# GCE AS/A level

985/01

# MATHEMATICS S3
## STATISTICS 3

P.M. TUESDAY, 22 June 2010

1½ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Jamie is given a coin and he wishes to estimate $p$, the probability of its landing 'heads' when tossed. He therefore tosses the coin 250 times and obtains 140 'heads'.

*(a)* Calculate an unbiased estimate of $p$. [1]

*(b)* Calculate an approximate 99% confidence interval for $p$. [5]

*(c)* State, with a reason, whether or not your results suggest that the coin is biased. [1]

**2.** A grower sells melons and claims that their mean weight is 1 kg. A shopkeeper buys a large number of these melons and he believes that the mean weight is less than 1 kg. In order to investigate his belief, he selects a random sample of 100 melons and he determines the weight, $x$ kg, of each one. He produces the following summary statistics.

$$\sum x = 99 \cdot 6, \ \sum x^2 = 99 \cdot 24$$

*(a)* State suitable hypotheses to test the shopkeeper's belief. [1]

*(b)* Calculate the $p$-value of these results and state your conclusion. [7]

*(c)* State what the Central Limit Theorem enabled you to assume in your solution to *(b)*. [1]

**3.** A bag contains six coins, of which one is a 20p coin, three are 10p coins and two are 5p coins. A random sample of three of these coins is taken **without replacement**. Determine the sampling distribution of the total value of the coins in the sample. [9]

**4.** A firm specialises in the manufacture of accurate watches. As part of a quality control procedure, 12 watches were selected and the number of seconds gained over a period of a week was recorded for each watch. The results were as follows.

6, 8, –5, 3, 4, –2, 6, 5, –8, 1, –4, 4

You may assume that this is a random sample from the $N(\mu, \sigma^2)$ distribution.

*(a)* Calculate unbiased estimates of $\mu$ and $\sigma^2$. [4]

*(b)* Calculate a 95% confidence interval for $\mu$. [5]

*(c)* The firm claims that 'on average, this type of watch is accurate to within 5 seconds after a week'. State, with a reason, whether or not your answer to *(b)* supports this claim. [1]

**5.** The director of a large chain of hotels wishes to compare the mean lifetimes of two types of electric light bulbs, Type A and Type B. He therefore determines the lifetime, $x$ thousand hours, of each of 75 randomly selected bulbs of Type A and the lifetime, $y$ thousand hours, of each of 75 randomly selected bulbs of Type B. He obtains the following results.

$$\sum x = 82{\cdot}6, \quad \sum x^2 = 92{\cdot}4, \quad \sum y = 86{\cdot}3, \quad \sum y^2 = 102{\cdot}2$$

*(a)* State suitable hypotheses for a two-sided test. [1]

*(b)* Calculate the $p$-value of these results. [10]

*(c)* Interpret your $p$-value in context. [1]

**6.** The probability distribution of the discrete random variable $X$ is given in the following table, where $0 < \theta < \frac{1}{3}$.

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $P(X = x)$ | $\theta$ | $2\theta$ | $1 - 3\theta$ |

*(a)* Obtain an expression for $E(X)$ and show that

$$\mathrm{Var}(X) = 2\theta(3 - 8\theta).$$ [3]

In order to estimate $\theta$, a random sample of $n$ observations of $X$ is taken.

*(b)* The mean of the observations in the sample is denoted by $\overline{X}$. Show that

$$U = \frac{1 - \overline{X}}{4}$$

is an unbiased estimator for $\theta$ and obtain an expression for the variance of $U$. [4]

*(c)* The number of observations in the sample equal to zero is denoted by $N$.
Show that

$$V = \frac{N}{2n}$$

is an unbiased estimator for $\theta$ and obtain an expression for the variance of $V$. [5]

*(d)* Show that

$$\mathrm{Var}(V) - \mathrm{Var}(U) > 0$$

State, with a reason, which is the better estimator, $U$ or $V$. [3]

# TURN OVER

**7.** The length, $y$ metres, of an elastic string and its tension, $x$ Newtons, are related by an equation of the form $y = \alpha + \beta x$. In order to estimate the values of $\alpha$ and $\beta$, the values of $y$ were measured for six different values of $x$. The following results were obtained.

| $x$ | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| $y$ | 2·02 | 2·23 | 2·39 | 2·56 | 2·77 | 2·95 |

The values of $x$ are exact but the values of $y$ are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·02 metres.

*(a)* Calculate least squares estimates for $\alpha$ and $\beta$. [8]

*(b)* Determine a 90% confidence interval for $\alpha$. [5]