GCE AS/A level

984/01

MATHEMATICS S2
Statistics 2

A.M. TUESDAY, 15 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A large batch of tomatoes is delivered to a packing station. The weights of these tomatoes may be assumed to be independent and normally distributed with mean 106 grams and standard deviation 8 grams.

(a) Find the probability that the weight of a randomly selected tomato exceeds 120 grams. [3]

(b) A pack contains 10 randomly selected tomatoes. Find the probability that the total weight of these 10 tomatoes is less than 1 kilogram. [6]

2. The number of computer breakdowns per day at a large office may be assumed to follow a Poisson distribution with mean μ. The IT Manager believes that the value of μ should be 1·5 but he decides to check this. He therefore defines the following hypotheses.

$$H_0: \mu = 1{\cdot}5; \quad H_1: \mu \neq 1{\cdot}5$$

(a) For one test, he decides to count the number of breakdowns, x, in a 10-day period and to define the critical region as $x \leqslant 9$ or $x \geqslant 22$. Find the significance level of this test. [5]

(b) For another test, he decides to count the number of breakdowns occurring during a 100-day period. Given that 170 breakdowns occur, calculate the approximate p-value and state your conclusion. [7]

3. When a weighing machine is used to weigh an object, the reading obtained, in grams, is a normally distributed random variable with mean equal to the actual weight of the object and standard deviation 0·2. Successive weighings are independent.

(a) When an object A was weighed three times, the readings obtained were 11·5, 11·7 and 11·6. Calculate a 95% confidence interval for the weight of object A. [5]

(b) Before an object B was weighed, Graham believed that it would weigh 12 grams but Jim believed that it would weigh more than that.

(i) State suitable hypotheses to test their beliefs.

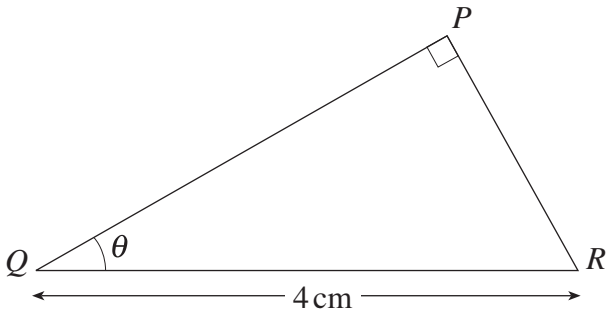
(ii) When the object B was weighed four times, the readings obtained were 12·1, 12·2, 12·4 and 12·1. Calculate the p-value of the four readings and state your conclusion. [7]

(c) Calculate a 90% confidence interval for the difference between the weights of objects A and B. [5]

4. *(a)* The random variable X has the binomial distribution B(n, p). Given that $E(X) = 3$ and $E(X^2) = 11{\cdot}1$, find the values of n and p. [6]

(b) The independent random variable Y has the binomial distribution B(15, 0·4) and $U = XY$. Find the mean and variance of U. [8]

5.



The above diagram shows a right-angled triangle in which the hypotenuse $QR = 4$ cm and $P\hat{Q}R = \theta$ radians, where θ is a continuous random variable uniformly distributed between 0 and $\frac{\pi}{4}$.

(a) Show that the area, A cm^2, of the triangle PQR is given by

$$A = 4 \sin 2\theta.$$ [1]

(b) Calculate $P(A \leqslant 2)$. [5]

(c) Determine $E(A)$. [4]

6. Ann and Brenda buy a packet of seeds which states that, on average, 75% of the seeds will germinate. They believe, however, that the germination rate is less than this so they plant a certain number of seeds and count how many germinate.

(a) State suitable hypotheses. [1]

(b) Ann plants 50 seeds and decides to reject the statement on the packet if less than 30 germinate.

(i) Find the significance level of this procedure.

(ii) Find the probability of accepting the statement on the packet if the actual germination rate is 50%. [6]

(c) Brenda plants 200 seeds and finds that 140 germinate. Find the approximate p-value of this result and state your conclusion in context. [6]