GCE AS/A level

984/01

MATHEMATICS S2
Statistics 2

A.M. THURSDAY, 11 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The number of telephone enquiries received per hour at a certain office may be assumed to follow a Poisson distribution with mean μ. Office records indicate that $\mu = 2$ but the office manager believes that the value of μ has increased.

(a) To test this belief, he counts the number of enquiries received during a 6-hour period. Given that 18 enquiries are received, calculate the *p*-value. [3]

(b) He now decides to count the number of enquiries received during a 50-hour period. Given that 125 enquiries are received, calculate the *p*-value and state your conclusion. [5]

2. Roger is a tennis player. When he serves, the speed of the ball may be assumed to be an independent normally distributed random variable with mean 140 km/h and standard deviation 8 km/h.

(a) He serves three times. Calculate the probability that

(i) the speed of the first serve exceeds 150 km/h,

(ii) the speed of all three serves exceeds 150 km/h. [5]

(b) Andy is another tennis player. When he serves, the speed of the ball may be assumed to be an independent normally distributed random variable with mean 145 km/h and standard deviation 6 km/h. Andy and Roger each serve once. Calculate the probability that the speed of Roger's serve is greater than the speed of Andy's serve. [5]

3. A gardener wishes to estimate the acidity level of the soil in his garden. He therefore takes ten readings of the acidity level with the following results.

6·62, 6·84, 6·77, 6·52, 6·86, 6·51, 6·82, 6·71, 6·49, 6·66

You may assume that this is a random sample from a normal distribution with standard deviation 0·1.

(a) Calculate a 99% confidence interval for the acidity level of his soil. [5]

(b) A friend uses the same data to calculate a confidence interval and obtains the following result.

[6·62, 6·74]

Calculate the confidence level of this interval. [4]

4. A teacher wishes to investigate whether or not boys and girls take the same time, on average, to solve jigsaw puzzles. She therefore gives the same jigsaw puzzle to the 6 girls and the 5 boys in her class. She records the time taken by each girl, x minutes, and the time taken by each boy, y minutes, to complete the puzzle. She finds that

$$\sum x = 94 \cdot 8, \sum y = 81 \cdot 0$$

You may assume that the times are random samples from normal distributions with common standard deviation 0·5 minutes.

(a) State suitable hypotheses for the investigation. [1]

(b) Determine the *p*-value of these results and state your conclusion in context. [7]

5. *(a)* The random variable X has the binomial distribution B (20, 0·4). Find the value of

(i) $E(X)$,

(ii) $E(X^2)$. [4]

(b) The random variable Y has a Poisson distribution with mean μ. Given that $E(Y^2) = 9{\cdot}36$, determine the value of μ. [4]

(c) The random variables X and Y are independent and $U = XY$. Determine the variance of U. [4]

6. The continuous random variable X is uniformly distributed on the interval [9, 16].

(a) (i) Write down an expression for $f(x)$, valid for $9 \leqslant x \leqslant 16$, where f denotes the probability density function of X.

(ii) Obtain an expression for $F(x)$, valid for $9 \leqslant x \leqslant 16$, where F denotes the cumulative distribution function of X. [4]

(b) The random variable Y is defined by $Y = \sqrt{X}$. Giving your answers correct to 2 decimal places, determine

(i) $E(Y)$,

(ii) the median of Y. [8]

7. A drug is known to cure 70% of patients suffering from a certain disease. A pharmaceutical company has developed a new drug which is claimed to cure a higher percentage than this. To test this claim, the new drug is given to 50 patients.

(a) State suitable hypotheses. [1]

(b) It is found that 40 of these patients are cured of the disease. Find the p-value of this result and state your conclusion in context. [5]

(c) The company decides to carry out a larger trial in which the new drug is to be given to 250 patients. Let x denote the number of patients cured. Given that the critical region is $x \geqslant 190$, find

(i) the significance level,

(ii) the probability of concluding that the new drug does not increase the percentage of patients cured when in fact the percentage cured has increased to 80%. [10]