GCE AS/A level

983/01

MATHEMATICS S1

Statistics

P.M. FRIDAY, 6 June 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A bag contains 9 sweets of which 2 are red, 3 are green and 4 are yellow. Jill chooses 2 of these sweets at random.

(a) Calculate the probability that these 2 sweets are of the same colour. [4]

(b) Hence, or otherwise, calculate the probability that these 2 sweets are of different colours. [2]

2. The two independent events A and B are such that

$$P(A) = 0{\cdot}2,\ P(A \cup B) = 0{\cdot}4.$$

(a) Evaluate $P(B)$. [4]

(b) Find the probability that exactly one of the two events occurs. [3]

(c) Given that exactly one of the two events occurs, calculate the probability that A occurs. [3]

3. Alan and Bill each throw a fair cubical die with faces numbered 1, 2, 3, 4, 5, 6 respectively.

(a) Calculate the probability that the score on Alan's die is

(i) equal to the score on Bill's die,

(ii) less than the score on Bill's die. [6]

(b) Given that the sum of the scores on the two dice is 6, find the probability that the two scores are equal. [3]

4. *(a)* The number of accidents occurring per week on a certain stretch of motorway has a Poisson distribution with mean 2·4.
Using tables, find the probability that, in a randomly chosen week, there are between 3 and 6 (both inclusive) accidents on this stretch of motorway. [3]

(b) The number of accidents occurring per week on another stretch of road has a Poisson distribution with mean 3·25.
Without the use of tables, find the probability that the number of accidents occurring on this stretch of road during a randomly chosen week is

(i) exactly 5,

(ii) less than 3. [5]

5. Ann is a tennis player.
When she plays a match against Brenda she has probability 0·3 of winning.
When she plays against Carol she has probability 0·4 of winning.
When she plays against Debbie she has probability 0·6 of winning.
Ann goes to the Tennis Club one morning and finds that Brenda, Carol and Debbie are all there waiting to play against her. Ann tosses 2 fair coins to decide her opponent.
If she obtains 2 heads she will play against Brenda.
If she obtains 2 tails she will play against Carol.
If she obtains 1 head and 1 tail she will play against Debbie.

(a) Find the probability that Ann wins her match. [5]

(b) Given that Ann wins her match, find the probability that she played against Debbie. [3]

6. The probability distribution of the discrete random variable X is given by

$$P(X = x) = k(1 + x) \quad \text{for } x = 1, 2, 3,$$
$$P(X = x) = 0 \quad \text{otherwise.}$$

(a) Show that $k = \frac{1}{9}$. [2]

(b) Evaluate $E(X)$. [3]

(c) Evaluate $E\left(\frac{1}{X}\right)$. [3]

7. A salesman makes 50 house calls during a particular week. You may assume that, independently for each house visited, the probability of a sale is 0·2.

(a) Find the probability that, during this week, he makes

(i) exactly 12 sales,

(ii) between 10 and 14 (both inclusive) sales.

(iii) his first sale on the third house visited. [9]

(b) At the end of the week, he is paid £100 plus a commission of £50 for every sale. Find the mean and standard deviation of his total pay for this week. [5]

8. The continuous random variable X has cumulative distribution function F given by

$$F(x) = 0 \quad \text{for } x < 0,$$
$$F(x) = 4x^3 - 3x^4 \quad \text{for } 0 \leqslant x \leqslant 1,$$
$$F(x) = 1 \quad \text{for } x > 1.$$

(a) Evaluate $P(0{\cdot}25 \leqslant X \leqslant 0{\cdot}75)$. [3]

(b) By evaluating $F(0{\cdot}6)$, determine whether the median of X is greater or less than 0·6. [3]

(c) Obtain an expression for $f(x)$, valid for $0 \leqslant x \leqslant 1$, where f denotes the probability density function of X. [2]

(d) Evaluate $E(X)$. [4]