GCE AS/A level

984/01

MATHEMATICS S2
Statistics 2

P.M. THURSDAY, 16 June 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The time taken by Alan to drive to work may be assumed to be normally distributed with mean 28 minutes and standard deviation 2 minutes.

(a) Find the probability that,

(i) on a particular day, he takes more than 30 minutes to drive to work,

(ii) in a particular 5-day week, the mean time taken to drive to work is less than 30 minutes. [8]

(b) The time taken by Brenda to drive to work may be assumed to be normally distributed with mean 25 minutes and standard deviation 3 minutes. Find the probability that, on a particular day, Brenda takes longer to drive to work than Alan. [5]

2. The random variable X has a normal distribution with unknown mean μ and standard deviation 0·5.

(a) A random sample of 60 values of X was taken and it was found that $\sum x = 1290$. Calculate a 95% confidence interval for μ, giving the end-points of your interval correct to two decimal places. [5]

(b) Determine the minimum sample size required for the width of a 95% confidence interval for μ to be less than 0·1. [4]

3. A factory manufactures screws and packs them in large bags. The number of defective screws in a bag can be modelled by a Poisson distribution whose mean is known to have been 0·5. However, new equipment has been installed which, it is hoped, will decrease this mean. The Quality Controller plans to take samples of bags to investigate whether or not there is a reduction in the mean.

(a) State suitable hypotheses. [1]

(b) He takes a random sample of 30 bags and finds that they contain a total of 12 defective screws. Calculate the p-value and state your conclusion. [4]

(c) He then takes a random sample of 200 bags and finds that they contain a total of 80 defective screws. Calculate an approximate p-value and state your conclusion. [6]

4. A zoologist believes that the mean weights of the adult males and females of a certain species of animal are equal. In order to test this belief, she weighs random samples of males and females with the following results.

Weights of males (kg)	14·3	15·8	13·9	13·4	14·5	15·1	13·6	14·2
Weights of females (kg)	13·2	14·8	13·7	14·7	15·0	13·1	13·5	

You may assume that these are random samples from normal populations with a common standard deviation of 0·5.

(a) State suitable hypotheses for carrying out a two-sided test. [1]

(b) Determine the p-value of these results and state whether or not the zoologist's belief is supported at the 5% level of significance. [9]

5. *(a)* The continuous random variable U is uniformly distributed on $[a, b]$.
Write down the probability density function of U and **hence** show that

$$E\left(U^2\right) = \frac{a^2 + ab + b^2}{3} \ .$$ [5]

(b) A piece of string of length 12 cm is cut at a random point. The length of the resulting shorter piece is denoted by X cm and the length of the longer piece by Y cm. You may assume that X is uniformly distributed on [0, 6].

(i) Find the mean and variance of X.

(ii) Express Y in terms of X and hence find the mean of XY.

(iii) Suppose now that 100 pieces of string of length 12 cm are each cut at a random point. Use the Central Limit Theorem to find, approximately, the probability that the total length of the 100 shorter pieces is greater than 280 cm. [11]

6. David is given a biased coin and is told that the probability of obtaining a head when the coin is tossed is either 0·3 or 0·6. To determine which, he defines the following hypotheses.

$$H_0 : p = 0{\cdot}3; \quad H_1 : p = 0{\cdot}6.$$

(a) He tosses the coin 20 times and denotes the number of heads obtained by x.
He will accept H_1 if $x \geqslant 9$ and he will accept H_0 if $x \leqslant 8$.
Calculate the probability of

(i) accepting H_1 when H_0 is true,

(ii) accepting H_0 when H_1 is true. [8]

(b) He now tosses the coin 80 times and denotes the number of heads obtained by y.
He will accept H_1 if $y \geqslant 36$ and he will accept H_0 if $y \leqslant 35$.
Using a normal approximation, calculate the probability of

(i) accepting H_1 when H_0 is true,

(ii) accepting H_0 when H_1 is true. [8]