



GCE AS/A level

0982/01

MATHEMATICS – M3
Mechanics

A.M. FRIDAY, 21 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9.8 ms^{-2} .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. At time $t = 0$, a particle of mass 6 kg is projected vertically upwards from a point A with a speed of 24.5 ms^{-1} . The resistance acting on the particle has magnitude $3v \text{ N}$, where $v \text{ ms}^{-1}$ is the speed of the particle at time $t \text{ s}$.

(a) (i) Show that v satisfies the equation

$$2 \frac{dv}{dt} = -19.6 - v.$$

(ii) Find an expression for v in terms of t . [8]

(b) Determine the time when the particle reaches its maximum height. [2]

(c) Find an expression for x in terms of t , where $x \text{ m}$ is the distance of the particle from A at time $t \text{ s}$. [4]

2. A particle P moves in a straight line with Simple Harmonic Motion about a fixed centre O with period 2 s. At time $t = 0 \text{ s}$, P is at a point A where $OA = 0.5 \text{ m}$ and its velocity is zero.

(a) Write down the amplitude of the motion. [1]

(b) Find the maximum magnitude of the acceleration of P and state the positions of P when this occurs. [4]

(c) Find the smallest positive value of time t for which AP is 0.75 m . [3]

(d) Determine the speed of P when it is 0.3 m from O . [3]

3. (a) A particle P , of mass 2 kg, moves along the horizontal x -axis under the action of a force directed towards the origin O . The magnitude of the force is equal to $8x \text{ N}$, where $x \text{ m}$ is the displacement of P from O . The particle is also subjected to a resistive force which is equal to $10v \text{ N}$, where $v \text{ ms}^{-1}$ is the speed of P at time $t \text{ s}$. When $t = 0 \text{ s}$, the particle P is at $x = 2 \text{ m}$ and it is moving away from O with speed 3 ms^{-1} .

(i) Show that the equation of motion of the particle is

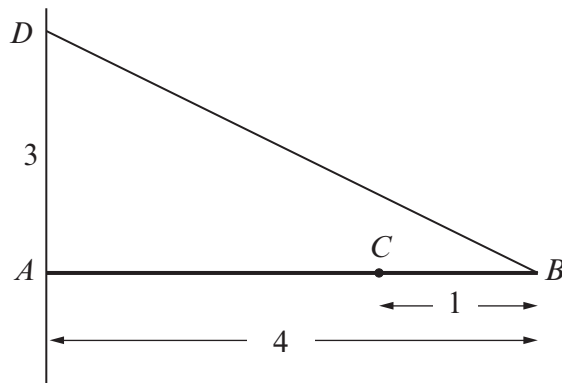
$$\frac{d^2x}{dt^2} = -4x - 5 \frac{dx}{dt}.$$

(ii) Find an expression for x in terms of t . [10]

(b) Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 4x = 12t - 3. \quad [4]$$

4. Two particles A and B , of masses 5 kg and 3 kg respectively, rest on a smooth horizontal surface. Particle A lies at the edge of the surface and particle B lies a distance of 0.2 m from the edge such that the line AB is perpendicular to the edge of the surface. The two particles are connected by a light inextensible string of length 1.8 m . Particle A is then allowed to drop from rest from the edge of the surface. Calculate the speed of B immediately after the string becomes taut and find the impulsive tension in the string. [9]
5. A particle P , of mass 0.25 kg , moves away from the origin along the positive x -axis under the action of a force of magnitude $\frac{5}{2x+1}\text{ N}$ directed away from the origin O . When P is at the origin, its speed is 4 ms^{-1} .
- (a) Find an expression for x in terms of the speed $v\text{ ms}^{-1}$. [8]
- (b) Find the value of x when the speed of P is 6 ms^{-1} . [2]
- (c) Find the speed of P when the acceleration of P is 5 ms^{-2} . [4]
6. The diagram shows a uniform rod AB , of mass 6 kg and length 4 m , held in a horizontal position by means of a light inextensible string BD , where D is a point 3 m vertically above A . The end A of the rod rests against a rough vertical wall. A particle of mass 3 kg is attached to the rod at C , where $BC = 1\text{ m}$. The rod is in limiting equilibrium in a vertical plane perpendicular to the wall.



- (a) Calculate the tension in the string. [4]
- (b) Find the vertical component and the horizontal component of the force exerted by the wall on the rod.
Hence find
- (i) the magnitude of the resultant force exerted by the wall on the rod,
- (ii) the coefficient of friction between the rod and the wall. [9]