GCE AS/A level

0982/01

S16-0982-01

MATHEMATICS – M3
Mechanics

A.M. MONDAY, 27 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle of mass 60 kg moves along the horizontal x-axis under the action of a horizontal constant force of 1800 N. The magnitude of the resistance to motion of the particle is $120v$ N, where v ms^{-1} is the velocity of the particle. At time $t = 0$ seconds, the particle is moving with velocity 8 ms^{-1}.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 30 - 2v.$$ [2]

(b) Find an expression for v at time t. Determine the limiting value of v. [7]

2. *(a)* A particle moves along the x-axis such that its position x m after time t seconds is given by

$$x = A\sin\omega t + B\cos\omega t.$$

Show that the motion of the particle is Simple Harmonic. State the value of x at the centre of motion and find the amplitude of the motion. [7]

(b) Another particle moves with Simple Harmonic Motion with centre *O*.
The particle has velocity 13 ms^{-1} when it is 3 m from *O* and 5 ms^{-1} when it is 5 m from *O*.

(i) Find the period and amplitude of the motion.

(ii) Given that the particle is at *O* at time $t = 0$, find the distance of the particle from *O* when $t = 0{\cdot}3$. [9]

3. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 27t,$$

where $x = \frac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$. Hence find the value of x when $t = 2$. [12]

4. A body of mass 8 kg starts from rest and falls vertically under gravity. At time t seconds, the body has fallen through a distance x metres, and its velocity is v ms^{-1}. During the downward motion, it experiences a resisting force of $0{\cdot}4v^2$ N.

(a) Show that v satisfies the differential equation

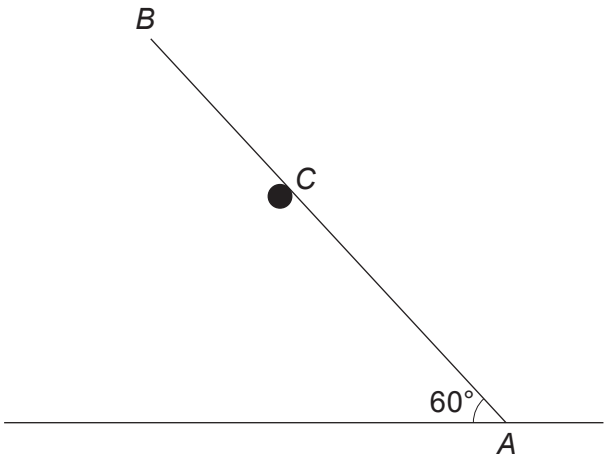
$$196 - v^2 = 20v\frac{\mathrm{d}v}{\mathrm{d}x}.$$ [2]

(b) Find an expression for x in terms of v and hence calculate the value of x when the speed of the body is 10 ms^{-1}. [6]

(c) Find an expression for v at time t and hence find the value of v when $t = 2$. [8]

5. A particle *A*, of mass 2 kg, lies on the edge of a horizontal surface. It is connected by means of a light inextensible string of length 1·8 m to another particle *B*, of mass 5 kg, which is lying on the surface 0·2 m from the edge such that *AB* is perpendicular to the edge. The surface is at a height of 2 m above the ground. Particle *A* is then pushed gently over the edge. Find the magnitude of the velocity with which *B* begins to move and the impulsive tension in the string. [8]

6. The diagram shows a uniform rod *AB*, of length 10 m and mass 25 kg, in limiting equilibrium with its end *A* on rough horizontal ground and point *C* resting against a smooth fixed peg. The rod is inclined at an angle of 60º to the ground.



The distance *AC* is x m and the coefficient of friction between the rod and the ground is 0·3.

(a) Draw a diagram showing all the forces acting on the rod. Label all points and forces clearly. [2]

(b) Determine the magnitude of the reaction at *C* and the magnitude of the normal reaction at *A*. [8]

(c) Find the value of x. [4]

END OF PAPER

