WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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983/01

MATHEMATICS S1

Statistics

P.M. MONDAY, 11 June 2007

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The independent events A, B are such that

$$P(A) = 0{\cdot}6,\ P(B) = 0{\cdot}3.$$

Find

(a) $P(A \cup B)$, [4]

(b) the probability that neither A nor B occurs, [3]

(c) $P(A \mid A \cup B)$. [3]

2. The number of letters, X, arriving per day at a house can be modelled by a Poisson distribution with mean 4·5.

(a) **Without the use of tables**, calculate

(i) $P(X = 5)$,

(ii) $P(X \leqslant 2)$. [5]

(b) **Using tables**, determine $P(3 \leqslant X \leqslant 7)$. [3]

3. The random variable X is such that $E(X) = 5$ and $\text{Var}(X) = 4$. The random variable Y is defined by $Y = aX - b$ where a, b are positive constants. Given that $E(Y) = 0$ and $\text{Var}(Y) = 1$, find the values of a and b. [6]

4. A fair cubical die is thrown twice. Let A denote the event that the score on the first throw is less than the score on the second throw and let B denote the event that the scores on the two throws differ by 1.

(a) Calculate $P(A)$. [3]

(b) Calculate $P(B)$. [2]

(c) Determine whether or not A and B are independent. [5]

5. Alan and Brenda play Scrabble against each other regularly.

(a) The probability that Alan wins a game is 0·6 and the probability that Brenda wins a game is 0·4, independently of all other games. During a weekend, they play 5 games. Let X denote the number of games won by Brenda.

(i) State the distribution of X.

(ii) Determine the mean and standard deviation of X.

(iii) Find the probability that Brenda wins at least 3 of the games. [6]

(b) The probability that one of their games takes more than 2 hours to complete is 0·05. During a school holiday, they play 24 games. Use a Poisson approximation to find the probability that less than 3 of these games take more than 2 hours to complete. [4]

6. The discrete random variable X has a probability distribution given by

$$P(X = x) = kx^2, \quad x = 1, 2, 3, 4,$$
$$P(X = x) = 0, \quad \text{otherwise.}$$

(a) Show that $k = \frac{1}{30}$. [2]

(b) Find the mean and variance of X. [7]

7. The continuous random variable X has probability density function f given by

$$f(x) = \frac{6}{5}x(x-1), \quad \text{for } 1 \leqslant x \leqslant 2,$$
$$f(x) = 0, \quad \text{otherwise.}$$

(a) Evaluate

$$E\left(\frac{1}{X}\right).$$ [4]

(b) (i) Find an expression for $F(x)$, for $1 \leqslant x \leqslant 2$, where F denotes the cumulative distribution function of X.

(ii) Evaluate $P(X \leqslant 1{\cdot}75)$.

(iii) Hence state, with a reason, whether the median of X is greater than or less than 1·75. [8]

8. Each of three boxes contains 5 cards. Box A contains 1 red card and 4 white cards. Box B contains 2 red cards and 3 white cards. Box C contains 3 red cards and 2 white cards. One of the boxes is chosen at random. A card is selected at random from this box and not replaced.

(a) Find the probability that the selected card is red. [3]

(b) Given that the selected card was red, find the probability that Box A was chosen. [3]

(c) Given that the selected card was red, find the probability that a second card selected at random from the chosen box will also be red. [4]