(4 - 16t)$ N.

(a) Show that x satisfies the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - \frac{\mathrm{d}x}{\mathrm{d}t} = 4t - 1 \ .$$ [3]

(b) Find an expression for x in terms of t. [12]

3. A piston of mass 0·1 kg is free to slide inside a smooth cylinder whose axis is horizontal. One end of a light spring, of modulus of elasticity 3·2 N and natural length 0·5 m, is attached to the piston, and the other end is attached to a fixed point A along the line of the axis of the cylinder. Initially the piston is at rest at the point O and $OA = 0{\cdot}5$ m.



The piston is given a velocity of $0{\cdot}8\,\text{ms}^{-1}$ along the axis of the cylinder away from the point A.

(a) Show that the subsequent motion of the piston is Simple Harmonic.

State its centre and show that the period is $\frac{\pi}{4}$ s. [6]

(b) Show that the amplitude of the motion is 0·1 m. [2]

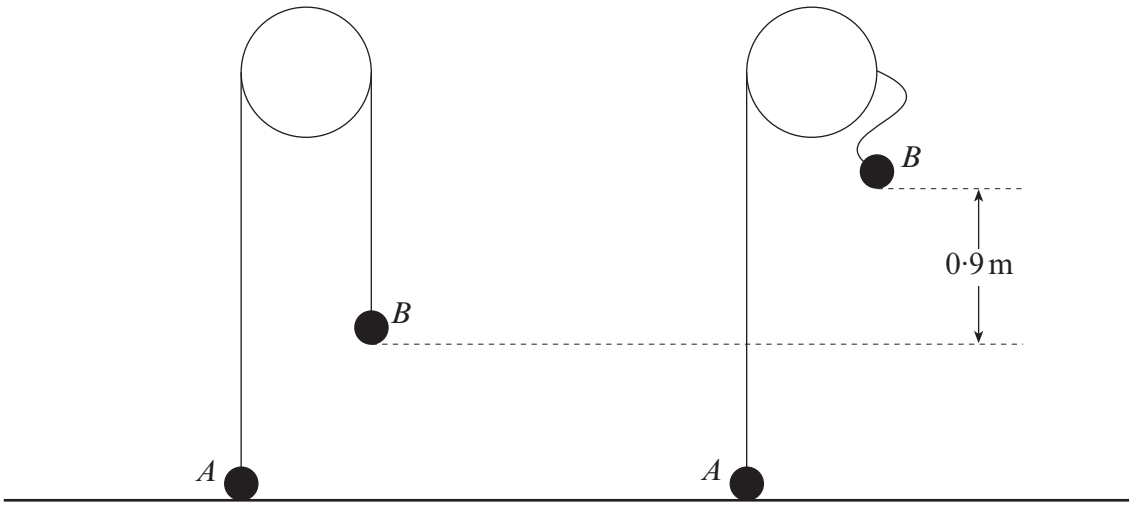
(c) Calculate the speed of the piston when it is 0·08 m from O. [3]

(d) Find the maximum magnitude of the acceleration of the piston. [2]

(e) Calculate the time taken for the piston to reach a point 0·05 m from O for the first time. [3]

4. A particle P moves along the x-axis. When the displacement of P from the origin O is x m, its acceleration is of magnitude $\left(\frac{9}{2x^2}\right)$ ms^{-2} and is directed towards O.
When $x = \frac{3}{4}$, the velocity of P is 3 ms^{-1}. Find the speed of P when $x = 2$ and the value of x when P comes to rest. [10]

5. The diagram shows two particles A and B, of masses 4 kg and 3 kg respectively, connected by a light inextensible string passing over a smooth light pulley fixed above a horizontal plane. Initially, the particle A is at rest on the plane and particle B hangs at a depth of 1·0 m below the pulley.



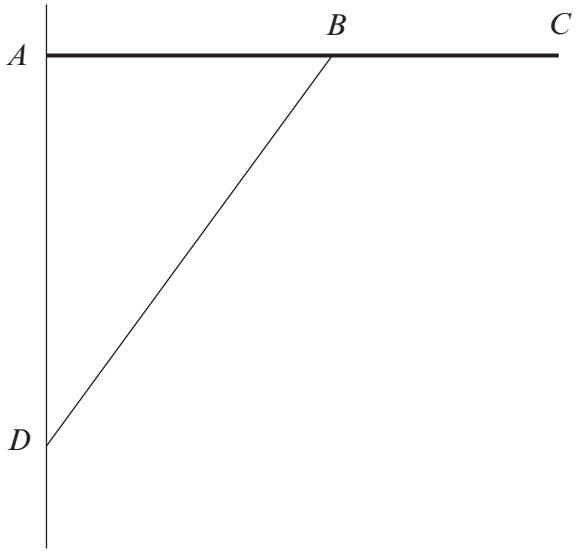
Particle B is then raised vertically a distance of 0·9 m and released from rest from that position.

(a) Calculate the speed of B immediately before the string tightens. [3]

(b) Determine the speed with which A leaves the plane and the impulsive tension in the string immediately after the string tightens. [7]

TURN OVER

6. The diagram shows a uniform plank AC, of mass 15 kg and length 1·2 m, hinged to a vertical wall at A. The plank is supported in a horizontal position by a fixed light rod BD, where D is on the wall and B is the midpoint of AC. The length AD is 0·8 m. A boy leans on the plank at C exerting a force of 20 N vertically downwards.



(a) Find the thrust in the rod BD. [5]

(b) Calculate the magnitude and direction of the reaction at the hinge A. [8]