GCE AS/A level

0985/01

MATHEMATICS – S3
Statistics

A.M. FRIDAY, 21 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A university Vice Chancellor wishes to estimate the proportion of students in the university who are fluent in Welsh. She therefore contacts a random sample of 300 students and finds that 87 of them are fluent in Welsh.
Determine an approximate 95% confidence interval for the proportion of the students in the university who are fluent in Welsh. [6]

2. A bag contains 10 balls, of which 3 are red and 7 are blue. A random sample of 4 balls is taken from the bag **without replacement**. Let X denote the number of red balls in the sample and let Y denote the number of blue balls in the sample. Find the sampling distribution of $|X - Y|$. [8]

3. The following nine numbers form a random sample from a normal distribution with mean μ and variance σ^2.

32·1 35·7 33·6 34·7 35·2 33·9 31·7 36·5 35·3

(a) Calculate unbiased estimates of μ and σ^2. [4]

(b) Determine a 90% confidence interval for μ. [5]

(c) Another confidence interval for μ based on this sample is given by

[32·7, 35·9].

Determine whether the confidence level of this interval is greater than or less than 99%. [4]

4. A firm manufacturing light bulbs wishes to determine whether or not a new type of bulb has a mean lifetime exceeding 2000 hours. The Quality Controller decides to record the lifetimes, in hours, of a randomly selected sample of 120 of these bulbs and to define the hypotheses

$$H_0 : \mu = 2000 \text{ versus } H_1 : \mu > 2000,$$

where μ hours denotes the mean lifetime. When he analyses his results, he finds that the unbiased estimate of the variance of the lifetimes is 2554 hours^2.

(a) Determine the range of values of $\bar{x}$, the mean of the lifetimes, which would result in acceptance of H_0 at the 5% significance level but rejection of H_0 at the 10% significance level. You should give the end-points of your range correct to 1 decimal place. [5]

(b) Explain whether or not your analysis requires the lifetimes to be normally distributed. [2]

5. A poultry farmer has two different breeds of chickens on his farm, breed A and breed B, and he wishes to know whether or not the mean weights of eggs produced by the two breeds are equal. He therefore selects random samples of 60 eggs from each breed and weighs them. Let x denote the weights (in grams) of eggs produced by chickens of breed A and let y denote the weights (in grams) of eggs produced by chickens of breed B. His results are summarised below.

$$\sum x = 3315, \quad \sum x^2 = 183345, \quad \sum y = 3345, \quad \sum y^2 = 186651$$

(a) State suitable hypotheses to investigate whether or not the mean weights of eggs produced by the two breeds are equal. [1]

(b) Calculate the p-value of the above results and interpret your value in context. [10]

6. The resistance, y ohms, of an electrical component is related to the temperature, x°C, by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants α and β , the following measurements were made.

x	10	15	20	25	30	35	40
y	12·3	13·9	15·1	16·6	18·6	20·1	21·5

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·1. Determine a 95% confidence interval for α. [5]

7. The random variable X has the binomial distribution B(n, p).

(a) Show that

$$\hat{p} = \frac{X}{n}$$

is an unbiased estimator for p, and find its standard error. [4]

(b) (i) Show that $\hat{p}^2$ is not an unbiased estimator for p^2.

(ii) By considering $E[X(X-1)]$, find an unbiased estimator for p^2. [8]

(c) Given that $q = 1 - p$,

(i) deduce an unbiased estimator for q^2,

(ii) find an unbiased estimator for pq, simplifying your answer as far as possible. [5]

