



GCE AS/A level

983/01

MATHEMATICS S1

Statistics

A.M. THURSDAY, 11 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A school committee contains 9 members of which 2 are teachers, 3 are boys and 4 are girls. A sub-committee of 3 members is to be formed and it is decided to choose the 3 members at random. Calculate the probability that the sub-committee contains

(a) no teachers, [2]

(b) 1 teacher, 1 boy and 1 girl. [3]

2. Events A and B are such that

$$P(A) = 0.2, P(B) = 0.3.$$

(a) Evaluate $P(A \cup B)$ when

(i) A and B are mutually exclusive,

(ii) A and B are independent. [5]

(b) Given that $P(A \cup B) = 0.4$, calculate $P(A|B)$. [4]

(c) What is the smallest possible value for $P(A \cup B)$ and when does this occur? [2]

3. The random variable X has the binomial distribution $B(25, 0.8)$.

(a) State the mean and variance of X . [2]

(b) The random variable Y is defined by

$$Y = aX - b$$

where a, b are positive constants.

(i) Given that $a = 2, b = 3$, find the mean and variance of Y .

(ii) Given that $E(Y) = 0$ and $\text{Var}(Y) = 1$, find the values of a and b . [8]

4. Dafydd is a fisherman. When he fishes in a certain lake, the number of fish that he catches in t hours has a Poisson distribution with mean $0.6t$.

(a) One morning, he fishes for 4 hours. Find the probability that he catches

(i) exactly 3 fish,

(ii) at least 3 fish. [5]

(b) One day, the probability of Dafydd catching no fish was 0.5. For how long did he fish? [4]

5. It is known that 5% of the population suffer from a certain disease. When a test is applied to a person with the disease, it gives a positive response with probability 0.99. When the test is applied to a person who does not have the disease, it gives a positive response with probability 0.02. The test is applied to a randomly selected member of the population.

- (a) Find the probability that a positive response is obtained. [3]
- (b) Given that a positive response is obtained, find the probability that the person has the disease. [3]

6. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3	4	5
$P(X = x)$	0.1	0.2	0.3	0.3	0.1

- (a) Evaluate

- (i) $E(X)$,
- (ii) $\text{Var}(X)$. [6]

- (b) Given that X_1, X_2 are independent observations on X , calculate

$$P(X_1 = X_2). \quad [3]$$

7. (a) Ann tosses 3 fair coins and Bob tosses 2 fair coins. Find the probability that Ann obtains more heads than Bob. [6]

- (b) Ceri and Mair now toss a fair coin alternately, starting with Ceri. The winner is the one who obtains the first head. Find the probability that Mair

- (i) wins on her first toss,
- (ii) wins on her second toss,
- (iii) is the winner. [7]

8. The continuous random variable X has probability density function f given by

$$f(x) = \frac{1}{2} (1 + 2x) \quad \text{for } 0 \leq x \leq 1,$$

$$f(x) = 0 \quad \text{otherwise.}$$

- (a) Calculate $E(X)$. [4]

- (b) Obtain an expression for $F(x)$, valid for $0 \leq x \leq 1$, where F denotes the cumulative distribution function of X . [3]

- (c) Calculate

- (i) $P(0.4 \leq X \leq 0.5)$,
- (ii) the median of X . [5]